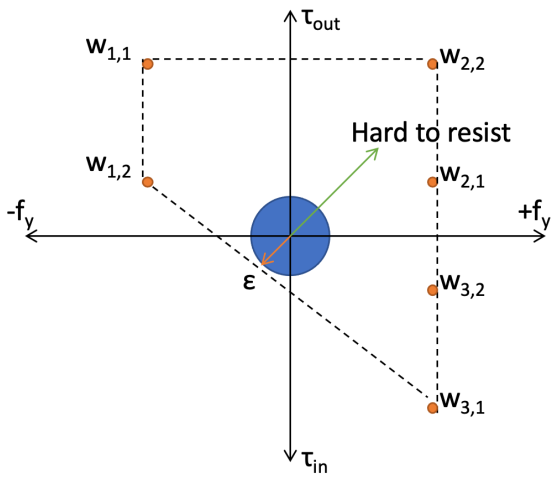
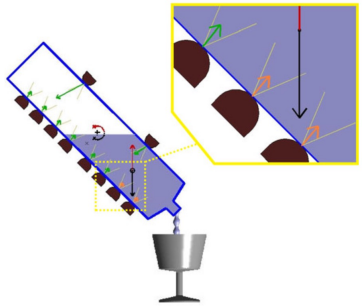
Principles of Robot Autonomy II

Fundamentals of Grasping and Manipulation II – Jeannette Bohg

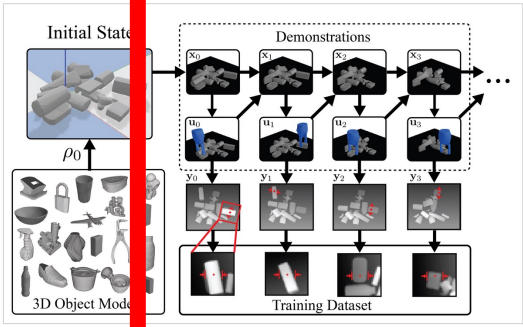
Stanford University

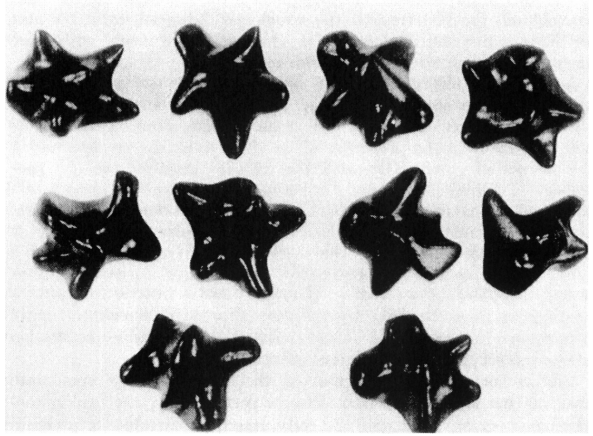
Learning Outcome for next four Lectures





Modeling and Evaluating Grasps





Use Manipulation to Perceive better

Apply Learning to Grasping and Manipulation

Modeling and Executing Manipulation

Today's itinerary

- Analytical Approach to Modeling a Grasp
  - Recap Last Lecture
  - Stability Analysis of a Grasp
    - Form Closure
    - Force Closure
    - Graphical method
- Generating a Grasp
- Grasp Force Optimization
- Modeling Push Manipulation

What is a grasp?

- Restraining an object's motion through application of forces and torques at a set of contact points

The Allegro Hand
http://www.simlab.co.kr/

What are good grasp characteristics?

Grasp Maintenance:

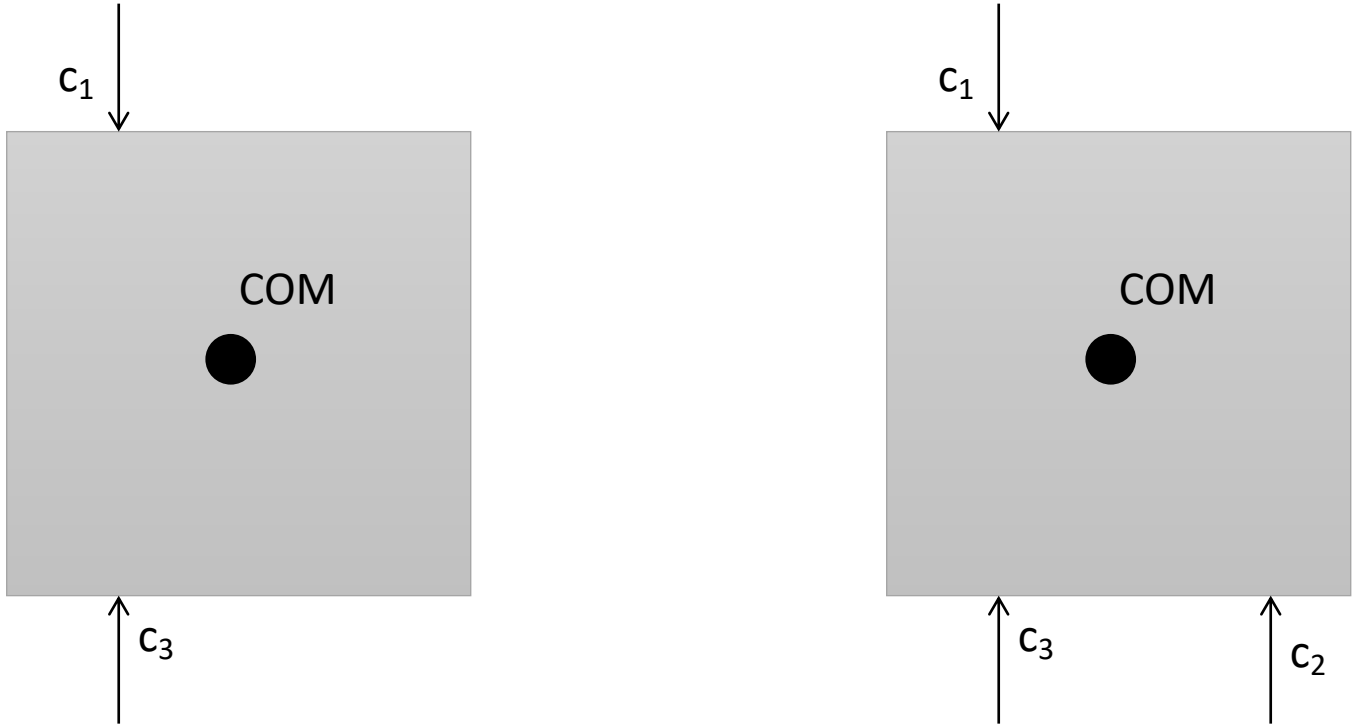
contact forces applied by the hand are such that they prevent contact separation and unwanted contact sliding

Closure:

Grasps that can be maintained for every possible disturbance load

Let's model a grasp analytically!

Restraining an object's motion through application of forces and torques at a set of contact points

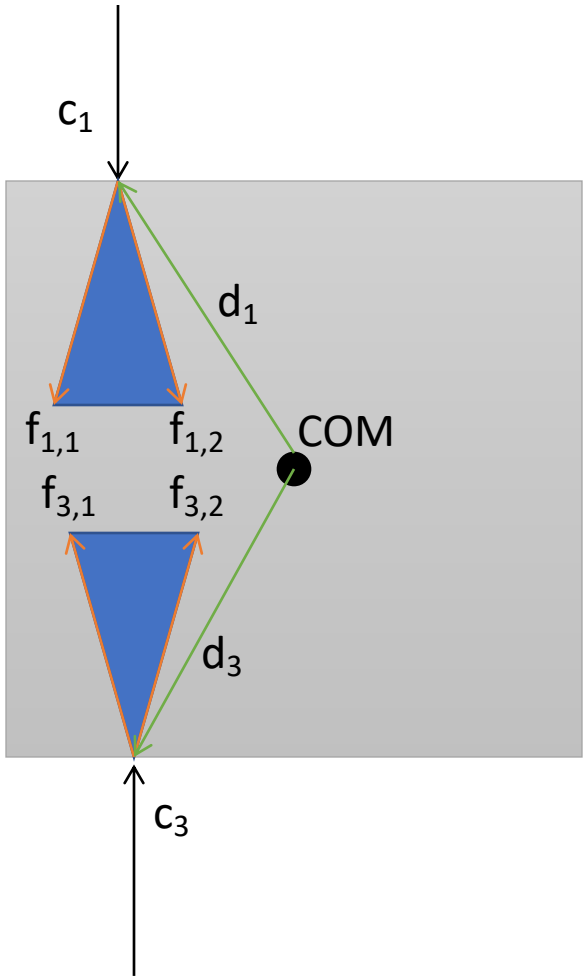


What's a wrench?

- Each point force also applies torque
  $\tau = d \times f$
- Wrench is a force-torque pair

$$\boldsymbol{w}_{i,j} = \begin{pmatrix} \boldsymbol{f}_{i,j} \\ \lambda(\boldsymbol{d}_i \times \boldsymbol{f}_{i,j}) \end{pmatrix}$$

  - The i-th point contact has m wrenches, one for each force in the pyramidal approximation of the friction cone
  - d is the vector from the point contact to the torque origin
  - λ is a constant relating force to torque



Terminology - Grasp

- = set of all possible wrenches that can be achieved through the contact points

$$\boldsymbol{w} = \sum_{i=1}^{k} G_i \boldsymbol{f}_i = G \begin{bmatrix} \boldsymbol{f}_1 \\ \vdots \\ \boldsymbol{f}_k \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & \dots & G_k \end{bmatrix},$$

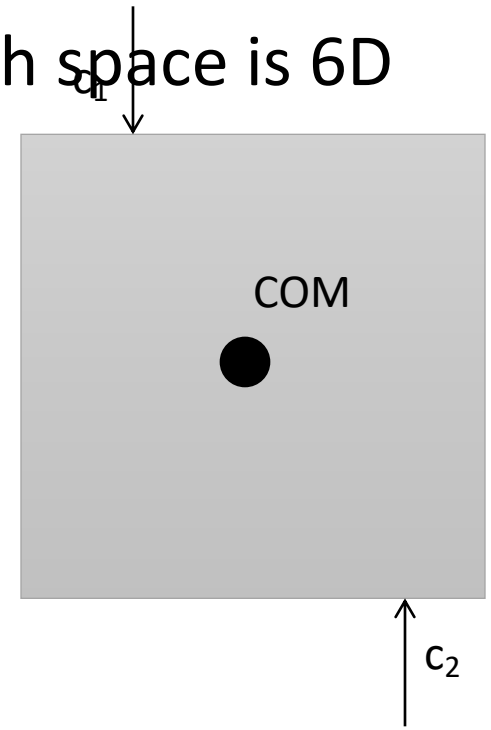
- G_i = wrench basis matrix including transformation from local contact-reference frame to global object-centric reference frame
- *G*=[G*1* ...G*k*] =grasp map
  - Transforming all applied forces and torques to achievable wrenches

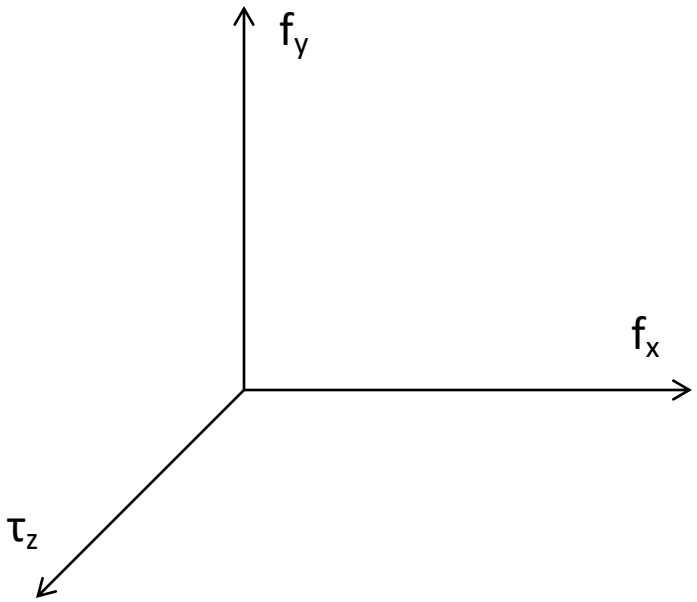
Terminology - Grasp Wrench Space

- = the set of all possible wrenches that can be applied to an object through admissible forces at k contact points

$$\mathcal{W} := \{\boldsymbol{w} \mid \boldsymbol{w} = \sum_{i=1}^{k} G_i \boldsymbol{f}_i, \quad \boldsymbol{f}_i \in \mathcal{F}_i, \quad i = 1, \ldots, k\}.$$

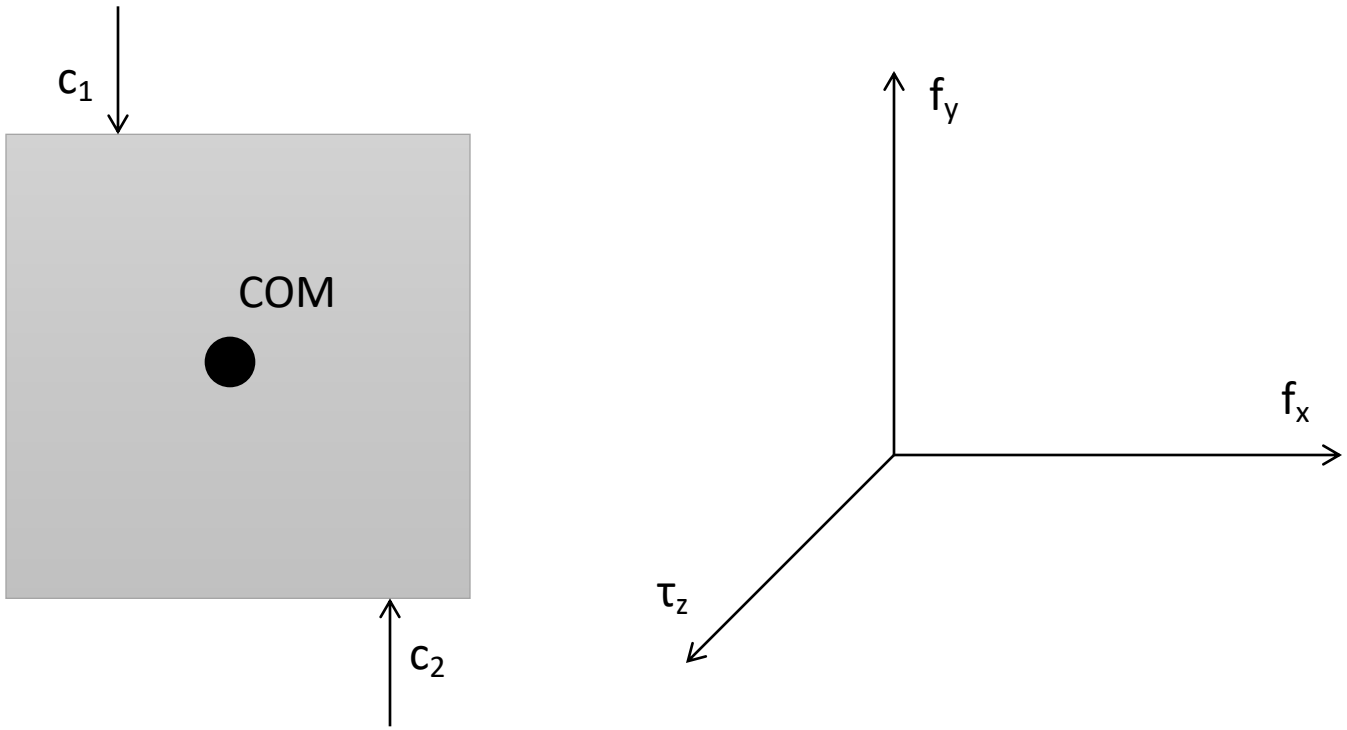
- For 3D objects, grasp wrench space is 6D
  - 3D for force, 3D for torque
  - For 2D objects, it's 3D





