Lecture 8: Policy Gradient I

Emma Brunskill

CS234 Reinforcement Learning.

Winter 2022

- Additional reading: Sutton and Barto 2018 Chp. 13

\(^1\)With many slides from or derived from David Silver and John Schulman and Pieter Abbeel
Consider doing experience replay over a finite, but extremely large, set of \((s,a,r,s')\) tuples. Q-learning is initialized to 0 everywhere and all rewards are positive. Select all that are true

1. Assume all tuples were gathered from a fixed, deterministic policy \(\pi\). Then in the tabular setting, if each tuple is sampled at random and used to do a Q-learning update, and this is repeated an infinite number of times, then there exists a learning rate schedule so that the resulting estimate will converge to the true \(Q^\pi\).

2. In situation (1) (the first option above) the resulting Q estimate will be identical to if one computed an estimated dynamics model and reward model using maximum likelihood evaluation from the tuples, and performed policy evaluation using the estimated dynamics and reward models.

3. If one uses DQN to populate the experience replay set of tuples, then doing experience replay with DQN is always guaranteed to converge to the optimal Q function.

4. Not sure
Consider doing experience replay over a finite, but extremely large, set of \((s,a,r,s')\) tuples. Q-learning is initialized to 0 everywhere and all rewards are positive. Select all that are true:

1. Assume all tuples were gathered from a fixed, deterministic policy \(\pi\). Then in the tabular setting, if each tuple is sampled at random and used to do a Q-learning update, and this is repeated an infinite number of times, then there exists a learning rate schedule so that the resulting estimate will converge to the true \(Q^\pi\).

2. In situation (1) (the first option above) the resulting Q estimate will be identical to if one computed an estimated dynamics model and reward model using maximum likelihood evaluation from the tuples, and performed policy evaluation using the estimated dynamics and reward models.

3. If one uses DQN to populate the experience replay set of tuples, then doing experience replay with DQN is always guaranteed to converge to the optimal Q function.

4. Not sure

Answer: 1 and 2 are true. 3 is false.
Last Time: We want RL Algorithms that Perform

- Optimization
- Delayed consequences
- Exploration
- Generalization
- And do it statistically and computationally efficiently
Can use structure and additional knowledge to help constrain and speed reinforcement learning
Class Structure

- Last time: Deep RL
- **This time:** Policy Search
- Next time: Policy Search Cont.

DNN + model free V
Today

- Introduction to policy search methods
- Gradient-free methods
- Finite difference methods
- Score functions and policy gradient
- REINFORCE
In the last lecture we approximated the value or action-value function using parameters $w$,

$$V_w(s) \approx V^\pi(s)$$

$$Q_w(s, a) \approx Q^\pi(s, a)$$

A policy was generated directly from the value function

- e.g. using $\epsilon$-greedy

$$\pi(s) = \text{argmax}_a Q^\pi(s, a)$$

In this lecture we will directly parametrize the policy, and will typically use $\theta$ to show parameterization:

$$\pi_\theta(s, a) = \mathbb{P}[a | s; \theta]$$

Goal is to find a policy $\pi$ with the highest value function $V^\pi$

We will focus again on model-free reinforcement learning
Value-Based and Policy-Based RL

- **Value Based**
  - learned Value Function
  - Implicit policy (e.g. $\epsilon$-greedy)

- **Policy Based**
  - No Value Function
  - Learned Policy

- **Actor-Critic**
  - Learned Value Function
  - Learned Policy
Types of Policies to Search Over

- So far have focused on deterministic policies (why?)
- Now we are thinking about direct policy search in RL, will focus heavily on stochastic policies
Example: Rock-Paper-Scissors

- Two-player game of rock-paper-scissors
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Let state be history of prior actions (rock, paper and scissors) and if won or lost
- Is deterministic policy optimal? Why or why not?
Example: Rock-Paper-Scissors, Vote

- Two-player game of rock-paper-scissors
  - Scissors beats paper
  - Rock beats scissors
  - Paper beats rock
- Let state be history of prior actions (rock, paper and scissors) and if won or lost
  Deterministic policy is easily exploited by an adversary. System is not Markov. A uniform random policy is optimal (Nash equilibrium).
Example: Aliased Gridword (1)

- The agent cannot differentiate the grey states
- Consider features of the following form (for all N, E, S, W)

\[
\phi(s, a) = 1 \text{(wall to N, } a = \text{move E)}
\]

