Learning Outcome for next four Lectures

Modeling and Evaluating Grasps

Apply Learning to Grasping and Manipulation

Modeling and Executing Manipulation

Use Manipulation to Perceive better
Today’s itinerary

• Analytical Approach to Modeling a Grasp
  • Terminology
  • Modeling of a Grasp
  • Stability Analysis of a Grasp
    • Form Closure
    • Force Closure

• Generating a Grasp
• Grasp Force Optimization
• Modeling Push Manipulation
What is a grasp?

• Restraining an object’s motion through application of forces and torques at a set of contact points
How to evaluate a grasp?

What are good grasp characteristics?

**Grasp Maintenance:**
contact forces applied by the hand are such that they prevent contact separation and unwanted contact sliding

**Closure:**
Grasps that can be maintained for every possible disturbance load
Let’s model a grasp!
What’s a wrench?

• Each point force also applies torque
  \[ \tau = d \times f \]
• Wrench is a force-torque pair

\[
\mathbf{w}_{ij} = \left( \frac{\mathbf{f}_{ij}}{\lambda (\mathbf{d}_i \times \mathbf{f}_{ij})} \right)
\]

• The i-th point contact has m wrenches, one for each force in the pyramidal approximation of the friction cone
• d is the vector from the point contact to the torque origin
• \( \lambda \) is a constant relating force to torque
What’s a grasp?

• = set of wrenches that can be achieved

\[ F_o = G_1 f_{c_1} + \cdots + G_k f_{c_k} = [G_1 \quad \cdots \quad G_k] \begin{bmatrix} f_{c_1} \\ \vdots \\ f_{c_k} \end{bmatrix} \]

\( f_c \in FC. \)

• \( G_i \) = wrench basis vectors transformed into single reference coordinate frame

• \( G = [G_1, \ldots, G_k] \) = grasp map
  • Transforming all applied forces and torques to achievable wrenches
Grasp Wrench Space

• Convex hull of all the wrenches from the contact points -- total possible range of wrenches that can be applied

• For 3D objects, wrench space is 6D
  • 3D for force, 3D for torque
  • For 2D objects, it’s 3D
How to evaluate a grasp?

• Quantify:
  • How many external wrenches can a grasp withstand?

• Force Closure:
  • withstand all external wrenches
Condition for Force-Direction Closure

Algebraic condition?
For force vectors \( p, q, r \),
there must exist \( \alpha, \beta, \gamma > 0 \)
s.t. \( \alpha p + \beta q + \gamma r = 0 \)
Grasp Analysis – Force Closure

- A grasp is a force-closure grasp IF for any external wrench $F_e$ there exist contact forces $f_c \in F_C$ such that

$$G f_c = -F_e$$

i.e., if able to apply sufficient force at each contact, every external wrench can be compensated for.

Practical test? Algebraic condition
How do you make the grasp stable?

• Ignore for now $f_x$

Does not contain origin, not stable

$w_{ij} = \left( \lambda (d_i \times f_{ij}) \right)$
In which quadrant of the wrench space do you add wrenches after adding the third contact point?

\[ w_{i,j} = \frac{f_{i,j}}{\lambda (d_i \times f_{i,j})} \]
What if there was a third point?

\[ w_{i,j} = \left( \lambda (d_i \times f_{i,j}) \right) \]
What if there was a third point?

Wrench hull

\( \mathbf{w}_{i,j} = \left( \frac{\mathbf{f}_{i,j}}{\lambda (\mathbf{d}_i \times \mathbf{f}_{i,j})} \right) \)
Both grasps are stable. Which one is better?
Grasp Quality

• Quality is how well a grip can resist disturbances
• Worst case scenario
  • How efficiently can a grip resist disturbance wrenches at its weakest point?
• Weakest means the direction (in wrench space) at which the sum normal force is converted to the desired wrench least efficiently
Worst Case Scenario

- The point on the wrench hull that is closest to the origin is the weakest point.
- Disturbances in the opposite direction are hardest to resist.
- Metric $\varepsilon$ = The radius of the largest ball that can be enclosed in the wrench hull.
  - Varies from 0 to 1 due to normalization of wrenches.
Physical Meaning

• In the worst case, the sum magnitude of the contact wrenches would need to be $\frac{1}{\varepsilon}$ times the disturbance wrench
Are these grasps equally good?
Average Case Scenario

• How efficiently can a grip resist a disturbance wrench on average?
• Metric $\nu = \text{Volume of the convex hull in wrench space}$
• The three-point contact has more volume, so it is more stable on average
Form Closure versus Force Closure

• Both are in a contact configuration that resists all external disturbances.