Terminology - Grasp Wrench Hull

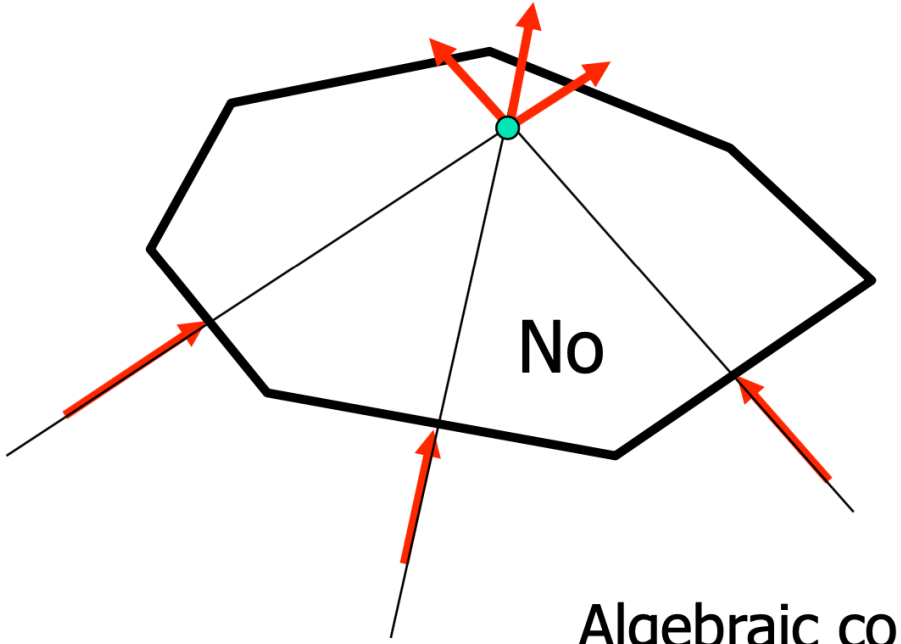
- = Convex hull of all the wrenches from the contact points
- Grasp Wrench hull ≤ Grasp Wrench Space
- Hull easier to compute

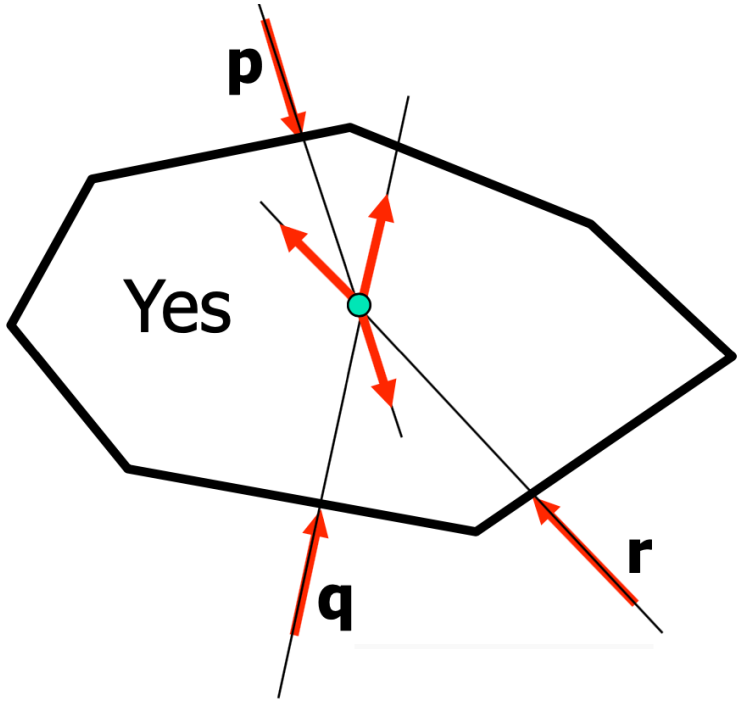


Grasp Maintenance

- Quantify:
  - How many external wrenches can a grasp withstand?
- Force Closure:
  - withstand all external wrenches
  - Friction forces help to counteract any external
- Form Closure
  - Frictionless force closure

Condition for Force-Direction Closure





Algebraic condition?
For force vectors **p**, **q**, **r**,
there must exist $\alpha, \beta, \gamma > 0$
s.t. $\alpha\mathbf{p} + \beta\mathbf{q} + \gamma\mathbf{r} = 0$

Grasp Analysis –Force Closure

- A grasp is a force-closure grasp IF for any external wrench F_e there exist contact forces $f_c \in F_C$ such that

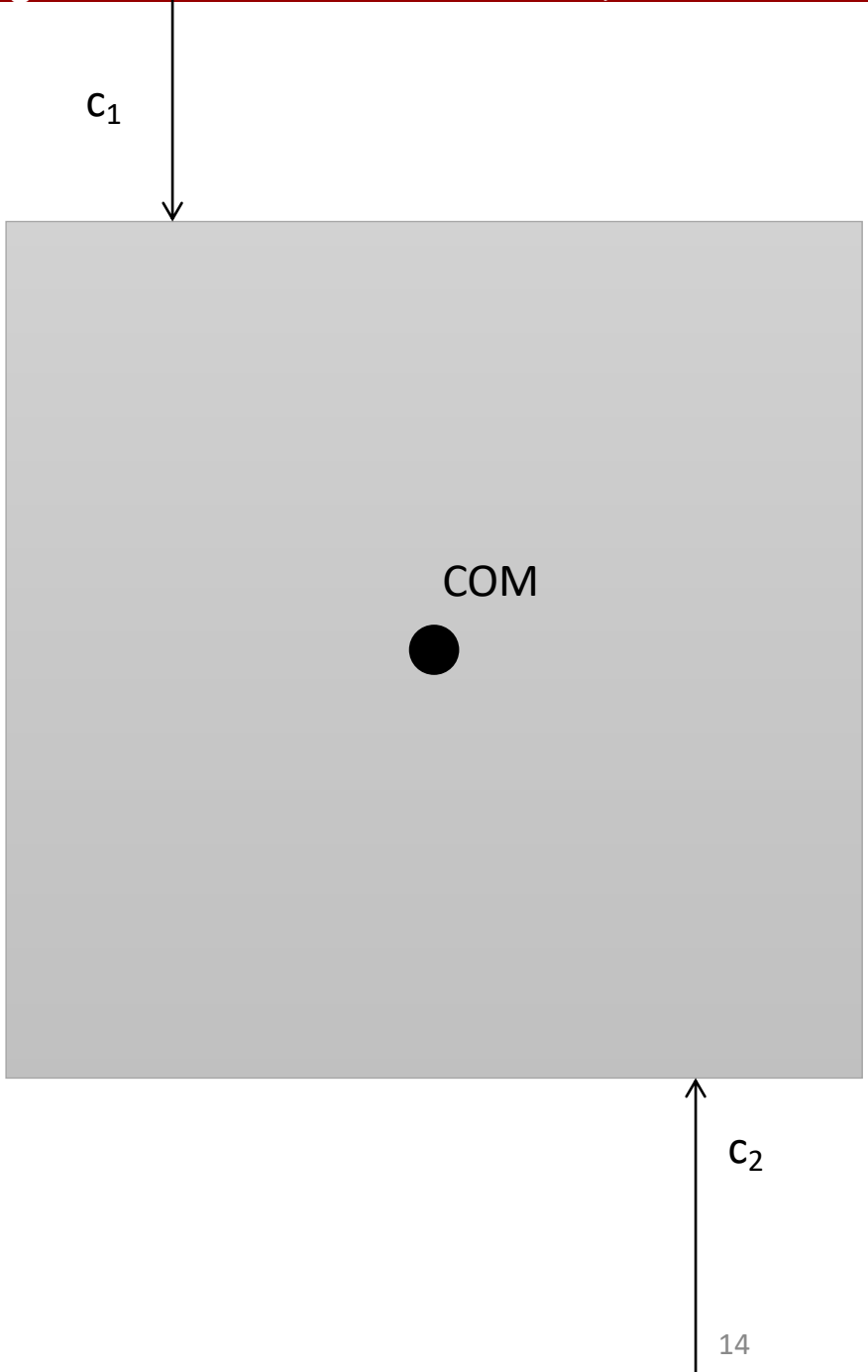
$$G\ f_c = -F_e$$

i.e., if able to apply sufficient force at each contact, every external wrench can be compensated for.

Practical test? Algebraic condition -> Solving a linear program (HW2)

Let's go through an example!

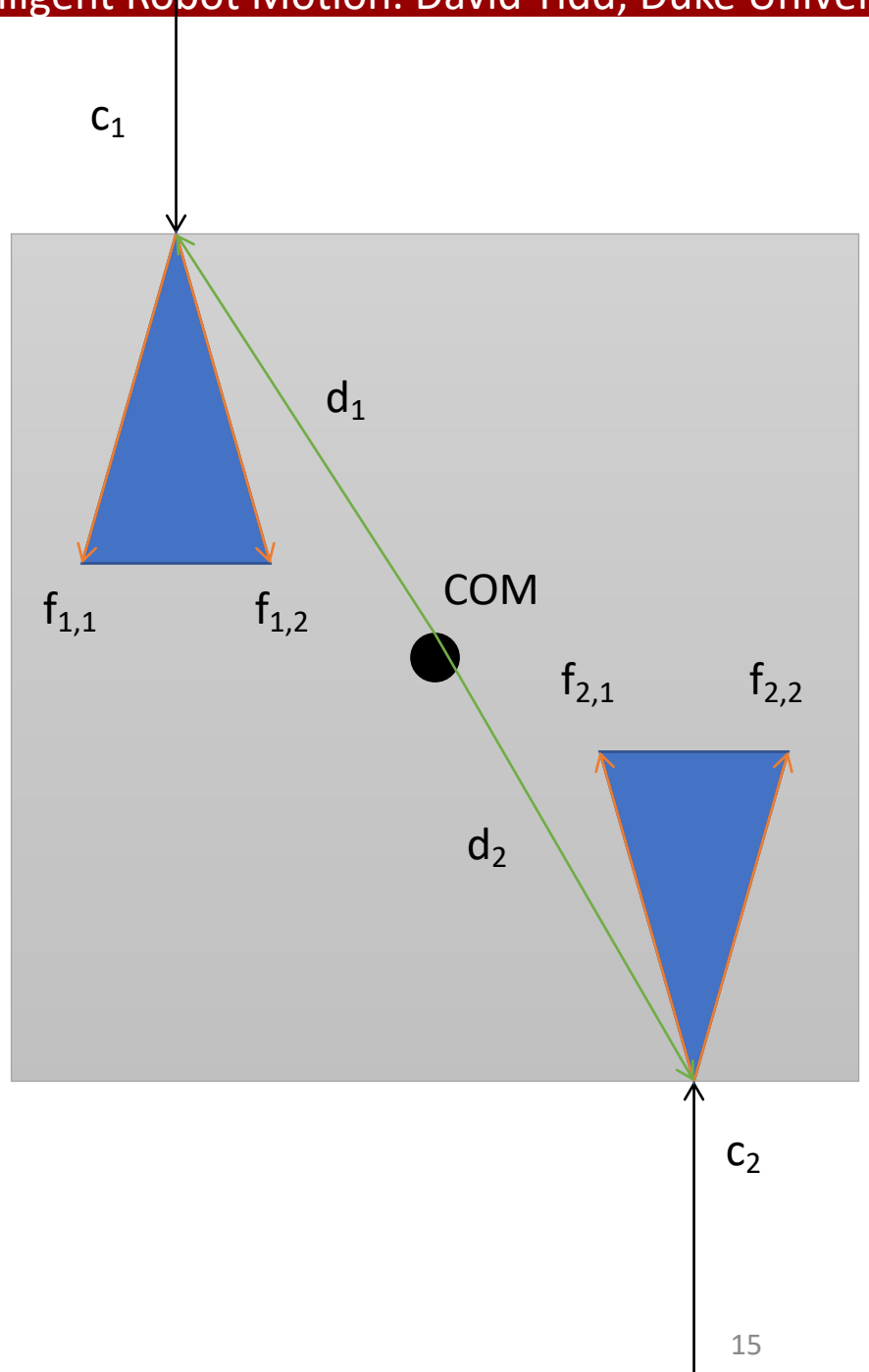
- 2-point contacts



Let's go through an example!

- 2-point contacts
- 4 wrenches

- Is this grasp stable?



