Principles of Robot Autonomy II

Fundamentals of Grasping – Jeannette Bohg
What is so important about autonomous grasping and manipulation?
Beyond Mobility
Contact Interaction
Service Robotics
Why is grasping and manipulation challenging?
Kids Making Robots Jealous
Learning Outcome for next four Lectures

Modeling and Evaluating Grasps

Apply Learning to Grasping and Manipulation

Modeling and Executing Manipulation

Use Manipulation to Perceive better
Today’s itinerary

• What is a Grasp?
  • Why is grasping hard?
  • How to generate a grasp?
  • How to evaluate a grasp?

• Analytical Approach to Modeling a Grasp
  • Terminology
  • Modeling of a Grasp
  • Stability Analysis of a Grasp
    • Form Closure
    • Force Closure

• Generating a Grasp
What is a grasp?

• Restraining an object’s motion through application of forces and torques at a set of contact points
Why is grasping hard?

• High-dimensional search problem
Why is grasping hard?

- High-dimensional search problem

All possible hand-object poses

Contact with object

Approach does not collide

Grasps that can be maintained

The Allegro Hand
http://www.simlab.co.kr/
Why is grasping hard?

• Grasp Planning / Synthesis / Generation
  • Searching the space of grasp parameters

• Grasp Acquisition
  • Executing reaching and grasping to acquire the grasp on the object

• Grasp Maintenance
  • Maintaining the grasp while lifting the object or manipulating something with it

• Grasp Evaluation
  • Objective function for guiding the search or reacting online during grasp acquisition/maintenance
How to generate a grasp?

- How do robots parameterize their grasp plan?
  - Approach vector
  - Wrist orientation of hand
  - Initial finger configuration
  - Points of contact
- How do you search this space?

http://openrave.org/docs/0.8.0/openravepy/examples.graspplanning/
How to evaluate a potential grasp?

What are good grasp characteristics?

**Grasp Maintenance:**
contact forces applied by the hand are such that they prevent contact separation and unwanted contact sliding

**Closure:**
Grasps that can be maintained for every possible disturbance load
Let’s model a grasp analytically!

Restraining an object’s motion through application of forces and torques at a set of contact points
Terminology - Contact Types

• Point: point on plane (stable), point on point or line (unstable)
• Line: line on plane or nonparallel line (stable), line on parallel line (unstable)
• Plane: plane on plane
Everything as a Point Contact

• Line contact -> 2 points
• Plane contact -> convex hull of points
• Any distribution of normal forces across a region can be represented as a weighted sum of point forces along that region’s convex hull
Everything as a Point Contact

• Line contact -> 2 points
• Plane contact -> convex hull of points
• Any distribution of normal forces across a region can be represented as a weighted sum of point forces along that region’s convex hull
Terminology - Contact Models

• The kind of contact determines what components of contact force $f$ and moment $\tau$ are transmitted through contact

• Local reference frame at contact point, $z$ aligned with normal pointing inward

$$ f = f_{\text{normal}} + f_{\text{tangent}}, \quad f_{\text{normal}} = [0, 0, f_z]^T \quad f_{\text{tangent}} = [f_x, f_y, 0]^T \quad f_z \geq 0 $$

• Three common contact models
  • Frictionless Point Contact
  • Point Contact with Friction
  • Soft-finger Contact
Terminology - Frictionless Point Contact

• Forces can only be applied in direction normal to the surface of the object

\[ F = \{ f_{\text{normal}} \mid f_z \geq 0 \} \]

\[ f = f_{\text{normal}} + f_{\text{tangent}}, \quad f_{\text{normal}} = [0, 0, f_z]^T \quad f_{\text{tangent}} = [f_x, f_y, 0]^T \quad f_z \geq 0 \]
Terminology - Point Contact with Friction

• A point contact with friction can apply more than just a normal force
• “Friction cone” is the vector space of all possible/admissable forces a point can apply due to friction (w/o slipping)
• Static Coefficient of friction $\mu_s$

$$f = f_{\text{normal}} + f_{\text{tangent}},$$

$$\mathcal{F} = \{ f \mid \|f_{\text{tangent}}\| \leq \mu_s\|f_{\text{normal}}\|, \quad f_z \geq 0 \}.$$
Terminology - Linearized Friction Cone