- Compare value-based RL, using an approximate value function

\[
Q_\theta(s, a) = f(\phi(s, a); \theta)
\]

- To policy-based RL, using a parametrized policy

\[
\pi_\theta(s, a) = g(\phi(s, a); \theta)
\]
Under aliasing, an optimal \textit{deterministic} policy will either
- move \textit{W} in both grey states (shown by red arrows)
- move \textit{E} in both grey states

Either way, it can get stuck and never reach the money

Value-based RL learns a near-deterministic policy
- e.g. greedy or \(\epsilon\)-greedy

So it will traverse the corridor for a long time
An optimal **stochastic** policy will randomly move E or W in grey states.

\[ \pi_{\theta}(\text{wall to N and S, move E}) = 0.5 \]

\[ \pi_{\theta}(\text{wall to N and S, move W}) = 0.5 \]

- It will reach the goal state in a few steps with high probability.
- Policy-based RL can learn the optimal stochastic policy.
Goal: given a policy $\pi_\theta(s, a)$ with parameters $\theta$, find best $\theta$

But how do we measure the quality for a policy $\pi_\theta$?

In episodic environments can use policy value at start state $V(s_0, \theta)$

For simplicity, today will mostly discuss the episodic case, but can easily extend to the continuing / infinite horizon case.
- Policy based reinforcement learning is an optimization problem
- Find policy parameters $\theta$ that maximize $V(s_0, \theta)$
Today

- Introduction to policy search methods
- **Gradient-free methods**
- Finite difference methods
- Score functions and policy gradient
- REINFORCE
Policy based reinforcement learning is an optimization problem.

Find policy parameters $\theta$ that maximize $V(s_0, \theta)$.

Can use gradient free optimization:
- Hill climbing
- Simplex / amoeba / Nelder Mead
- Genetic algorithms
- Cross-Entropy method (CEM)
- Covariance Matrix Adaptation (CMA)
Human-in-the-Loop Exoskeleton Optimization (Zhang et al. Science 2017)

Optimization was done using CMA-ES, variation of covariance matrix evaluation.

Figure: Zhang et al. Science 2017
Can often work embarrassingly well: "discovered that evolution strategies (ES), an optimization technique that’s been known for decades, rivals the performance of standard reinforcement learning (RL) techniques on modern RL benchmarks (e.g. Atari/MuJoCo)" (https://blog.openai.com/evolution-strategies/)
Gradient Free Policy Optimization

- Often a great simple baseline to try

**Benefits**
- Can work with any policy parameterizations, including non-differentiable
- Frequently very easy to parallelize

**Limitations**
- Typically not very sample efficient because it ignores temporal structure
Today

- Introduction to policy search methods
- Gradient-free methods
  - Finite difference methods
- Score functions and policy gradient
- REINFORCE
Policy optimization

- Policy based reinforcement learning is an optimization problem
- Find policy parameters $\theta$ that maximize $V(s_0, \theta)$
- Can use gradient free optimization:
  - Greater efficiency often possible using gradient
    - Gradient descent
    - Conjugate gradient
    - Quasi-newton
- We focus on gradient descent, many extensions possible
- And on methods that exploit sequential structure
Policy Gradient

\[ \theta \rightarrow \pi \rightarrow V^\pi \theta \]

- Define \( V(\theta) = V(s_0, \theta) \) to make explicit the dependence of the value on the policy parameters [but don’t confuse with value function approximation, where parameterized value function]

- Assume episodic MDPs (easy to extend to related objectives, like average reward)
Policy Gradient

- Define $V^\pi_\theta = V(s_0, \theta)$ to make explicit the dependence of the value on the policy parameters.
- Assume episodic MDPs.
- Policy gradient algorithms search for a local maximum in $V(s_0, \theta)$ by ascending the gradient of the policy, w.r.t parameters $\theta$.

$$\Delta \theta = \alpha \nabla_\theta V(s_0, \theta)$$

- Where $\nabla_\theta V(s_0, \theta)$ is the policy gradient.

$$\nabla_\theta V(s_0, \theta) = \left( \begin{array}{c} \frac{\partial V(s_0, \theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial V(s_0, \theta)}{\partial \theta_n} \end{array} \right)$$

- and $\alpha$ is a step-size parameter.
Simple Approach: Compute Gradients by Finite Differences