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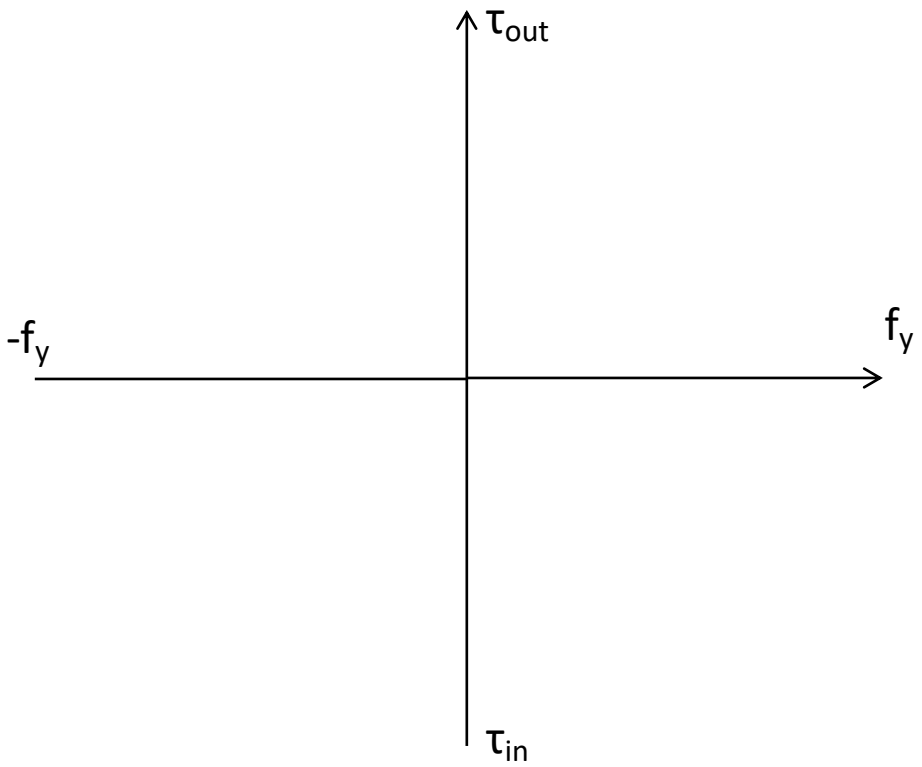
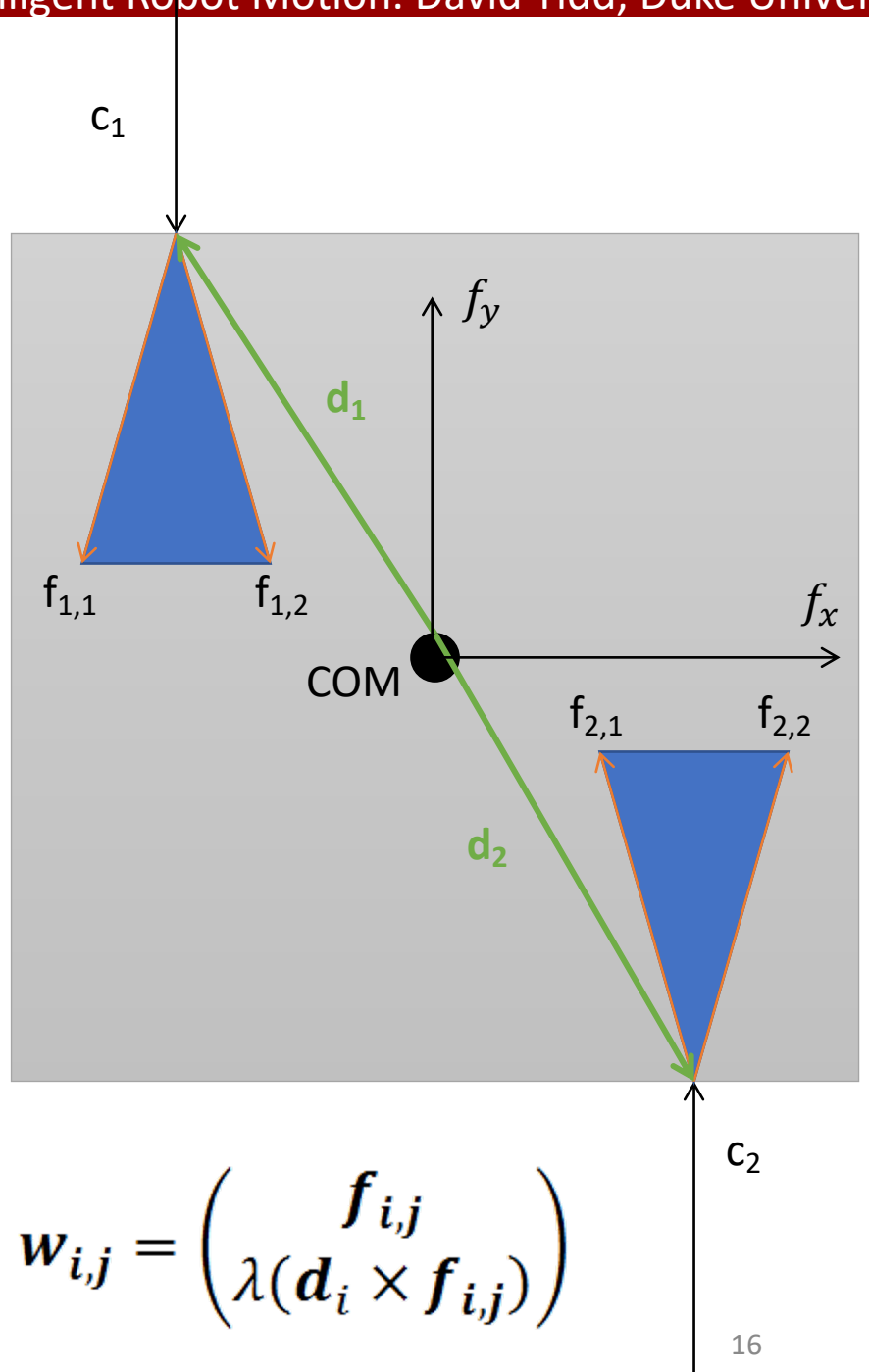




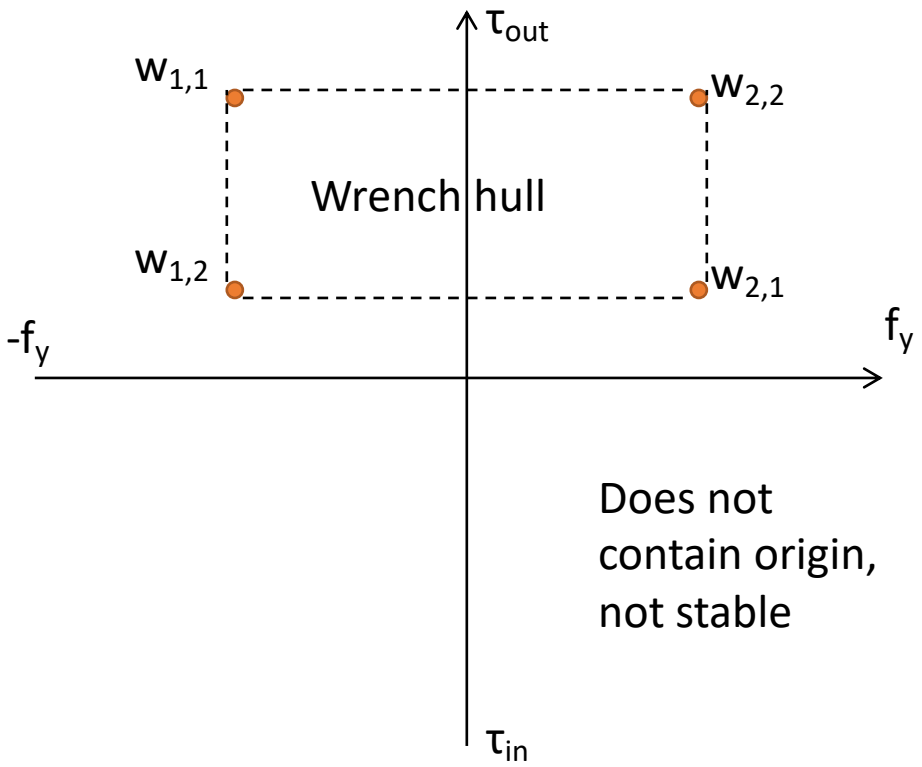
- Ignore for now f_x

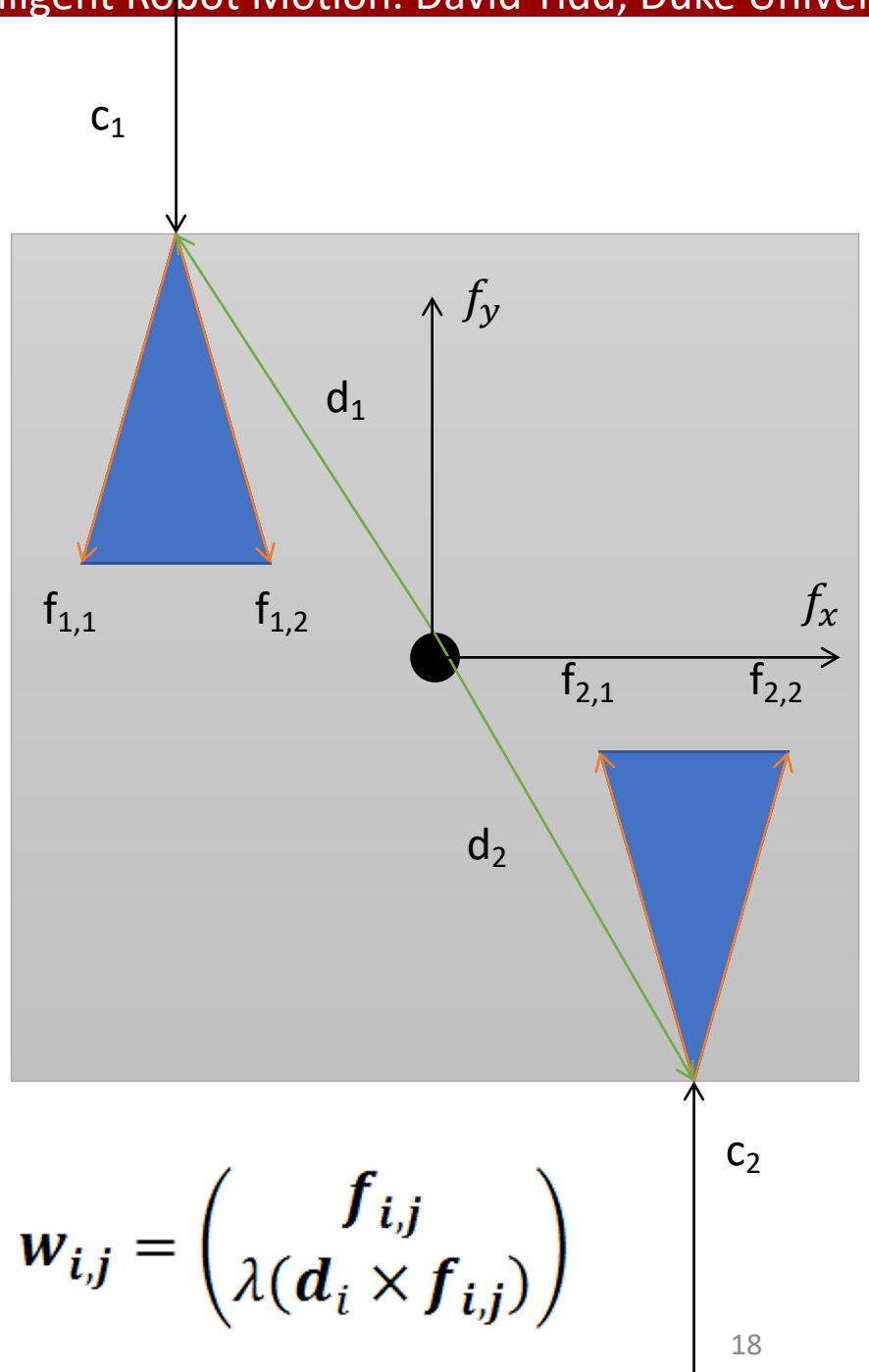
$$\boldsymbol{w}_{i,j} = \begin{pmatrix} \boldsymbol{f}_{i,j} \\ \lambda(\boldsymbol{d}_i \times \boldsymbol{f}_{i,j}) \end{pmatrix}$$

Cross Product for Torque

The cross product **a** × **b** (vertical, in purple) changes as the angle between the vectors **a** (blue) and **b** (red) changes. The cross product is always orthogonal to both vectors, and has magnitude zero when the vectors are parallel and maximum magnitude ‖**a**‖‖**b**‖ when they are orthogonal.

How do you make the grasp stable?





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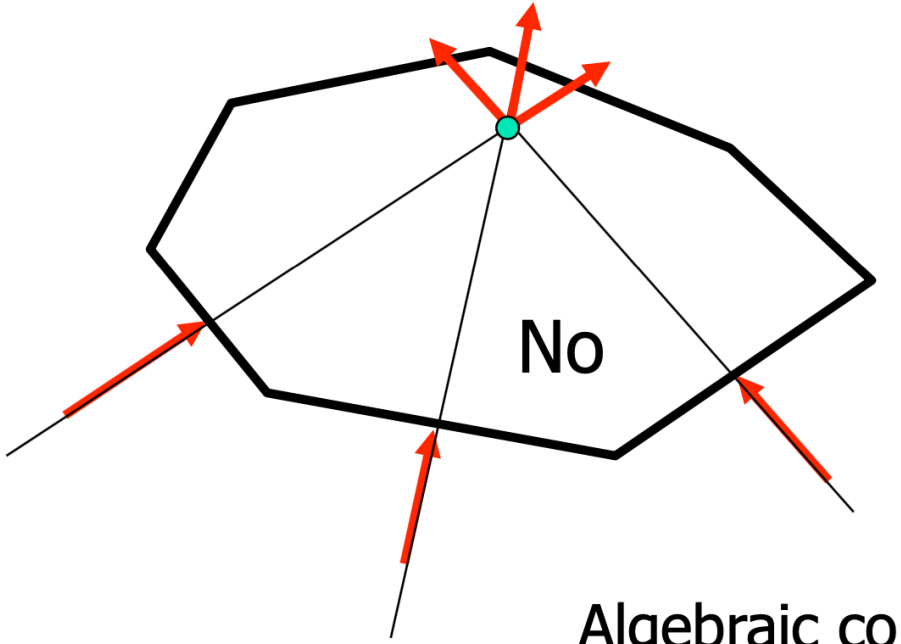
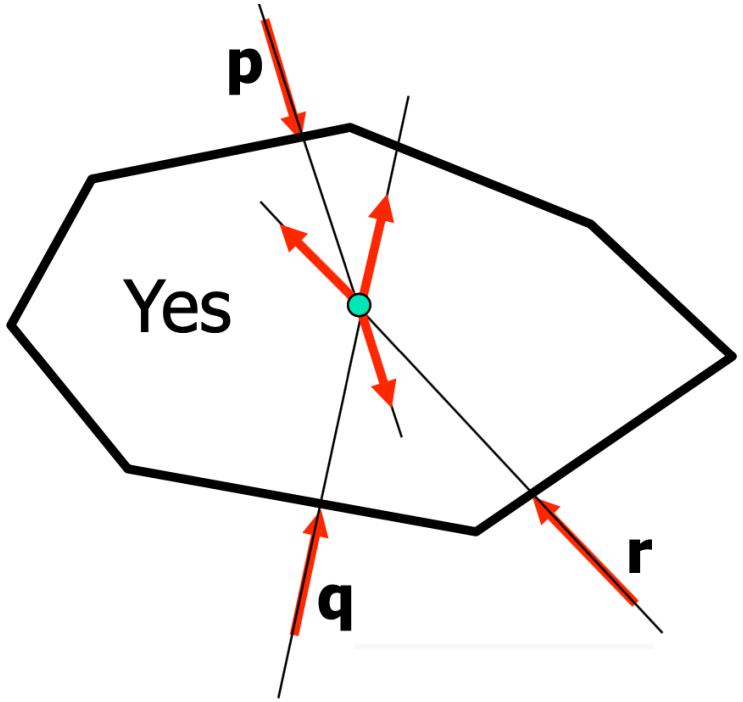
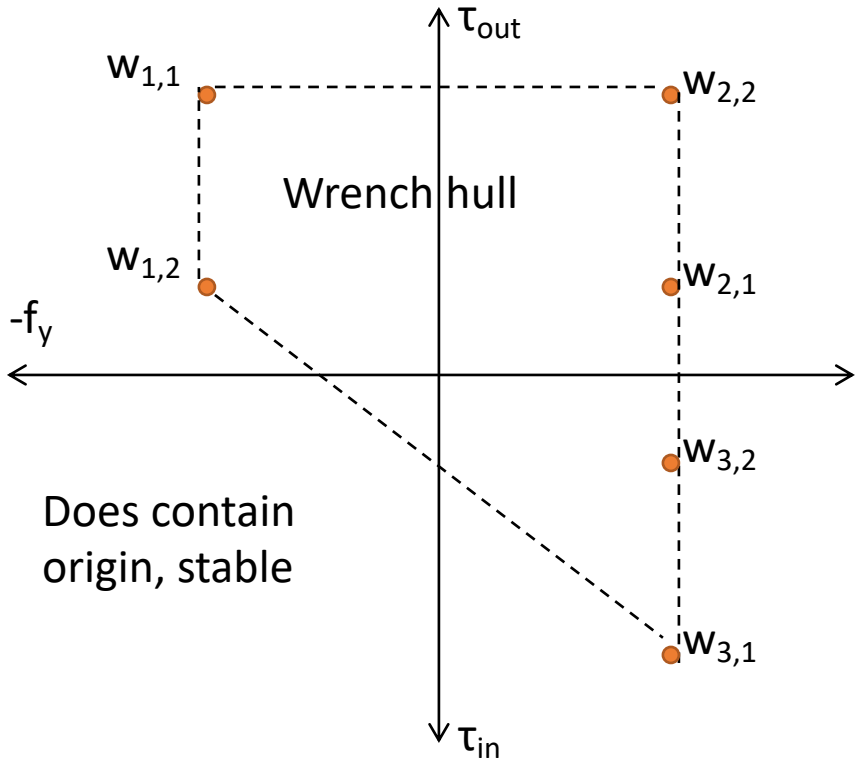
Algebraic Condition