• Note: Every form closure grasp is also in force closure

• Why do I need less contact points to be in Force closure?
How do we generate a grasp?
Suggested Reading

• Constructing Force Closure Grasps. Van-Duc Nguyen. IJRR 1988
• Planning Optimal Grasps. Carlo Ferrari & John Canny. ICRA 1992
Grasp Force Optimization

- In force closure, you can **theoretically** resist any wrench.
- But what forces do you need to apply at each contact to generate the desired wrench?
Motivating Example

Figure adapted from *A Grasping Force Optimization Algorithm for Multiarm Robots With Multifingered Hands*. Lippiello et al. Transactions on Robotics. 2013

Fig. 3. Sequence of significant configurations of the bottle and of the forces during task execution with $n = 10$. 
Formalizing the problem

- **M** contact points at \( c^{(i)} \)
- \( f^{(i)} \) is the contact force applied at contact point
- Local coordinate system where \( x, y \) are tangent to surface and \( z \) is aligned with surface normal pointing inward
- \( f^i = (f_x^{(i)}, f_y^{(i)}, f_z^{(i)}) \)
- Friction cone

\[
\sqrt{f_x^{(i)}^2 + f_y^{(i)}^2} \leq \mu_i f_z^{(i)}
\]

- or in planar case: \( f_x^{(i)} \leq \mu_i f_y^{(i)} \)

Friction Cone Constraints

\[
\sqrt{f_x(i)^2 + f_y(i)^2} \leq \mu_i f_z(i)
\]

- or in planar case: \( f_x(i) \leq \mu_i f_y(i) \)

- Second-order cone constraints

\[
K_i = \left\{ x \in \mathbb{R}^3 \mid \sqrt{x_1^2 + x_2^2} \leq \mu_i x_3 \right\}, \quad i = 1, \ldots, M
\]

- Compact notation \( f(i) \in K_i, \quad i = 1, \ldots, M. \)

Equilibrium Constraints – Force

- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $Q^{(i)} f^{(i)}$ = force applied to object
- Applied forces need to generate a force that compensates external force

\[ f^{\text{ext}} \in \mathbb{R}^3 \]

\[ Q^{(1)} f^{(1)} + \cdots + Q^{(M)} f^{(M)} + f^{\text{ext}} = 0 \]

Equilibrium Constraints – Torque

- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $c^{(i)} \times Q^{(i)} f^{(i)} = \text{torque applied to object}$
- Applied forces need to generate a force that compensates external force
- $c^{(1)} \times Q^{(1)} f^{(1)} + \cdots + c^{(M)} \times Q^{(M)} f^{(M)} + \tau^{ext} = 0$

Matrix Notation of Cross Product

• $c^{(1)} \times Q^{(1)} f^{(1)} + \ldots + c^{(M)} \times Q^{(M)} f^{(M)} + \tau^{ext} = 0$
• $S^{(1)} Q^{(1)} f^{(1)} + \ldots + S^{(M)} Q^{(M)} f^{(M)} + \tau^{ext} = 0$

• Where

$$\begin{pmatrix}
0 & -c_z^{(i)} & c_y^{(i)} \\
-c_z^{(i)} & 0 & -c_x^{(i)} \\
-c_y^{(i)} & c_x^{(i)} & 0
\end{pmatrix} \in skew(3) \text{ where } S^i x = c^i \times x$$
Equilibrium Constraints – Force Closure

Compact notation

• Contact force vector \( f \in \mathbb{R}^{3M} \)
  \[ f = (f^{(1)}, \ldots, f^{(M)}) \]

• Contact Matrices \( G_i \in \mathbb{R}^{6 \times 3} \)
  \[ G_i = \frac{Q^{(i)}}{S^{(i)}Q^{(i)}}, i = 1 \ldots M \]

• Grasp matrix
  \[ G = [G_1, \ldots, G_M] \in \mathbb{R}^{6 \times 3M} \]

• External Wrench \( \omega^{ext} = (f^{ext}, \tau^{ext}) \)

• Equilibrium conditions
  \[ Gf + \omega^{ext} = 0 \]

Other Constraints Constraints

Hardware constraints (max torque, kinematic limits).

\[ f \in C^{\text{other}} \]

Convex Optimization Problem

- Second-order cone program because friction cones are quadratic.