- To evaluate policy gradient of $\pi_\theta(s, a)$
- For each dimension $k \in [1, n]$ (for each policy param dim)
  - Estimate $k$th partial derivative of objective function w.r.t. $\theta$
  - By perturbing $\theta$ by small amount $\epsilon$ in $k$th dimension

$$\frac{\partial V(s_0, \theta)}{\partial \theta_k} \approx \frac{V(s_0, \theta + \epsilon u_k) - V(s_0, \theta)}{\epsilon}$$

where $u_k$ is a unit vector with 1 in $k$th component, 0 elsewhere.
Computing Gradients by Finite Differences

- To evaluate policy gradient of $\pi_\theta(s, a)$
- For each dimension $k \in [1, n]$
  - Estimate $k$th partial derivative of objective function w.r.t. $\theta$
  - By perturbing $\theta$ by small amount $\epsilon$ in $k$th dimension

$$\frac{\partial V(s_0, \theta)}{\partial \theta_k} \approx \frac{V(s_0, \theta + \epsilon u_k) - V(s_0, \theta)}{\epsilon}$$

where $u_k$ is a unit vector with 1 in $k$th component, 0 elsewhere.

- Uses $n$ evaluations to compute policy gradient in $n$ dimensions
- Simple, noisy, inefficient - but sometimes effective
- Works for arbitrary policies, even if policy is not differentiable
Goal: learn a fast AIBO walk (useful for Robocup)
Adapt these parameters by finite difference policy gradient
Evaluate performance of policy by field traversal time

AIBO Policy Parameterization

- AIBO walk policy is open-loop policy
- No state, choosing set of action parameters that define an ellipse
- Specified by 12 continuous parameters (elliptical loci)
  - The front locus (3 parameters: height, x-pos., y-pos.)
  - The rear locus (3 parameters)
  - Locus length
  - Locus skew multiplier in the x-y plane (for turning)
  - The height of the front of the body
  - The height of the rear of the body
  - The time each foot takes to move through its locus
  - The fraction of time each foot spends on the ground
- New policies: for each parameter, randomly add ($\epsilon$, 0, or $-\epsilon$)
"All of the policy evaluations took place on actual robots... only human intervention required during an experiment involved replacing discharged batteries ... about once an hour.”

- Ran on 3 Aibos at once
- Evaluated 15 policies per iteration.
- Each policy evaluated 3 times (to reduce noise) and averaged
- Each iteration took 7.5 minutes
Authors discuss that performance is likely impacted by: initial starting policy parameters, \( \epsilon \) (how much policies are perturbed), learning rate for how much to change policy, as well as policy parameterization.
Check Your Understanding

- Finite difference policy gradient (select all)
  1. Is guaranteed to converge to a local optima  **(T)**
  2. Is guaranteed to converge to a global optima  **(F)**
  3. Relies on the Markov assumption  **(F)**
  4. Uses a number of evaluations to estimate the gradient that scales linearly with the state dimensionality  **(F)**
  5. Not sure  **(F)**
Check Your Understanding

Finite difference policy gradient (select all)

- Is guaranteed to converge to a local optima
- Is guaranteed to converge to a global optima
- Relies on the Markov assumption
- Uses a number of evaluations to estimate the gradient that scales linearly with the state dimensionality
- Not sure

Answer: Is guaranteed to converge to a local optima (not global), does not rely on the Markov assumption, uses a number of evaluations that scales linearly with the policy feature dimension (not state)
Summary of Benefits of Policy-Based RL

Advantages:
- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies

Disadvantages:
- Typically converge to a local rather than global optimum
- Evaluating a policy is typically inefficient and high variance

Shortly will see some ideas to help with this last limitation
Today

- Introduction to policy search methods
- Gradient-free methods
- Finite difference methods
- Score functions and policy gradient
- REINFORCE
Computing the gradient analytically

- We now compute the policy gradient \textit{analytically}
- Assume policy \( \pi_{\theta} \) is differentiable whenever it is non-zero
- Assume we can calculate gradient \( \nabla_{\theta} \pi_{\theta}(s, a) \) analytically
- What kinds of policy classes can we do this for?
Many choices of differentiable policy classes including:
- Softmax
- Gaussian
- Neural networks
A score function is the derivative of the log of a parameterized probability / likelihood.