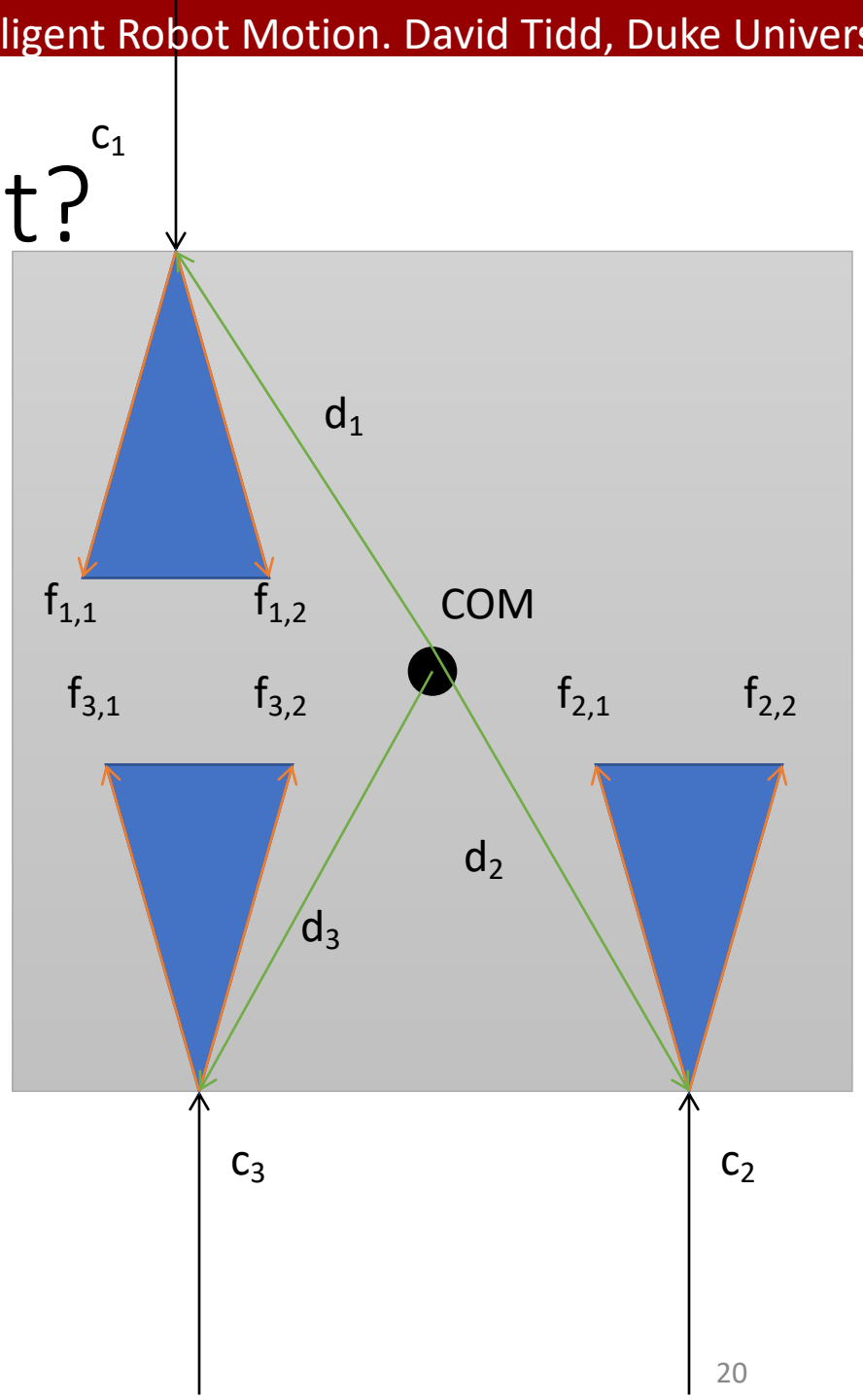




Algebraic condition?
For force vectors **p**, **q**, **r**,
there must exist $\alpha, \beta, \gamma > 0$
s.t. $\alpha\mathbf{p} + \beta\mathbf{q} + \gamma\mathbf{r} = 0$

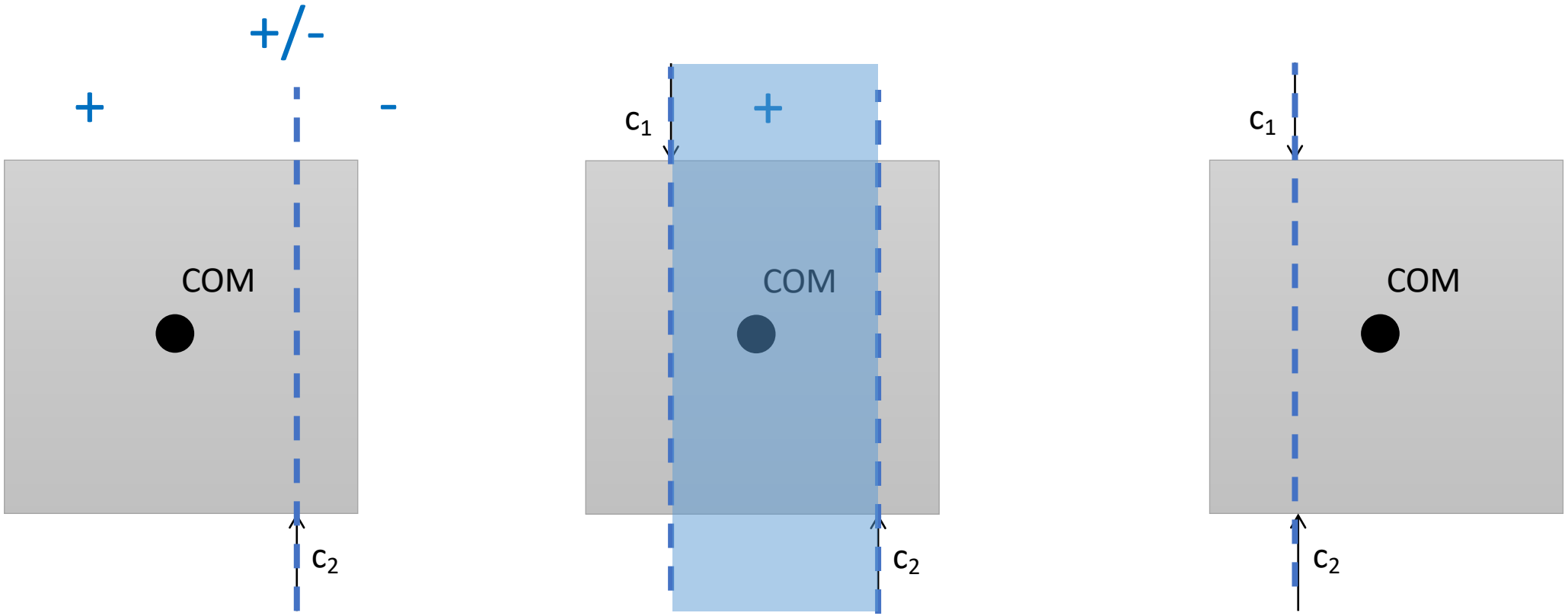
What if there was a third point?





$$\boldsymbol{w}_{i,j} = \begin{pmatrix} \boldsymbol{f}_{i,j} \\ \lambda(\boldsymbol{d}_i \times \boldsymbol{f}_{i,j}) \end{pmatrix}$$

Planar Graphical Method with Moment Labels

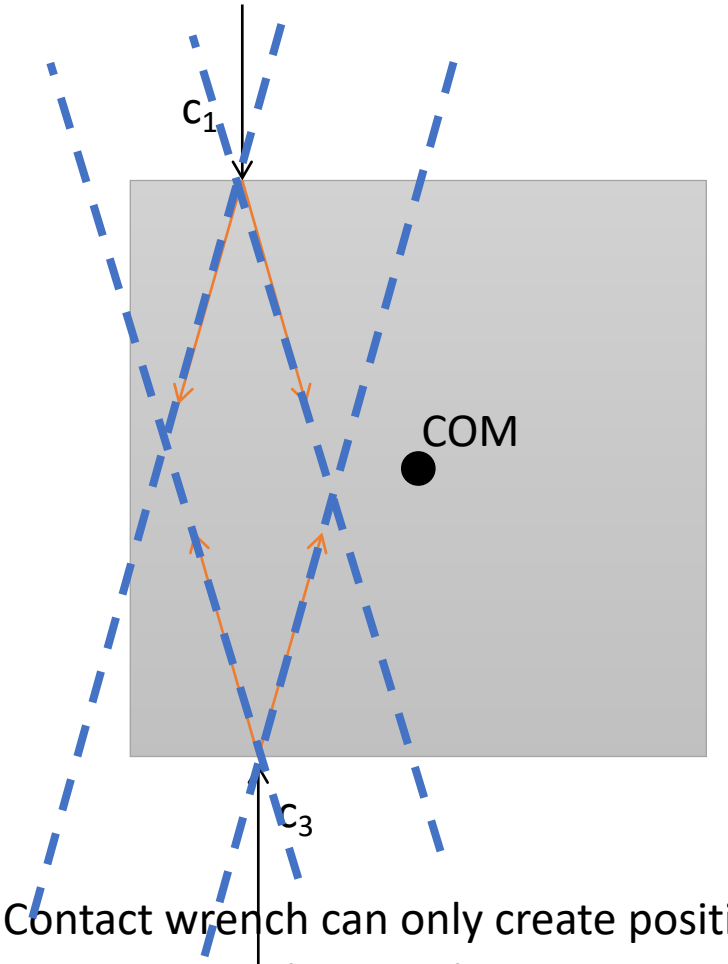


+ Contact wrench can only create positive moment around a point in that area

- Contact wrench can only create negative moment around a point in that area

+/- Contact wrench can neither create negative/positive moment around a point on that line

Planar Graphical Method



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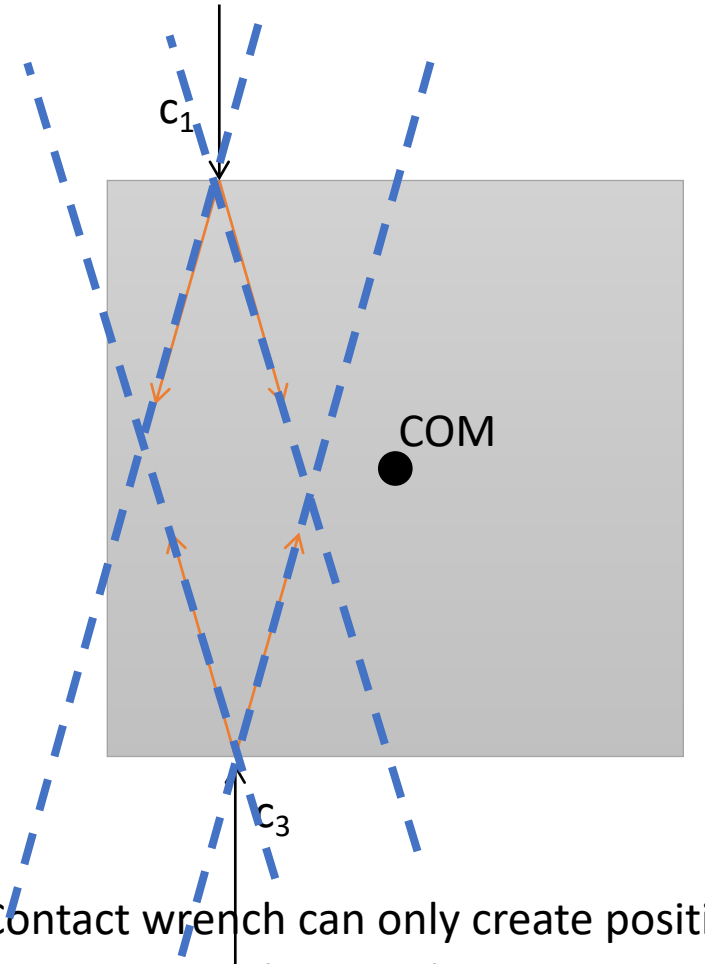
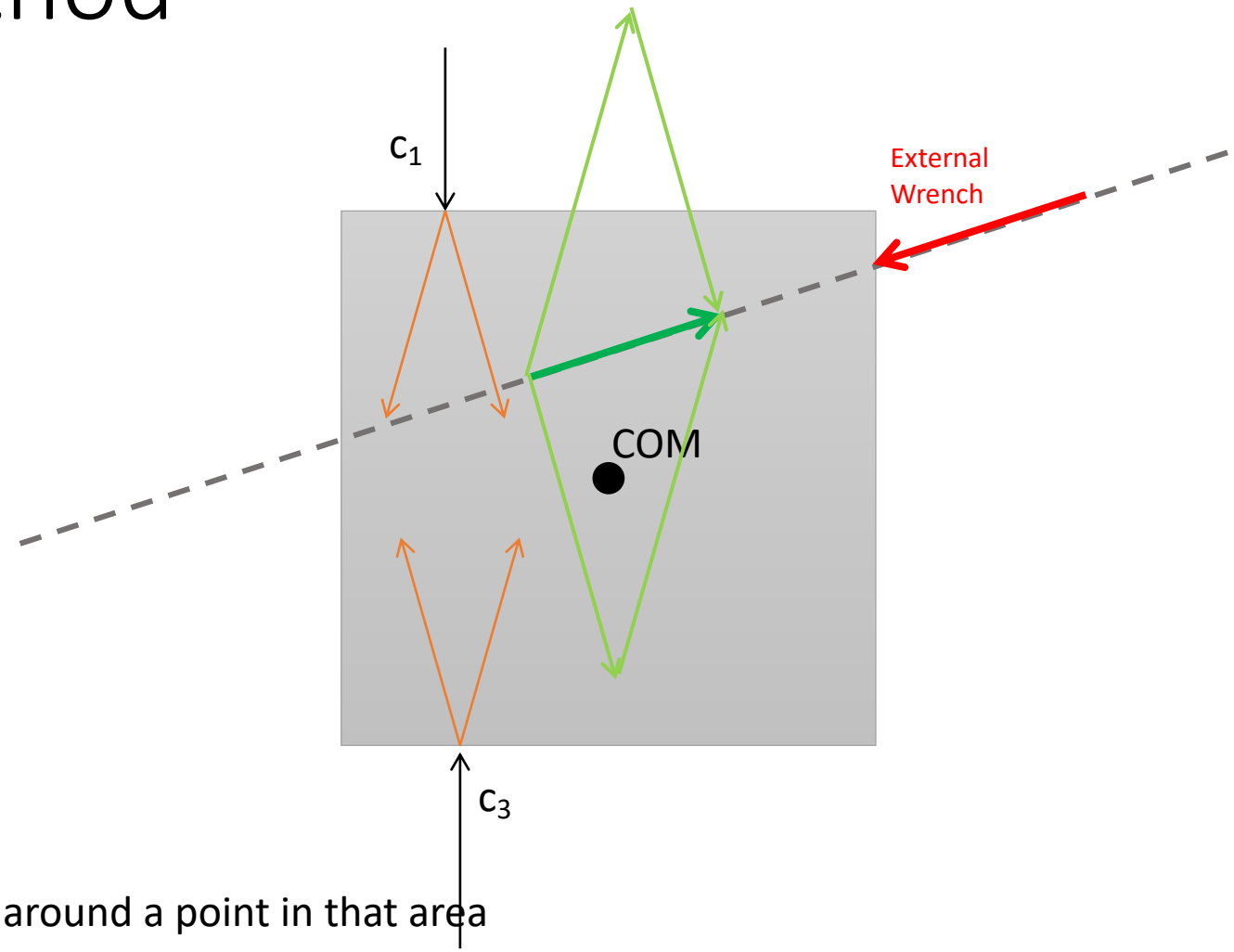
Planar Graphical Method