- Objective function:
  \[ F^{\text{max}} = \max \left\{ \| f^{(1)} \|, \ldots, \| f^{(M)} \| \right\} \]
  \[ = \max_{i=1,\ldots,M} \sqrt{f_x^{(i)} + f_y^{(i)} + f_z^{(i)}} \]

- Optimization problem:
  - minimize \( F^{\text{max}} \)
  - subject to \( f^{(i)} \in K_i, i = 1 \ldots M \)
  - \( Gf + \omega^{\text{ext}} = 0 \)

Motivating Example

Figure adapted from A Grasping Force Optimization Algorithm for Multiarm Robots With Multifingered Hands. Lipiello et al. Transactions on Robotics. 2013
Manipulation through Contact

Learning Hierarchical Control for Robust In-hand Manipulation

ICRA 2020 Submission

Tingguang Li, Krishnan Srinivasan, Max Q.-H. Meng, Wenzhen Yuan, Jeannette Bohg

A Data-Efficient Approach to Precise and Controlled Pushing

Hogan et al. CORL 2018.

Figure adapted from A Grasping Force Optimization Algorithm for Multiarm Robots With Multifingered Hands. Lippiello et al. Transactions on Robotics. 2013

Learning Hierarchical Control for Robust In-Hand Manipulation. Li et al. ICRA 2020.

A Data-Efficient Approach to Precise and Controlled Pushing. Hogan et al. CORL 2018.
Case Study – Planar Pushing

Reorient parts
- Mason 1986

Transport large objects
- Meričli 2015

Push-grasp under clutter
- Dogar 2010

Track object pose
- Koval 2015
Modeling Planar Pushing

**Friction limit surface:** describes friction forces occurring when part slides over support.

When pushed with a wrench within the limit surface: **no motion.**

For **quasi-static pushing:** wrench on the limit surface; object twist normal to limit surface where **twist** = linear and angular velocity: \( t_i = (v^i_x, v^i_y, \omega^i_z) \)

If **object translates without rotation** the friction force magnitude \( \mu mg \) where \( \mu = \) friction coefficient, \( m = \) object mass, \( g = \) gravitational acceleration.

Relation between wrench cone, limit surface and unit twist sphere. Adopted from Chapter 37, Fig 37.10 in Springer Handbook of Robotics.
Modeling Planar Pushing

\( o \) position of the object
\( v_o \) linear and angular object velocity
\( v_p \) linear velocity at the contact point - effective push velocity
\( p \) position of the pusher
\( u \) linear pusher velocity - action
\( c \) contact point (global)
\( c' \) contact point relative to \( o \)
\( n \) surface normal at \( c \)
\( l \) ratio between maximal torsional and linear friction force
\( \mu \) friction coefficient pusher-object

\( f_b \) left or right boundary force of the friction cone
\( m_b \) torques corresponding to the boundary forces
\( v_{o,b} \) object velocities resulting from boundary forces
\( v_{p,b} \) effective push velocities corresponding to the boundary forces
\( b = l, r \) placeholder for left or right boundary
\( s \) contact indicator, \( s \in [0, 1] \)
\( k \) rotation axis

Fig. 1: Overview and illustration of the terminology for pushing.
Validating Models for Planar Pushing

IROS 2016, "More than a Million Ways to Be Pushed: A High-Fidelity Experimental Dataset of Planar Pushing" by Peter Yu, Maria Bauza et al.
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2/1/21
AA 274B | Lecture 6
Validating Models for Planar Pushing

More than a Million Ways to Be Pushed.
A High-Fidelity Experimental Dataset of Planar Pushing

Kuan-Ting Yu, Maria Bauza, Nima Fazeli, and Alberto Rodriguez
Computer Science and Artificial Intelligence Lab & Mechanical Engineering Department, MIT

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Suggested Reading

• *Fast Computation of Optimal Contact Forces* by Boyd and Wegbreit. TRO 2007
Next time

• Learning-based approaches to Grasping and Manipulation