Example: let $p(s; \theta)$ be the probability of state $s$ under parameter $\theta$.

Then the score function would be

$$\nabla_\theta \log p(s; \theta)$$ (1)
Softmax Policy

- Weight actions using linear combination of features $\phi(s, a)^T \theta$
- Probability of action is proportional to exponentiated weight

$$\pi_\theta(s, a) = \frac{e^{\phi(s,a)^T \theta}}{\sum_a e^{\phi(s,a)^T \theta}}$$

The score function is

$$\nabla_\theta \log \pi_\theta(s, a) =$$

$$\nabla_\theta \log \left[ \frac{e^{\phi(s,a)^T \theta}}{\sum_a e^{\phi(s,a)^T \theta}} \right]$$

$$= \nabla_\theta \log e^{\phi(s,a)^T \theta} - \nabla_\theta \log \sum_a e^{\phi(s,a)^T \theta}$$

$$= \phi(s,a)^T \theta - \frac{1}{\sum_a e^{\phi(s,a)^T \theta}} \sum_a e^{\phi(s,a)^T \theta} \phi(s,a)^T$$

$$= \phi(s,a) - \sum_a \pi_\theta(s,a) \phi(s,a)$$

$$\sum_a \pi_\theta(s,a) = 1$$
Softmax Policy

- Weight actions using linear combination of features $\phi(s, a)^T \theta$
- Probability of action is proportional to exponentiated weight
  \[
  \pi_\theta(s, a) = e^{\phi(s, a)^T \theta} / \left( \sum_a e^{\phi(s, a)^T \theta} \right)
  \]
- The score function is
  \[
  \nabla_\theta \log \pi_\theta(s, a) = \phi(s, a) - \mathbb{E}_{\pi_\theta}[\phi(s, \cdot)]
  \]
In continuous action spaces, a Gaussian policy is natural.

Mean is a linear combination of state features \( \mu(s) = \phi(s)^T \theta \)

Variance may be fixed \( \sigma^2 \), or can also be parametrised.

Policy is Gaussian \( a \sim \mathcal{N}(\mu(s), \sigma^2) \)

The score function is

\[
\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s))\phi(s)}{\sigma^2}
\]
Value of a Parameterized Policy

- Now assume policy $\pi_\theta$ is differentiable whenever it is non-zero and we know the gradient $\nabla_\theta \pi_\theta(s, a)$

- Recall policy value is $V(s_0, \theta) = \mathbb{E}_{\pi_\theta} \left[ \sum_{t=0}^{T} R(s_t, a_t); \pi_\theta, s_0 \right]$ where the expectation is taken over the states & actions visited by $\pi_\theta$

- We can re-express this in multiple ways
  - $V(s_0, \theta) = \sum_a \pi_\theta(a|s_0) Q(s_0, a, \theta)$
Value of a Parameterized Policy

- Now assume policy $\pi_\theta$ is differentiable whenever it is non-zero and we know the gradient $\nabla_\theta \pi_\theta(s, a)$

- Recall policy value is $V(s_0, \theta) = \mathbb{E}_{\pi_\theta} \left[ \sum_{t=0}^{T} R(s_t, a_t); \pi_\theta, s_0 \right]$ where the expectation is taken over the states & actions visited by $\pi_\theta$

- We can re-express this in multiple ways
  - $V(s_0, \theta) = \sum_a \pi_\theta(a|s_0) Q(s_0, a, \theta)$
  - $V(s_0, \theta) = \sum_\tau P(\tau; \theta) R(\tau)$
    - where $\tau = (s_0, a_0, r_0, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$ is a state-action trajectory,
    - $P(\tau; \theta)$ is used to denote the probability over trajectories when executing policy $\pi(\theta)$ starting in state $s_0$, and
    - $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ the sum of rewards for a trajectory $\tau$

- To start will focus on this latter definition. See Chp 13.1-13.3 of SB for a nice discussion starting with the other definition
Likelihood Ratio Policies

- Denote a state-action trajectory as 
  \[ \tau = (s_0, a_0, r_0, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T) \]
- Use \( R(\tau) = \sum_{t=0}^{T} R(s_t, a_t) \) to be the sum of rewards for a trajectory \( \tau \)
- Policy value is
  \[
  V(\theta) = \mathbb{E}_{\pi_{\theta}} \left[ \sum_{t=0}^{T} R(s_t, a_t); \pi_{\theta} \right] = \sum_{\tau} P(\tau; \theta) R(\tau)
  \]
- where \( P(\tau; \theta) \) is used to denote the probability over trajectories when executing policy \( \pi(\theta) \)
- In this new notation, our goal is to find the policy parameters \( \theta \):
  \[
  \arg \max_{\theta} V(\theta) = \arg \max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau)
  \]
**Likelihood Ratio Policy Gradient**