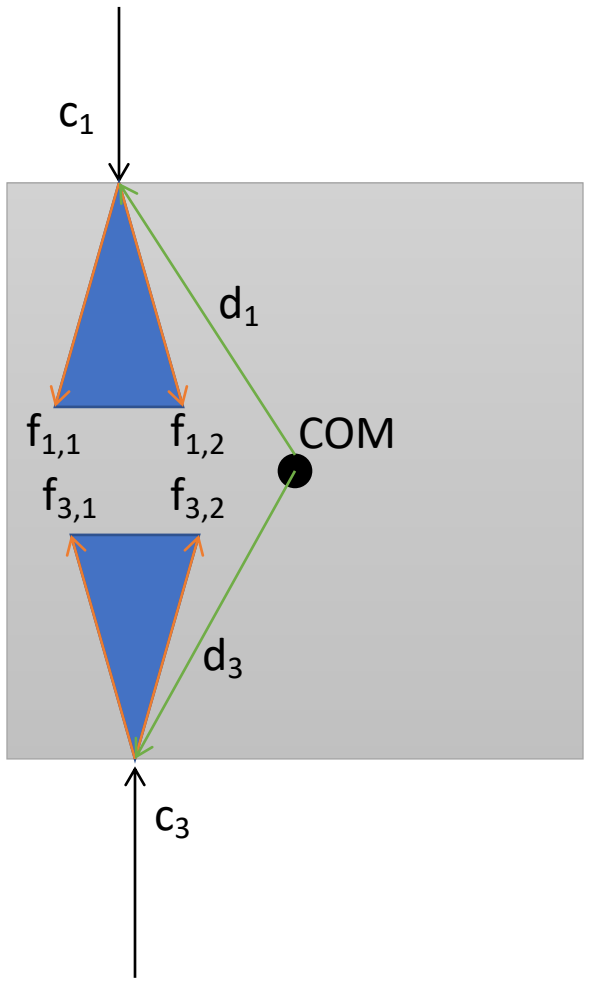


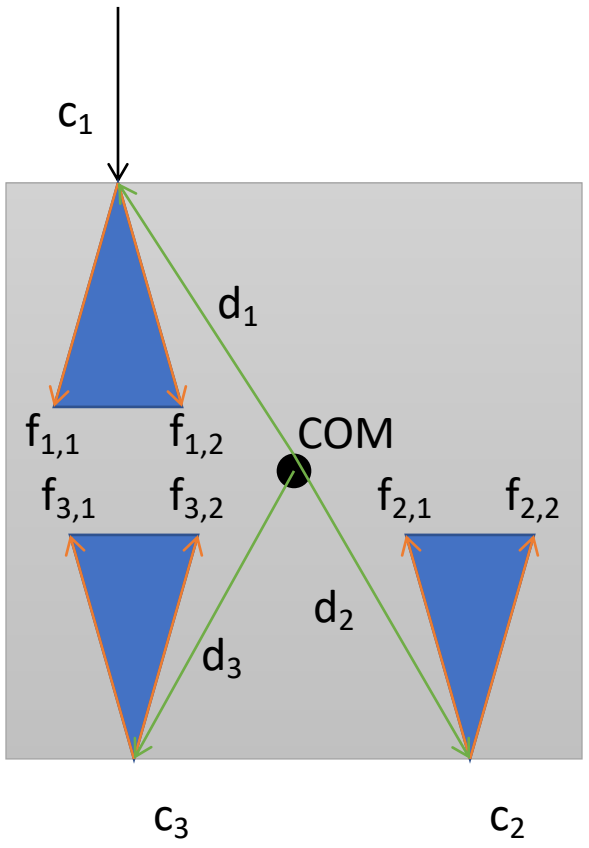
+ Contact wrench can only create positive moment around a point in that area

- Contact wrench can only create negative moment around a point in that area

+/- Contact wrench can neither create negative/positive moment around a point on that line

Both grasps are stable. Which one is better?



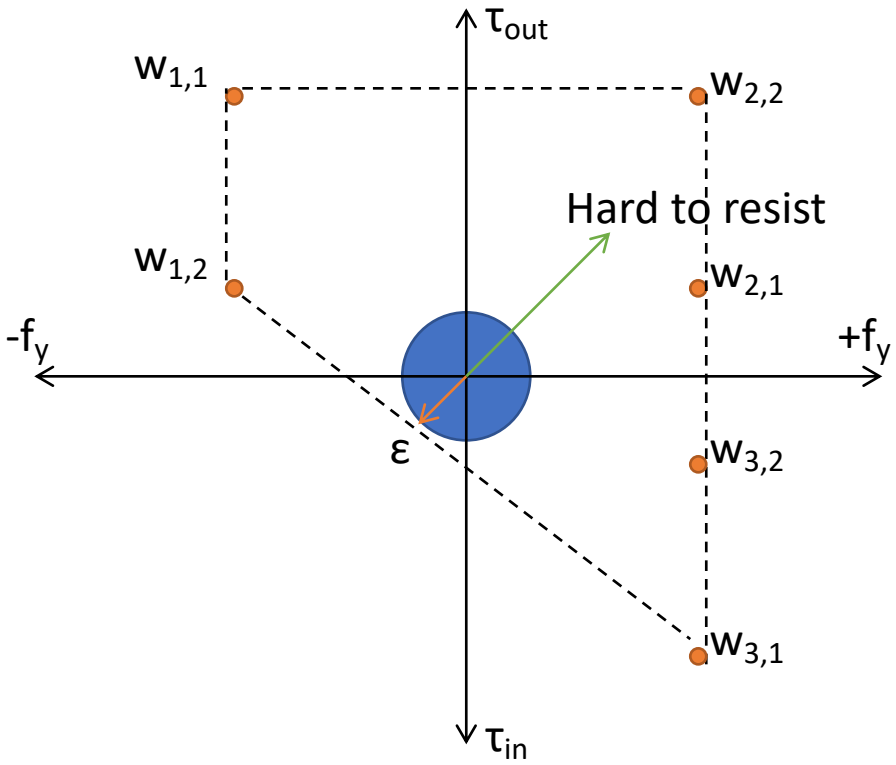


Grasp Quality

- Quality is how well a grip can resist disturbances
- Worst case scenario
  - How efficiently can a grip resist disturbance wrenches at its weakest point?
- Weakest means the direction (in wrench space) at which the sum normal force is converted to the desired wrench least efficiently

Worst Case Scenario

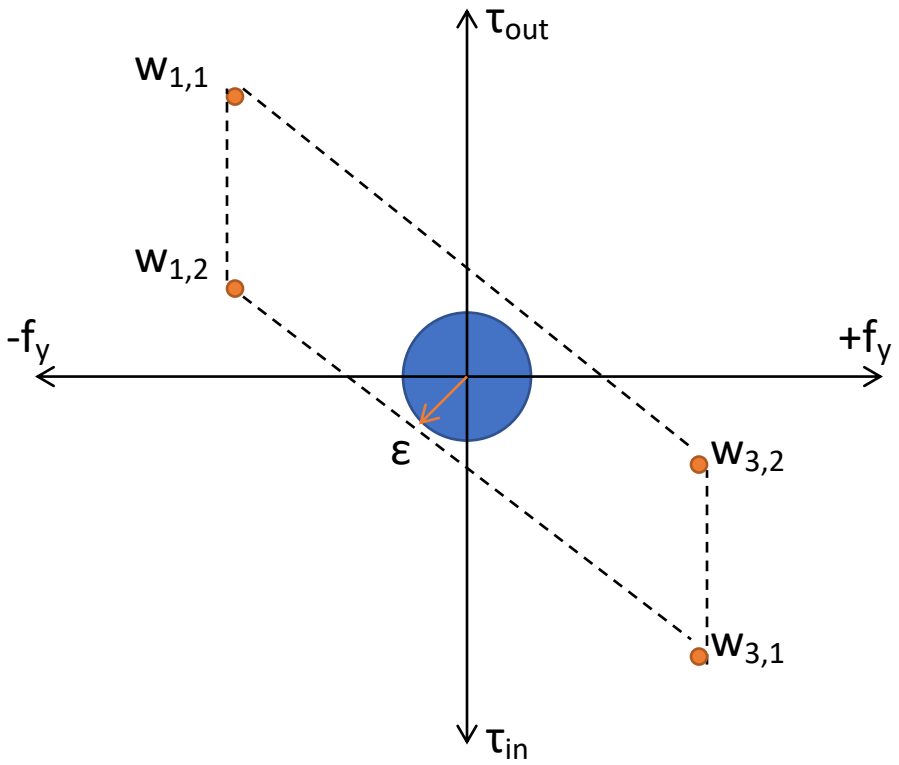
- The point on the wrench hull that is closest to the origin is the weakest point
- Disturbances in the opposite direction are hardest to resist
- Metric ε = The radius of the largest ball that can be enclosed in the wrench hull
  - Varies from 0 to 1 due to normalization of wrenches

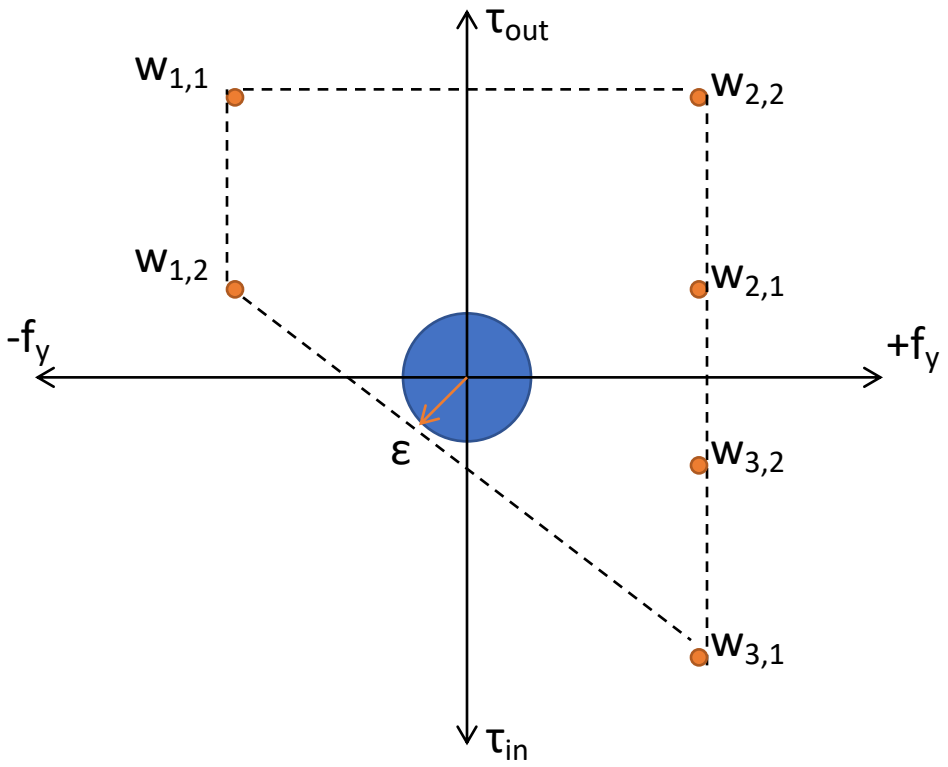


Physical Meaning

- In the worst case, the sum magnitude of the contact wrenches would need to be $1/\varepsilon$ times the disturbance wrench

Are these grasps equally good?