- Pyramidal approximation converts vector space to finite set of vectors
- Space of possible applied wrenches
Terminology - Soft-finger Contact Model

• Also allows for torque around the normal
• Space of possible applied wrenches:

\[ \mathcal{F} = \{ (f, \tau_{\text{normal}}) \mid \| f_{\text{tangent}} \| \leq \mu_s \| f_{\text{normal}} \|, \; f_z \geq 0, \; |\tau_{\text{normal}}| \leq \gamma f_z \}. \]

\( \gamma \)  Torsional friction coefficient
## Summary of Contact Models in Wrench Space

<table>
<thead>
<tr>
<th>Contact type</th>
<th>Picture</th>
<th>Wrench basis</th>
<th>FC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frictionless point contact</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Matrix" /></td>
<td>$f_3 \geq 0$</td>
</tr>
<tr>
<td>Point contact with friction</td>
<td><img src="image3.png" alt="Image" /></td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$\sqrt{f_1^2 + f_2^2} \leq \mu f_3$; $\frac{f_3}{f_3 \geq 0}$</td>
</tr>
<tr>
<td>Soft-finger</td>
<td><img src="image4.png" alt="Image" /></td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$\sqrt{f_1^2 + f_2^2} \leq \mu f_3$; $</td>
</tr>
</tbody>
</table>

Wrench is a 6D vector of forces and torques applied at a contact.

$$\mathbf{w} = \begin{bmatrix} \mathbf{f} \\ \mathbf{\tau} \end{bmatrix} \in \mathbb{R}^6,$$

$$\mathbf{w} = \text{Wrench basis} \ast \mathcal{F}$$

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Murray, Li, Sastry, A Mathematical Introduction to Robotic Manipulation, Chapter 5
Terminology – Object Wrench

• Each contact point applies forces and torques (a wrench) to the object frame
• Stacked as a 6D vector
Terminology – Object Wrench

• Forces $f \in \mathbb{R}^3$ and torques $\tau \in \mathbb{R}^3$ applied at a contact that act on the object center of mass

• Stacked as 6D Vector written wrt to frame in object

$$w = \begin{bmatrix} f \\ \tau \end{bmatrix} \in \mathbb{R}^6,$$

$$w_i = \begin{bmatrix} f_i \\ \lambda(d_i \times f_i) \end{bmatrix}.'$$

$i$ Index of contact $c$

d Vector defining position of $i$th contact wrt to COM

$f$ Force

$\lambda$ Arbitrary scaling of torque component

Index of contact $c$
Terminology - Grasp

- \( \mathcal{G} = \) set of all possible wrenches that can be achieved through the contact points

\[
\mathbf{w} = \sum_{i=1}^{k} \mathbf{G}_i \mathbf{f}_i = \mathbf{G} \begin{bmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_k \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & \cdots & \mathbf{G}_k \end{bmatrix}
\]

- \( \mathbf{G}_i = \) wrench basis matrix including transformation from local contact-reference frame to global object-centric reference frame

- \( \mathbf{G} = [\mathbf{G}_1 \ldots \mathbf{G}_k] = \) grasp map
  - Transforming all applied forces and torques to achievable wrenches
Terminology - Grasp Wrench Space

• = the set of all possible wrenches that can be applied to an object through admissible forces at k contact points

\[ \mathcal{W} := \{ \mathbf{w} \mid \mathbf{w} = \sum_{i=1}^{k} G_i \mathbf{f}_i, \quad \mathbf{f}_i \in \mathcal{F}_i, \quad i = 1, \ldots, k \}. \]

• For 3D objects, grasp wrench space is 6D
  • 3D for force, 3D for torque
  • For 2D objects, it’s 3D
Terminology - Grasp Wrench Hull

- = Convex hull of all the wrenches from the contact points
- Grasp Wrench hull $\leq$ Grasp Wrench Space
- Hull easier to compute

$\mathbf{c}_1$ $\mathbf{c}_2$

COM

$f_x$ $\tau_z$ $f_y$
Visualization of Hull from GraspIt! Simulator
How to evaluate a grasp? Stability Analysis

What are good grasp characteristics?