- Goal is to find the policy parameters $\theta$:

$$\arg\max_\theta V(\theta) = \arg\max_\theta \sum_\tau P(\tau; \theta)R(\tau)$$

- Take the gradient with respect to $\theta$:

$$\nabla_\theta V(\theta) = \nabla_\theta \sum_\tau P(\tau; \theta)R(\tau)$$

$$= \sum_\tau \nabla_\theta P(\tau; \theta)R(\tau)$$

$$= \sum_\tau R(\tau) \frac{P(\tau; \theta)}{P(\tau; \theta)} \nabla_\theta \frac{P(\tau; \theta)}{P(\tau; \theta)}$$

$$= \sum_\tau R(\tau)P(\tau; \theta) \nabla_\theta \log P(\tau; \theta)$$
Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters $\theta$:
  \[ \arg \max_{\theta} V(\theta) = \arg \max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau) \]

- Take the gradient with respect to $\theta$:
  \[ \nabla_{\theta} V(\theta) = \nabla_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau) \]
  \[ = \sum_{\tau} \nabla_{\theta} P(\tau; \theta) R(\tau) \]
  \[ = \sum_{\tau} \frac{P(\tau; \theta)}{P(\tau; \theta)} \nabla_{\theta} P(\tau; \theta) R(\tau) \]
  \[ = \sum_{\tau} P(\tau; \theta) R(\tau) \nabla_{\theta} \log P(\tau; \theta) \]

$\nabla_{\theta} \log P(\tau; \theta)$ is known as the likelihood ratio.
Likelihood Ratio Policy Gradient

- Goal is to find the policy parameters $\theta$:

$$
\arg \max_{\theta} V(\theta) = \arg \max_{\theta} \sum_{\tau} P(\tau; \theta)R(\tau)
$$

- Take the gradient with respect to $\theta$:

$$
\nabla_{\theta} V(\theta) = \sum_{\tau} P(\tau; \theta)R(\tau) \nabla_{\theta} \log P(\tau; \theta)
$$

- Approximate with empirical estimate for $m$ sample trajectories under policy $\pi_\theta$:

$$
\nabla_{\theta} V(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} R(\tau^{(i)}; \theta) \nabla_{\theta} \log P(\tau^{(i)}; \theta)
$$
Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for $m$ sample paths under policy $\pi_\theta$:

$$\nabla_\theta V(\theta) \approx \hat{g} = \left(\frac{1}{m}\right) \sum_{i=1}^m R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)})$$

$$\nabla_\theta \log P(\tau^{(i)}; \theta) = \frac{\nabla_\theta \log \sum_{s_0} \prod_{t=0}^{T-1} \pi_\theta(a_t | s_t) P(s_{t+1} | h_0:t-1)}{m}$$

$$= \nabla_\theta \log \mu(s_0) + \nabla_\theta \sum_{t=0}^{T-1} \log \pi_\theta(a_t | s_t) + \log P(s_{T+1} | h_0:T-1)$$

$$= 0$$

$$= \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t | s_t)$$
Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for \( m \) sample paths under policy \( \pi_\theta \):

\[
\nabla_\theta V(\theta) \approx \hat{g} = \left( \frac{1}{m} \right) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)})
\]

\[
\nabla_\theta \log P(\tau^{(i)}; \theta) = \nabla_\theta \log \left[ \begin{array}{c}
\mu(s_0) \\
\prod_{t=0}^{T-1} \pi_\theta(a_t|s_t) P(s_{t+1}|s_t, a_t)
\end{array} \right]
\]

\[
= \nabla_\theta \left[ \log \mu(s_0) + \sum_{t=0}^{T-1} \log \pi_\theta(a_t|s_t) + \log P(s_{t+1}|s_t, a_t) \right]
\]

\[
= \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)
\]

no dynamics model required!
Consider score function as $\nabla_{\theta} \log \pi_{\theta}(s, a)$
Putting this together