Average Case Scenario

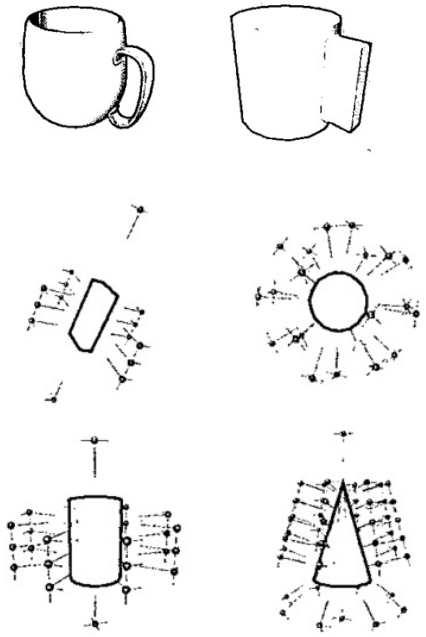
- How efficiently can a grip resist a disturbance wrench on average?
- Metric v = Volume of the convex hull in wrench space
- The three-point contact has more volume, so it is more stable on average

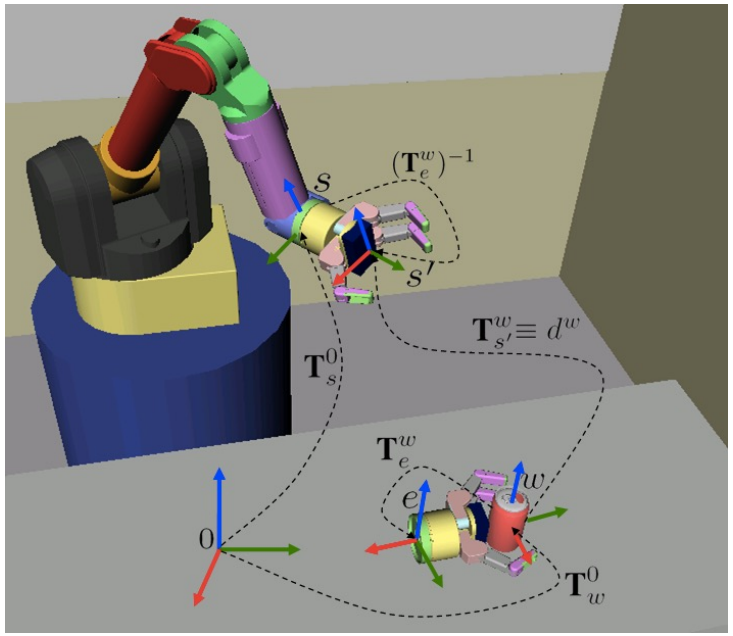
Form Closure versus Force Closure

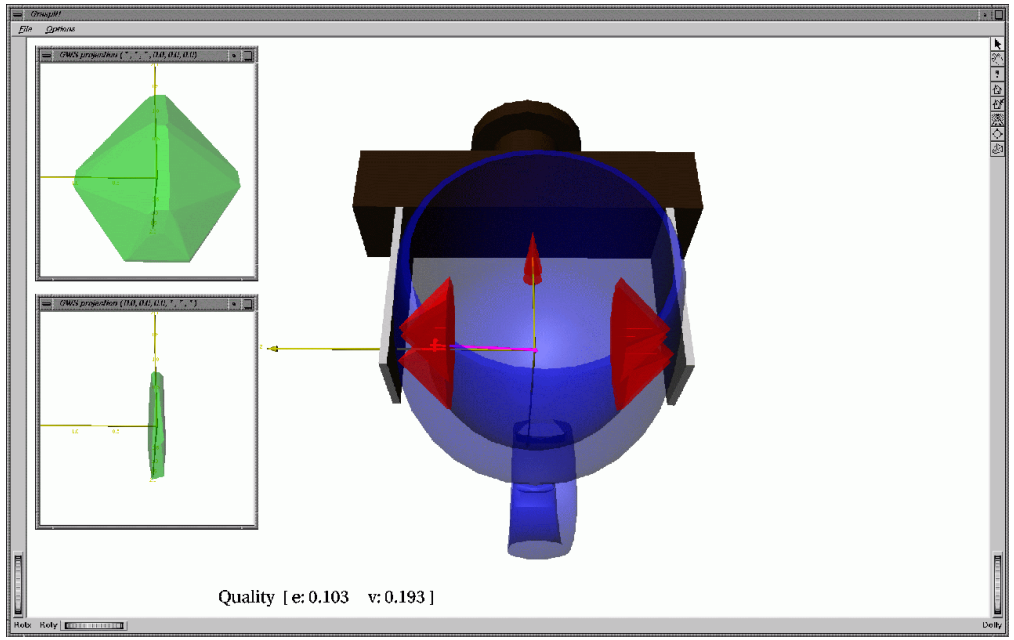
- Both are in a contact configuration that resists all external disturbances.
- Note: Every form closure grasp is also in force closure

- Why do I need less contact points to be in Force closure?

How do we generate a grasp?







Suggested Reading

- Lecture Notes!
- Constructing Force Closure Grasps. Van-Duc Nguyen. IJRR 1988
- Planning Optimal Grasps. Carlo Ferrari & John Canny. ICRA 1992
- Springer Handbook of Robotics. Chapter - Grasping. 2016.
  - Check out graphical method for homework assignment

If there are moving contacts whose motion is prescribed, e.g., robot fingers, the constraints on the motion of the remaining parts will no longer be homogeneous. As a result, the convex polyhedral feasible velocity space is no longer a cone rooted at the origin.

37.2.3 Graphical Planar Methods

When a part is confined to move in the x–y-plane, the twist $\boldsymbol{t}$ reduces to $\boldsymbol{t} = (\omega_x, \omega_y, \omega_z, v_x, v_y, v_z)^{\mathrm{T}} = (0, 0, \omega_z, v_x, v_y, 0)^{\mathrm{T}}$. The point $(-v_y/\omega_z, v_x/\omega_z)$ is called the center of rotation (COR) in the projective plane, and we can represent any planar twist by its COR and rotational velocity ω_z. (Note that the case $\omega_z = 0$ must be treated with care, as it corresponds to a COR at infinity.) This is sometimes useful for graphical purposes: for a single part constrained by stationary fixtures, at least, we can easily draw the feasible twist cone as CORs [37.14, 31].

As an example, Fig. 37.2a shows a planar part standing on a table and being contacted by a robot finger.

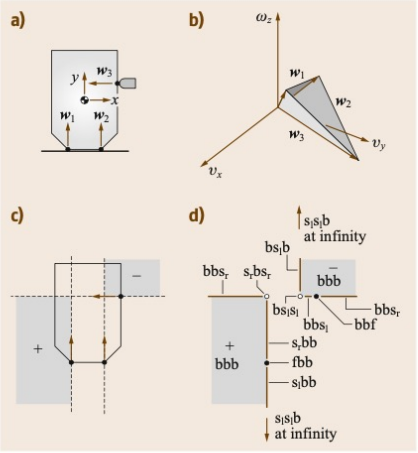


Fig. 37.2 **(a)** A part resting on a table and the three contact constraints. **(b)** The twist cone satisfying the contact constraints. The contact normals are shown with their associated constraint planes. **(c)** The equivalent representation as CORs, shown in *grey*. Note that the lines that extend off to the *left* and to the *bottom wrap around* at infinity and come back in from the *right* and the *top*, respectively, so this COR region should be interpreted as a single connected convex region, equivalent to the twist cone representation in **(b)**. **(d)** The contact modes assigned to each feasible motion. The zero velocity contact mode is fff

This finger is currently stationary, but we will later set it in motion. The finger defines one constraint on the part's motion and the table defines two more. (Note that contact points in the interior of the edge between the part and the table provide redundant kinematic constraints.) The constraint wrenches can be written

$$\boldsymbol{w}_1 = (0, 0, -1, 0, 1, 0)^{\mathrm{T}},$$
$$\boldsymbol{w}_2 = (0, 0, 1, 0, 1, 0)^{\mathrm{T}},$$
$$\boldsymbol{w}_3 = (0, 0, 1, -1, 0, 0)^{\mathrm{T}}.$$

For a stationary finger, the kinematic constraints yield the feasible twist cone shown in Fig. 37.2b. This region can also be easily visualized in the plane by using the following method: at each contact, draw the contact normal line. Label all points on the normal $\pm$, points to the left of the inward normal $+$, and points to the right $-$. For each contact constraint, all the points labeled $+$ can serve as CORs with positive angular velocity, and all the points labeled $-$ can serve as CORs with negative angular velocity, without violating the contact constraint. After doing this for all the contact normals, keep only the CORs that are consistently labeled. These CORs are a planar representation of the feasible twist cone (Fig. 37.2c).

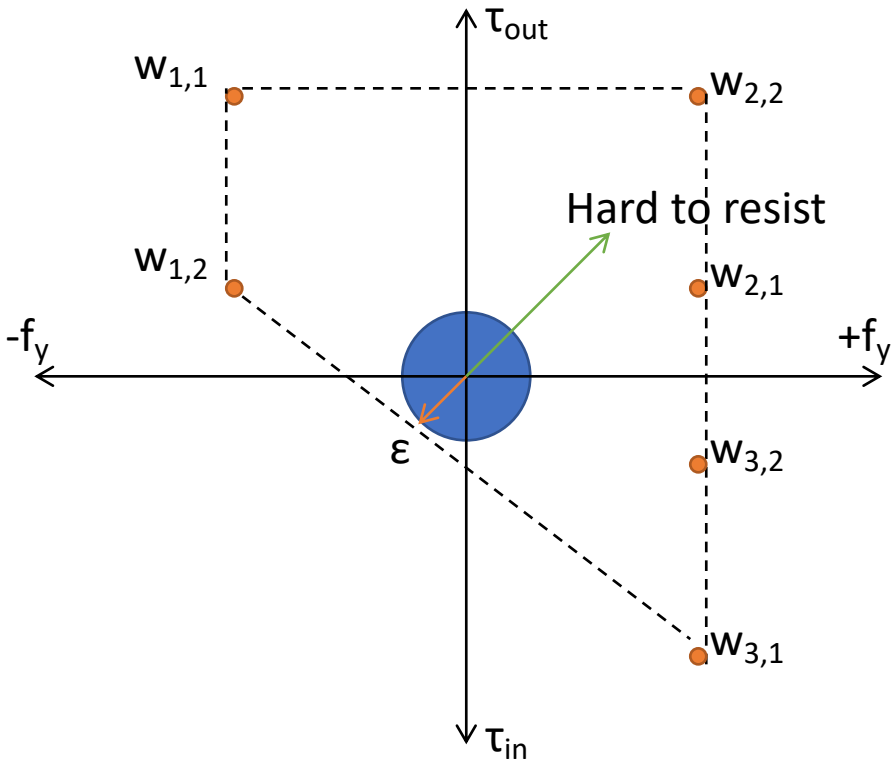
We can refine this method by assigning contact modes to each feasible COR. For each contact normal, label the COR at the contact point f for fixed, other CORs on the normal line s for slipping, and all other CORs b for breaking contact. The concatenation of these labels gives the part's contact mode for a particular part motion. In the planar case, the label s on the contact normal line can be further refined into s_r or s_l, indicating whether the part is slipping right or left relative to the constraint. An s COR labeled $+$ above the contact (in the direction of the contact normal) or $-$ below the contact should be relabeled s_r, and an s COR labeled $-$ above the contact or $+$ below should be relabeled s_l (Fig. 37.2d).

This method can be used readily to determine if the part is in form closure. If there is no COR labeled consistently, then the feasible velocity cone consists of only the zero velocity point, and the part is immobilized by the stationary contacts. This method also makes it clear that at least four contacts are necessary to immobilize the part by the first-order analysis (Chap. 38). This is a weakness of the first-order analysis – curvature effects can be used to immobilize a part with three or even two contacts [37.24]. This weakness can also be seen in Fig. 37.2d. A pure rotation about the COR labeled $\{+, s_r b s_r\}$ is actually not feasible, but it would be if the part had a small radius of curvature at the contact with the finger. The first-order analysis ignores this curvature.

Part D | 37.2

Grasp Force Optimization

- In force closure, you can **theoretically** resist any wrench
- But what forces do you need to apply at each contact to generate the desired wrench?



Motivating Example

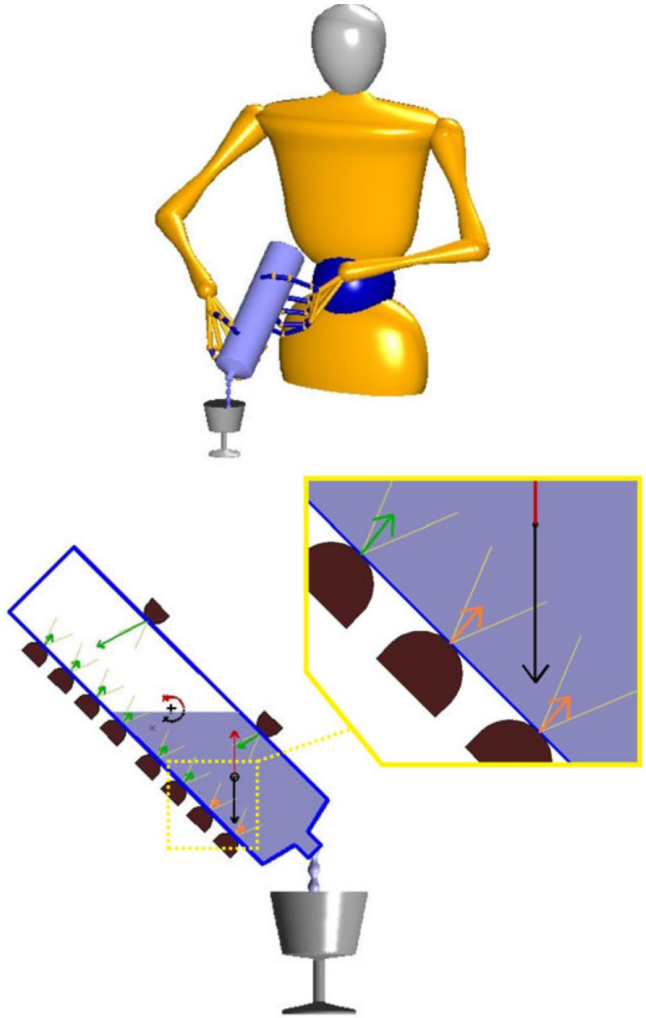


Fig. 3. Sequence of significant configurations of the bottle and of the forces during task execution with $n = 10$.

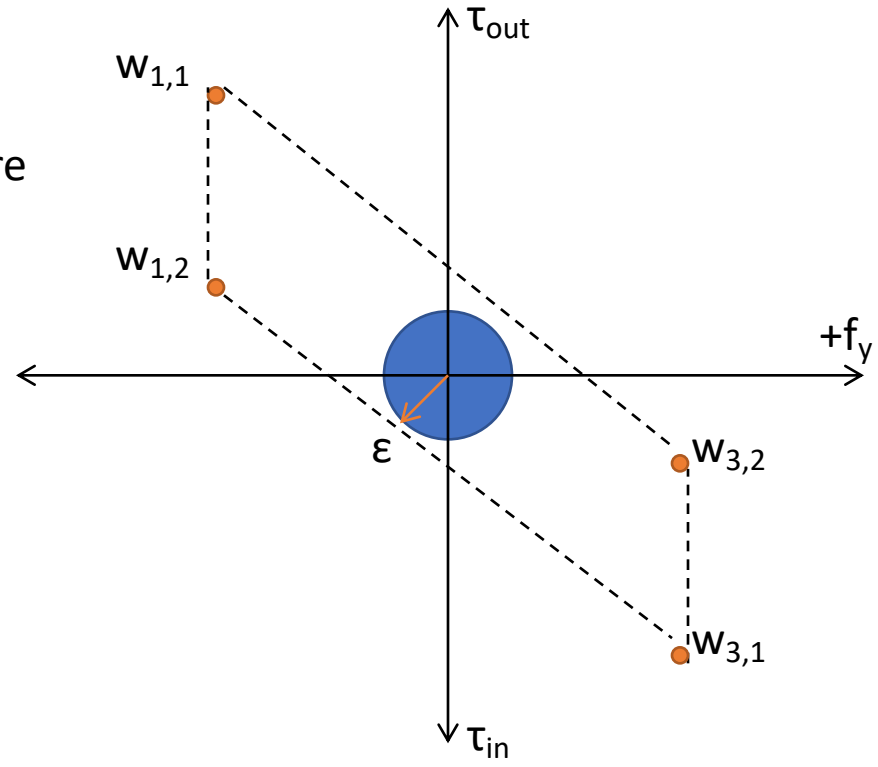
Figure adapted from *A Grasping Force Optimization Algorithm for Multiarm Robots With Multifingered Hands*. Lipiello et al. Transactions on Robotics. 2013