**Grasp Maintenance:**
contact forces applied by the hand are such that they prevent contact separation and unwanted contact sliding

**Closure:**
Grasps that can be maintained for every possible disturbance load
Form Closure

- Joint angles locked
- Palm fixed in space
- Impossible to move the object even infinitesimally under arbitrary external wrenches
- No wiggle room
- Power grasps, enveloping grasps
Force Closure

- Grasp can be maintained under any object wrench
- Forces can be applied at the contact points to withstand the external wrench
- Friction forces help balance the wrench
- Fewer contacts needed compared to Form Closure
How to evaluate a grasp?

- Quantify:
  - How many external wrenches can a grasp withstand?
Let’s break this down!

• Force direction closure with frictionless contacts
• When will set of point contact forces resist arbitrary translation?
How many contacts needed? C-Space Analysis

Cartesian space

Configuration space

$(x, y)$
What does a point contact imply in C-Space?

Cartesian space

Configuration space

(x, y)
Condition for Force-Direction Closure

Algebraic condition?
Condition for Force-Direction Closure

Algebraic condition?
For force vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$, there must exist $\alpha$, $\beta$, $\gamma > 0$
$s.t. \quad \alpha \mathbf{p} + \beta \mathbf{q} + \gamma \mathbf{r} = 0$
What is the minimum number of contacts for force-direction closure on 3D object?
Grasp Closure Analysis

- **Force-direction closure**
  Translate forces to O; they compose to generate any desired resultant force

- **Torque closure**
  Translate forces to intersection Points; they can be adjusted to point at each other and away from each other to generate torque
Grasp Analysis – Force Closure

• A grasp is a force-closure grasp IF for any external wrench $F_e$ there exist contact forces $f_c \in F_C$ such that

$$G f_c = -F_e$$

i.e., if able to apply sufficient force at each contact, every external wrench can be compensated for.

Practical test? Algebraic condition and using the Wrench hull
Let’s go through an example!

• 2-point contacts
Let’s go through an example!

- 2-point contacts
- 4 wrenches
- Is this grasp stable?
Let’s go through an example!

• Ignore for now \( f_x \)

\[
\begin{align*}
\tau_{\text{out}} & \downarrow \\
-f_y & \quad f_y \\
\tau_{\text{in}} & \uparrow \\
\end{align*}
\]

\[
w_{i,j} = \left( \frac{f_{i,j}}{\lambda(d_i \times f_{i,j})} \right)
\]
The cross product $\mathbf{a} \times \mathbf{b}$ (vertical, in purple) changes as the angle between the vectors $\mathbf{a}$ (blue) and $\mathbf{b}$ (red) changes. The cross product is always orthogonal to both vectors, and has magnitude zero when the vectors are parallel and maximum magnitude $\|\mathbf{a}\|\|\mathbf{b}\|$ when they are orthogonal.
How do you make the grasp stable?

- Ignore for now $f_x$

\[ w_{i,j} = \left( \lambda (d_i \times f_{i,j}) \right) \]
Algebraic Condition

Algebraic condition? For force vectors $p$, $q$, $r$, there must exist $\alpha$, $\beta$, $\gamma > 0$ s.t. $\alpha f_1 + \beta f_2 + \gamma f_3 = 0$
What if there was a third point?

Wrench hull

Does contain origin, stable

\[ w_{i,j} = \left( \frac{f_{i,j}}{\lambda(d_i \times f_{i,j})} \right) \]

\[ \tau_{\text{out}} \]

\[ \tau_{\text{in}} \]
Both grasps are stable. Which one is better?
Grasp Quality

• Quality is how well a grip can resist disturbances
• Worst case scenario
  • How efficiently can a grip resist disturbance wrenches at its weakest point?
• Weakest means the direction (in wrench space) at which the sum normal force is converted to the desired wrench least efficiently
Worst Case Scenario

• The point on the wrench hull that is closest to the origin is the weakest point.

• Disturbances in the opposite direction are hardest to resist.