Goal is to find the policy parameters $\theta$:

$$\arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau)$$

Approximate with empirical estimate for $m$ sample paths under policy $\pi_\theta$ using score function:

$$\nabla_\theta V(\theta) \approx \hat{g} = (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \nabla_\theta \log P(\tau^{(i)}; \theta)$$

$$= (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)}|s_t^{(i)})$$

Do not need to know dynamics model
Consider generic form of $R(\tau^{(i)})(\nabla_{\theta} \log P(\tau^{(i)}; \theta))$:

$$\hat{g}_i = f(x_i)(\nabla_{\theta} \log p(x_i|\theta))$$

- $f(x)$ measures how good the sample $x$ is.
- Moving in the direction $\hat{g}_i$ pushes up the logprob of the sample, in proportion to how good it is.
- **Valid even if $f(x)$ is discontinuous, and unknown, or sample space (containing $x$) is a discrete set**
\[ \hat{g}_i = f(x_i) \nabla_\theta \log p(x_i|\theta) \]
\hat{g}_i = f(x_i) \nabla_\theta \log p(x_i | \theta)
The policy gradient theorem generalizes the likelihood ratio approach.

**Theorem**

For any differentiable policy $\pi_\theta(s, a)$, for any of the policy objective function $J = J_1$, (episodic reward), $J_{avR}$ (average reward per time step), or $\frac{1}{1-\gamma}J_{avV}$ (average value), the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

Chapter 13.2 in SB has a nice derivation of the policy gradient theorem for episodic tasks and discrete states.
Today

- Introduction to policy search methods
- Gradient-free methods
- Finite difference methods
- Score functions and policy gradient
- REINFORCE
\[ \nabla_\theta V(\theta) \approx \frac{1}{m} \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)}|s_t^{(i)}) \]

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline
- Next time will discuss some additional tricks
Policy Gradient: Use Temporal Structure

- Previously:

\[ \nabla_{\theta} \mathbb{E}_{\tau}[R] = \mathbb{E}_{\tau} \left[ \left( \sum_{t=0}^{T-1} r_t \right) \left( \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right) \right] \]

- We can repeat the same argument to derive the gradient estimator for a single reward term \( r_{t'} \).

\[ \nabla_{\theta} \mathbb{E}[r_{t'}] = \mathbb{E} \left[ r_{t'} \sum_{t=0}^{t'} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right] \]

- Summing this formula over \( t \), we obtain

\[ V(\theta) = \nabla_{\theta} \mathbb{E}[R] = \mathbb{E} \left[ \sum_{t' = 0}^{T-1} r_{t'} \sum_{t=0}^{t'} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right] \]

\[ = \mathbb{E} \left[ \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(a_t, s_t) \sum_{t' = t}^{T-1} r_{t'} \right] \]
Recall for a particular trajectory $\tau^{(i)}$, $\sum_{t'=t}^{T-1} r^{(i)}_{t'}$ is the return $G^{(i)}_t$

$$\nabla_{\theta} \mathbb{E}[R] \approx \frac{1}{m} \sum_{i=1}^{m} \sum_{t=0}^{T-1} \nabla_{\theta} \log \pi_{\theta}(a_t, s_t) G^{(i)}_t$$
Monte-Carlo Policy Gradient (REINFORCE)

- Leverages likelihood ratio / score function and temporal structure

\[ \Delta \theta_t = \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t \]

REINFORCE:
Initialize policy parameters \( \theta \) arbitrarily

\[ \text{for each episode } \{s_1, a_1, r_2, \cdots, s_{T-1}, a_{T-1}, r_T \} \sim \pi_\theta \text{ do} \]

\[ \text{for } t = 1 \text{ to } T - 1 \text{ do} \]

\[ \theta \leftarrow \theta + \alpha \nabla_\theta \log \pi_\theta(s_t, a_t) G_t \]

\[ \text{endfor} \]

\[ \text{endfor} \]

return \( \theta \)
\[ \nabla_\theta V(\theta) \approx (1/m) \sum_{i=1}^{m} R(\tau^{(i)}) \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^{(i)} | s_t^{(i)}) \]

- Unbiased but very noisy
- Fixes that can make it practical
  - Temporal structure
  - Baseline
- Next time will discuss some additional tricks
Class Structure

- Last time: Imitation Learning in Large State Spaces
- This time: Policy Search
- Next time: Policy Search Cont.