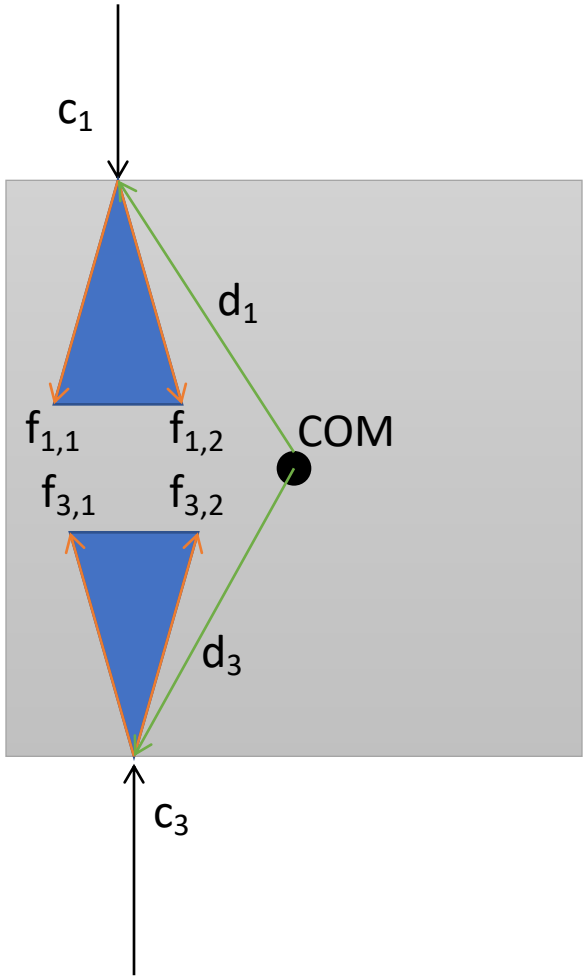
Formalizing the problem

- M contact points at $c^{(i)}$
- $f^{(i)}$ is the contact force applied at contact point
- Local coordinate system where x, y are tangent to surface and z is aligned with surface normal pointing inward
- $f^i = (f_x^{(i)}, f_y^{(i)}, f_z^{(i)})$
- Friction cone

$$\sqrt{f_x^{(i)2} + f_y^{(i)2}} \leq \mu_i f_z^{(i)}$$

- or in planar case: $f_x^{(i)} \leq \mu_i f_y^{(i)}$





Following Approach in *Fast Computation of Optimal Contact Forces* by Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.

Friction Cone Constraints

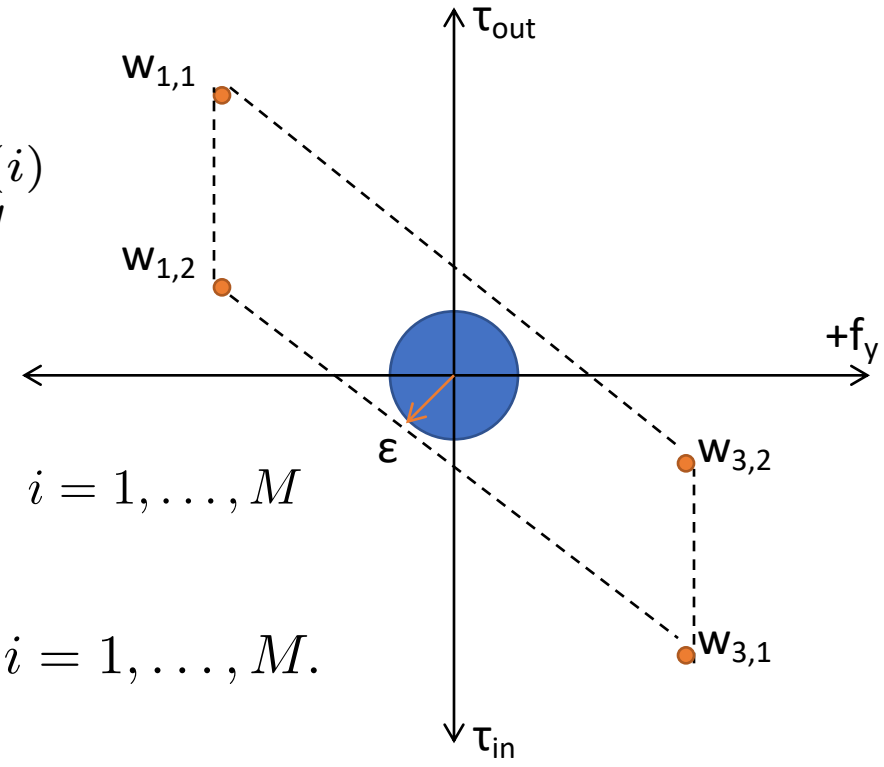
$$\sqrt{f_x^{(i)2} + f_y^{(i)2}} \leq \mu_i f_z^{(i)}$$

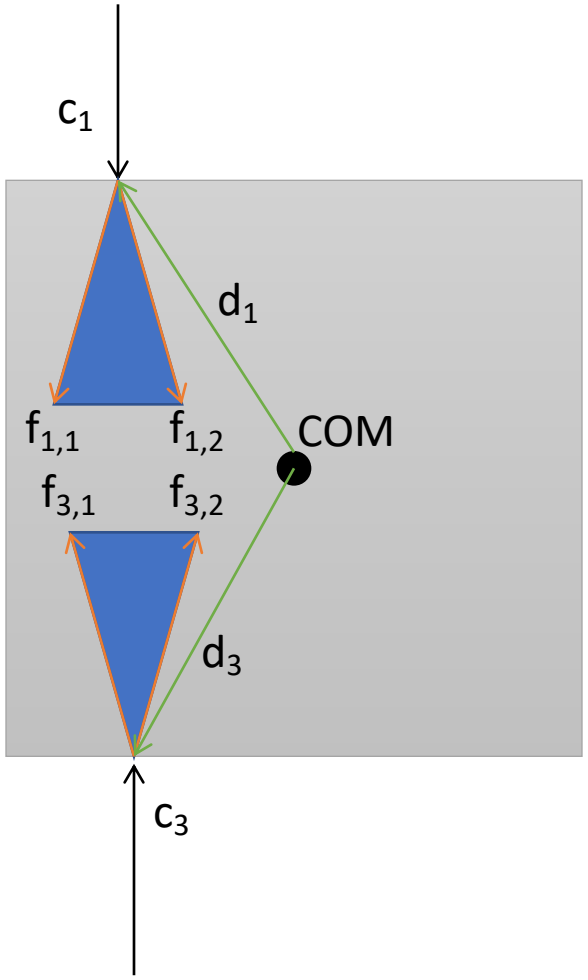
- or in planar case: $f_x^{(i)} \leq \mu_i f_y^{(i)}$

- Second-order cone constraints

$$K_i = \left\{ x \in \mathbf{R}^3 \;\middle|\; \sqrt{x_1^2 + x_2^2} \leq \mu_i x_3 \right\}, \qquad i = 1, \ldots, M$$

- Compact notation $f^{(i)} \in K_i, \qquad i = 1, \ldots, M.$





Following Approach in *Fast Computation of Optimal Contact Forces* by Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.

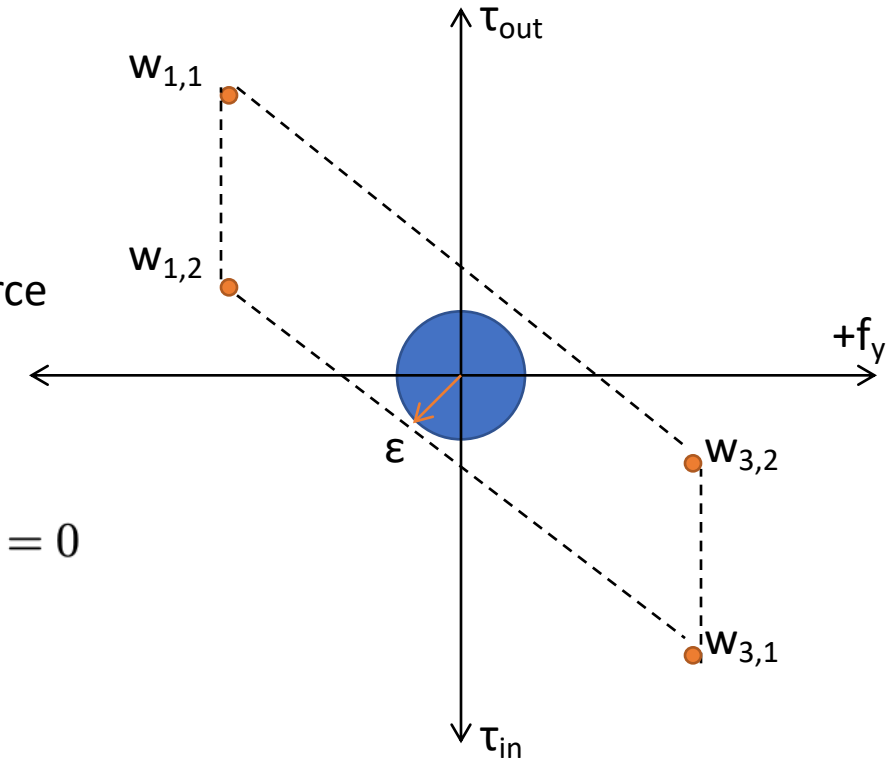
Equilibrium Constraints – Force

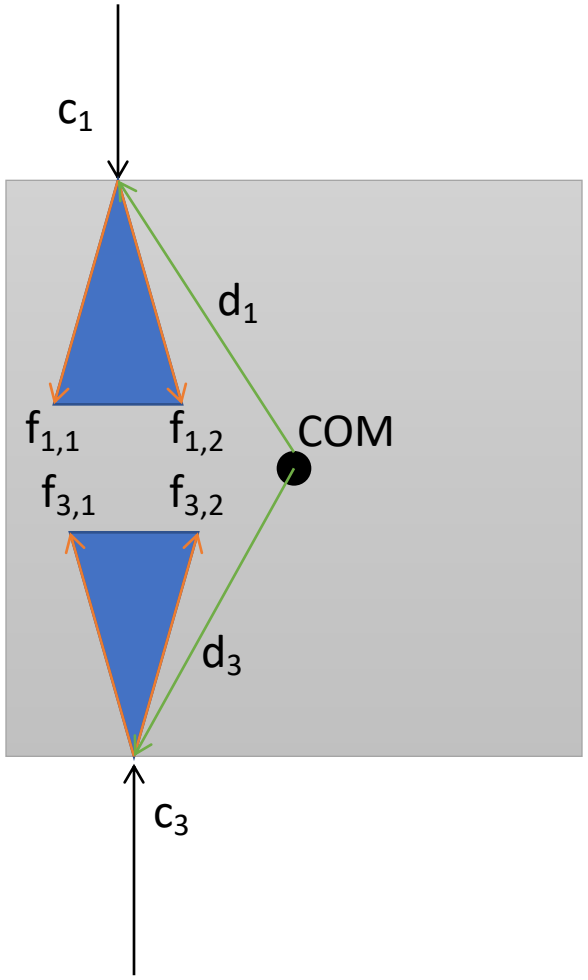
- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $Q^{(i)} f^{(i)}$ = force applied to object

- Applied forces need to generate a force that compensates external force

$$f^{\text{ext}} \in \mathbf{R}^3$$

$$Q^{(1)} f^{(1)} + \cdots + Q^{(M)} f^{(M)} + f^{\text{ext}} = 0$$

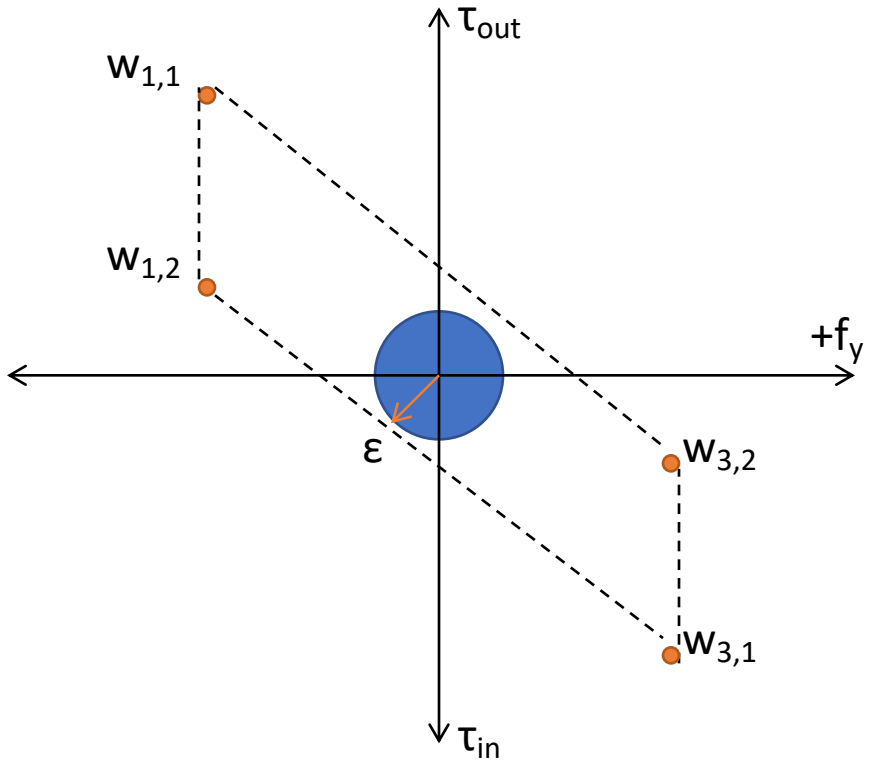


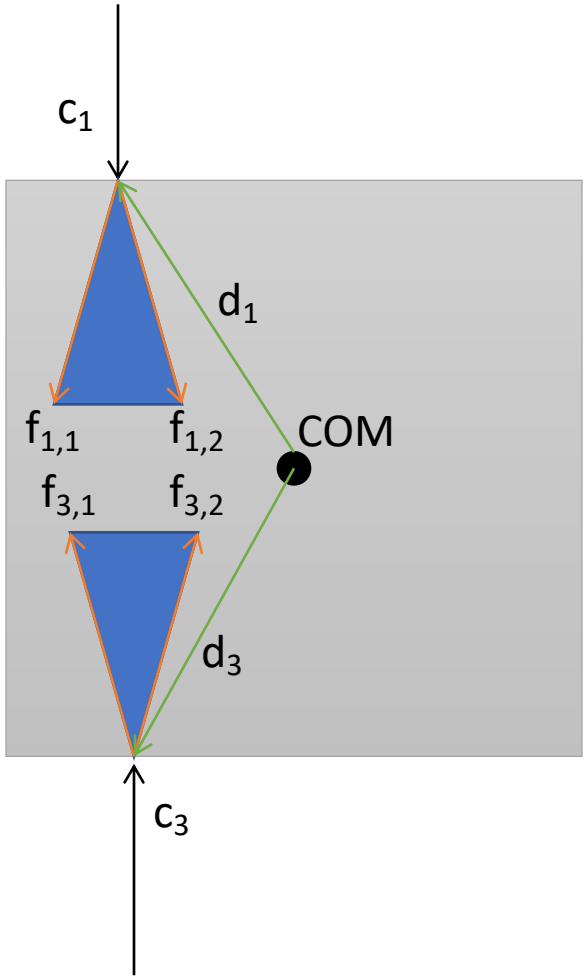


Following Approach in *Fast Computation of Optimal Contact Forces* by Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.