• Metric $\varepsilon = \text{The radius of the largest ball that can be enclosed in the wrench hull}$
  • Varies from 0 to 1 due to normalization of wrenches.

\[ \begin{align*}
\tau_{\text{out}} & = w_{1,1} + f_y \\
\tau_{\text{in}} & = w_{1,2} + f_y \\
\varepsilon & = w_{2,1} + w_{2,2} + w_{3,1} + w_{3,2}
\end{align*} \]
Physical Meaning

- In the worst case, the sum magnitude of the contact wrenches would need to be \( \frac{1}{\varepsilon} \) times the disturbance wrench.
Are these grasps equally good?
Average Case Scenario

• How efficiently can a grip resist a disturbance wrench on average?
• Metric \( v = \) Volume of the convex hull in wrench space
• The three-point contact has more volume, so it is more stable on average
Form Closure versus Force Closure

• Both are in a contact configuration that resists all external disturbances.
• Note: Every form closure grasp is also in force closure

• Why do I need less contact points to be in Force closure?
How do we generate a grasp?
Suggested Reading

• Lecture Notes!
• Constructing Force Closure Grasps. Van-Duc Nguyen. IJRR 1988
• Planning Optimal Grasps. Carlo Ferrari & John Canny. ICRA 1992
• Check out graphical method for homework assignment

If there are moving contacts whose motion is prescribed, e.g., robot fingers, the constraints on the motion of the remaining parts will no longer be homogeneous. As a result, the convex polyhedral feasible velocity space is no longer a cone rooted at the origin.

### 37.2.3 Graphical Planar Methods

When a part is confined to move in the $x$-$y$-plane, the twist $\tau$ reduces to $\tau = (a, v_x, v_y)^T = (0, a, v_x, v_y)^T$. The point $(-v_x/a, v_y/a)$ is called the center of rotation (COR) in the projective plane, and we can represent any planar twist by its COR and rotational velocity $a$. (Note that the case $a = 0$ must be treated with care, as it corresponds to a COR at infinity.) This is sometimes useful for graphical purposes: for a single part constrained by stationary fixtures, at least, we can easily draw the feasible twist cone as CORs [37.14, 31].

As an example, Fig. 37.2a shows a planar part standing on a table and being contacted by a robot finger.

![Diagram of a planar part being contacted by a robot finger.](image)

This finger is currently stationary, but we will later set it in motion. The finger defines one constraint on the part’s motion and the table defines two constraints, that contact points in the interior of the edge between the part and the table provide (redundant kinematic constraints). The constraint wrenches can be written

$$w_1 = (0, 0, -1, 0, 1, 0)^T,$$

$$w_2 = (0, 0, 1, 0, 1, 0)^T,$$

$$w_3 = (0, 0, 1, -1, 0, 0)^T.$$

For a stationary finger, the kinematic constraints yield the feasible twist cone shown in Fig. 37.2b. This region can also be easily visualized in the plane by using the following method: at each contact, draw the contact normal line. Label all points on the normal as points to the left of the traced normal $\nu$, and points to the right $\nu$. For each contact constraint, all the points labeled $\nu$ can serve as CORs with positive angular velocity, and all the points labeled $-\nu$ can serve as CORs with negative angular velocity, without violating the contact constraint. After doing this for all the contact normals, keep only the CORs that are consistently labeled. These CORs are a planar representation of the feasible twist cone (Fig. 37.2c).

We can refine this method by assigning contact modes to each feasible COR. For each contact normal, label the COR at the contact point $f$ for fixed, other CORs on the normal line $n$ for sliding, and all other CORs $b$ for breaking contact. The concatenation of these labels gives the part’s contact mode for a particular part motion. In the planar case, the label $a$ on the contact normal line can be further refined into $a_0$ or $a_1$, indicating whether the part is slipping right or left relative to the constraint. An a COR labeled $+a$ and above the contact (in the direction of the contact normal) or below the contact should be labeled $a_0$, and an a COR labeled $-a$ below the contact or $+a$ should be labeled $a_1$ (Fig. 37.2d).

This method can be used readily to determine if the part is in form closure. If there is no COR labeled consistently, then the feasible velocity space consists of only the zero velocity point, and the part is immobilized by the stationary contacts. This method also makes it clear that at least four contacts are necessary to immobilize the part by the first-order analysis (Chap. 38). This is a weakness of the first-order analysis – curvature effects can be used to immobilize a part with three or even two contacts [37.24]. This weakness can also be seen in Fig. 37.2d. A pure rotation about the COR labeled $\pm a_0, n_1$ is actually not feasible, but it would be if the part had a small radius of curvature at the contact with the finger. The first-order analysis ignores this curvature.
Next time

- Fundamentals of Manipulation