Equilibrium Constraints – Torque

- $Q \in SO(3)$ transforms forces from local to global coordinate system
- $d^{(i)} \times Q^{(i)} f^{(i)}$ = torque applied to object
- Applied forces need to generate a torque that compensates external torque
- $d^{(1)} \times Q^{(1)} f^{(1)} + \ldots + d^{(M)} \times Q^{(M)} f^{(M)} + \tau^{ext} = 0$





Following Approach in *Fast Computation of Optimal Contact Forces* by Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.

Matrix Notation of Cross Product

- $d^{(1)} \times Q^{(1)} f^{(1)} + \ldots + d^{(M)} \times Q^{(M)} f^{(M)} + \tau^{ext} = 0$
- $S^{(1)} \quad Q^{(1)} f^{(1)} + \ldots + S^{(M)} \quad Q^{(M)} f^{(M)} + \tau^{ext} = 0$
- Where
  - $S^{(i)} = \begin{matrix} 0 & -d_z^{(i)} & d_y^{(i)} \\ d_z^{(i)} & 0 & -d_x^{(i)} \\ -d_y^{(i)} & d_x^{(i)} & 0 \end{matrix} \in skew(3)$ where $S^i x = d^i \times x$

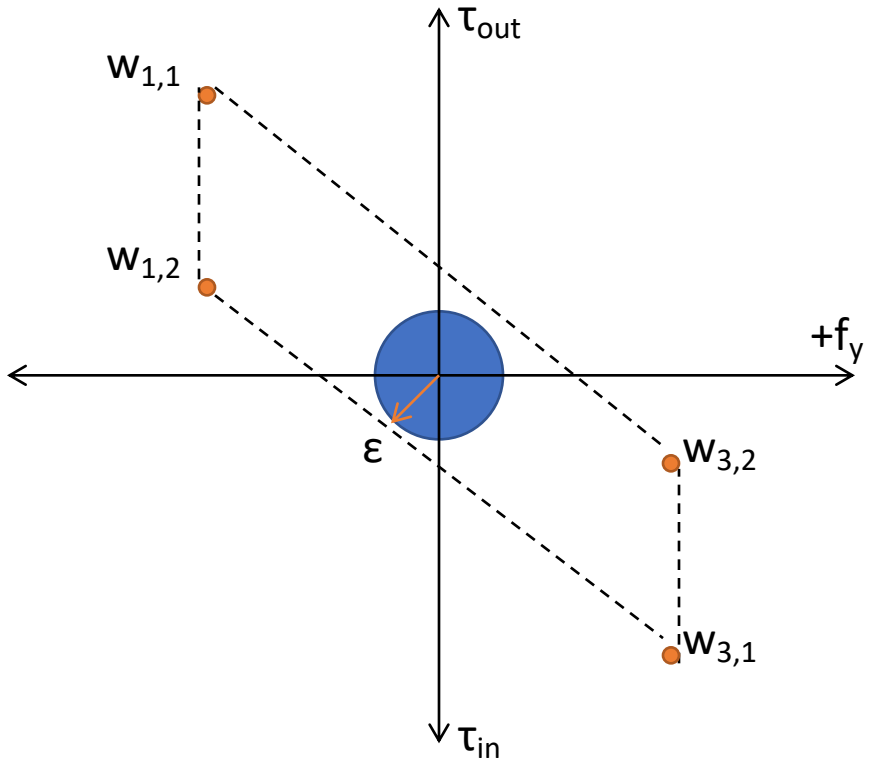
Equilibrium Constraints – Force Closure

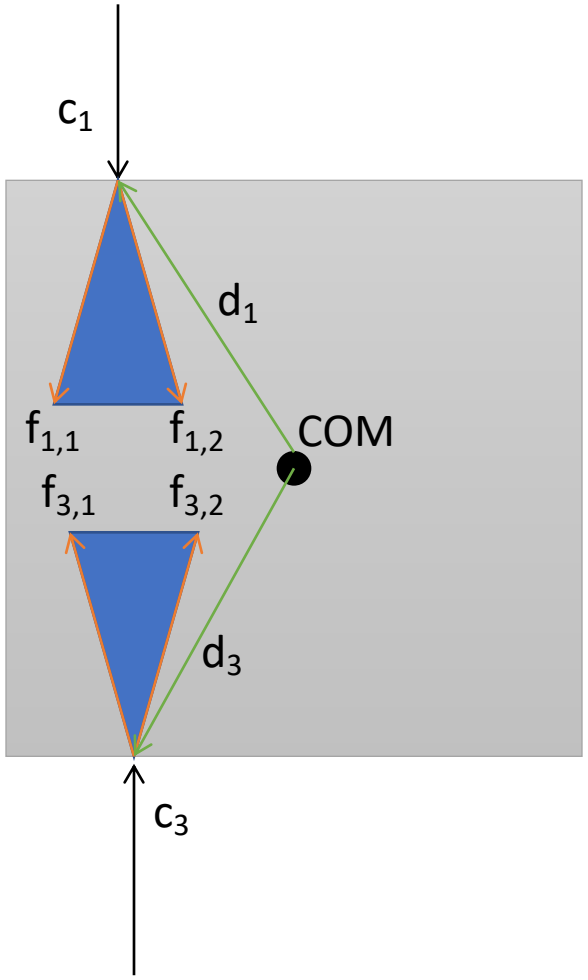
Compact notation

- Contact force vector $f \in \boldsymbol{R}^{3M}$

$$f = (f^{(1)}, \ldots, f^{(M)})$$

- Contact Matrices $G_i \in \boldsymbol{R}^{6x3}$
  - $G_i = \begin{matrix} Q^{(i)} \\ S^{(i)}Q^{(i)} \end{matrix}, i = 1 \ \ldots M$
- Grasp matrix
  - $G = [G_1, \ldots, G_M] \in \boldsymbol{R}^{6x3M}$
- External Wrench $\omega^{ext} = (f^{ext}, \tau^{ext})$
- Equilibrium conditions
  - $Gf + \omega^{ext} = 0$





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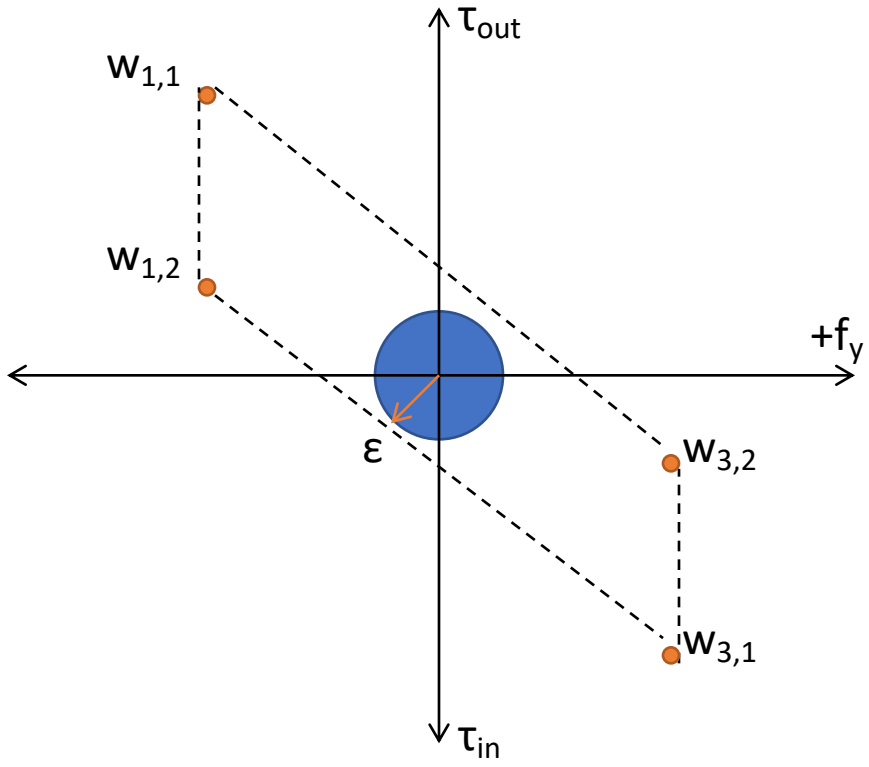
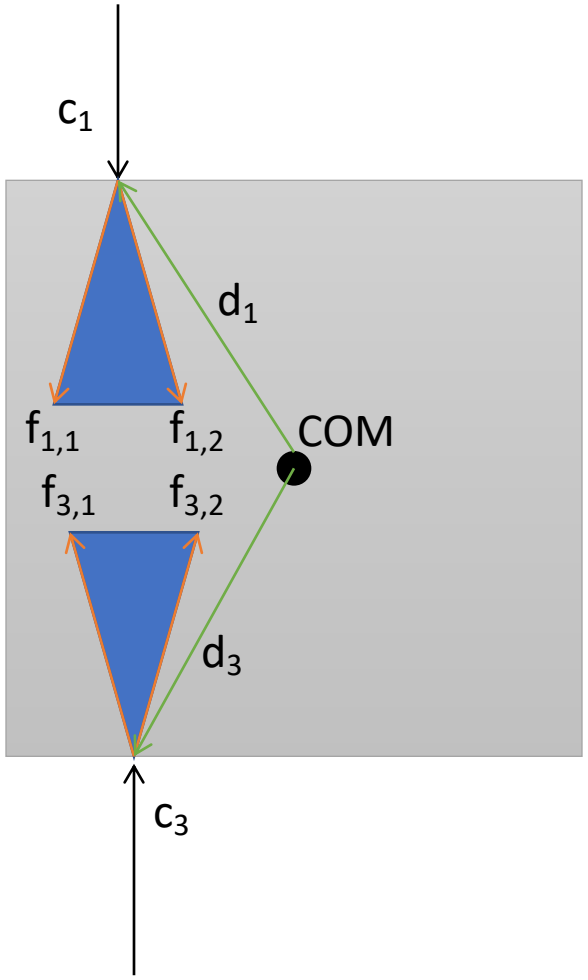
Other Constraints Constraints

Hardware constraints (max torque, kinematic limits).

$$f \in \mathcal{C}^{\text{other}}$$





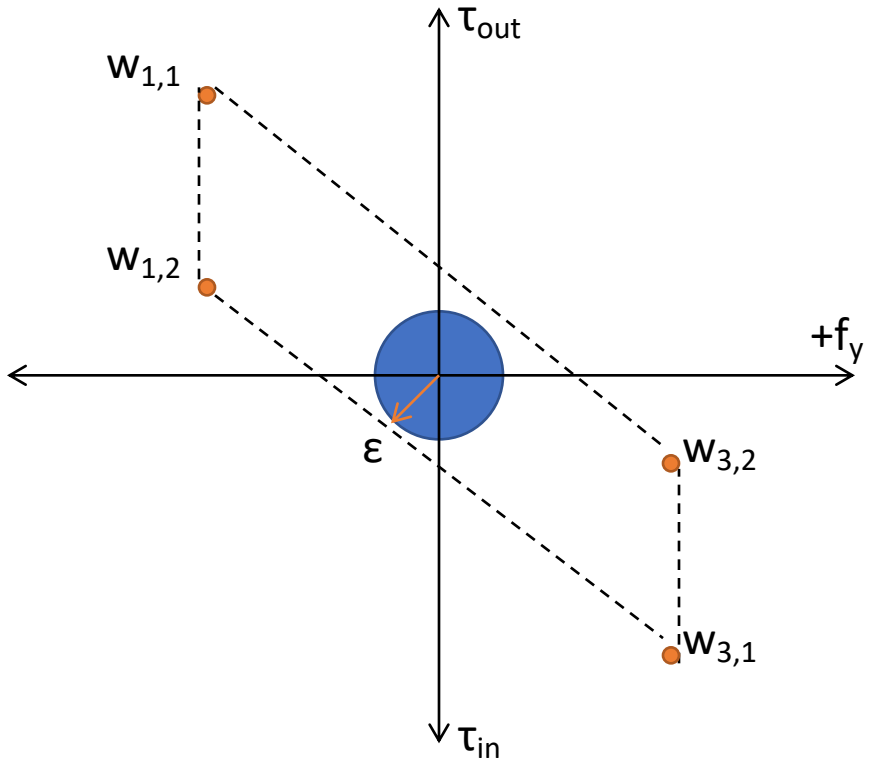
Following Approach in *Fast Computation of Optimal Contact Forces* by Stephen P. Boyd and Ben Wegbreit. Transactions on Robotics. 2007.

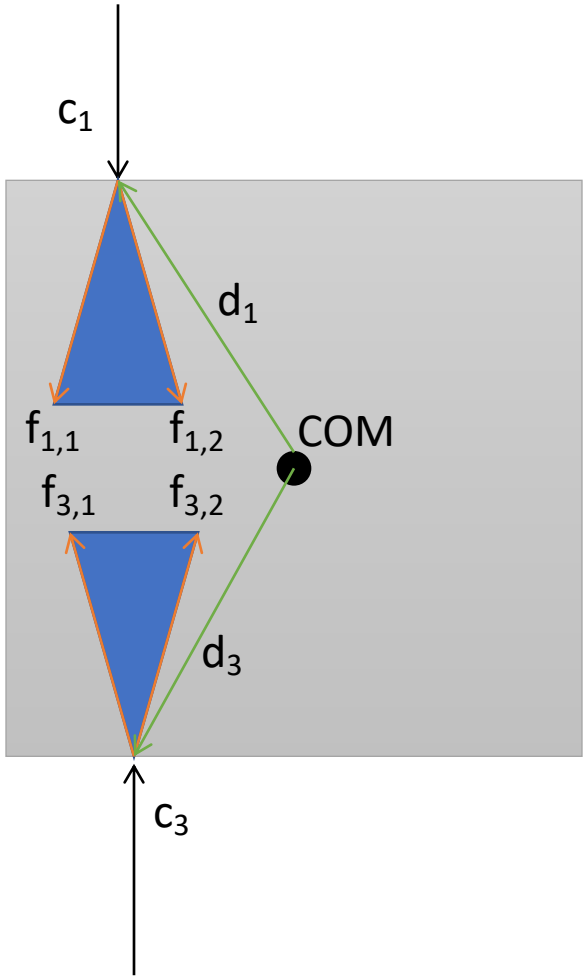
Convex Optimization Problem

- Second-order cone program because friction cones are quadratic.
- Objective function:

$$F^{\max} = \max\{\|f^{(1)}\|, \ldots, \|f^{(M)}\|\}$$
$$= \max_{i=1,\ldots,M} \sqrt{f_x^{(i)2} + f_y^{(i)2} + f_z^{(i)2}}$$

- Optimization problem:
  - $minimize\ F^{max}$
  - $subject\ to\ f^{(i)} \in K_i, i = 1 \ldots M$
  - $Gf + \omega^{ext} = 0$





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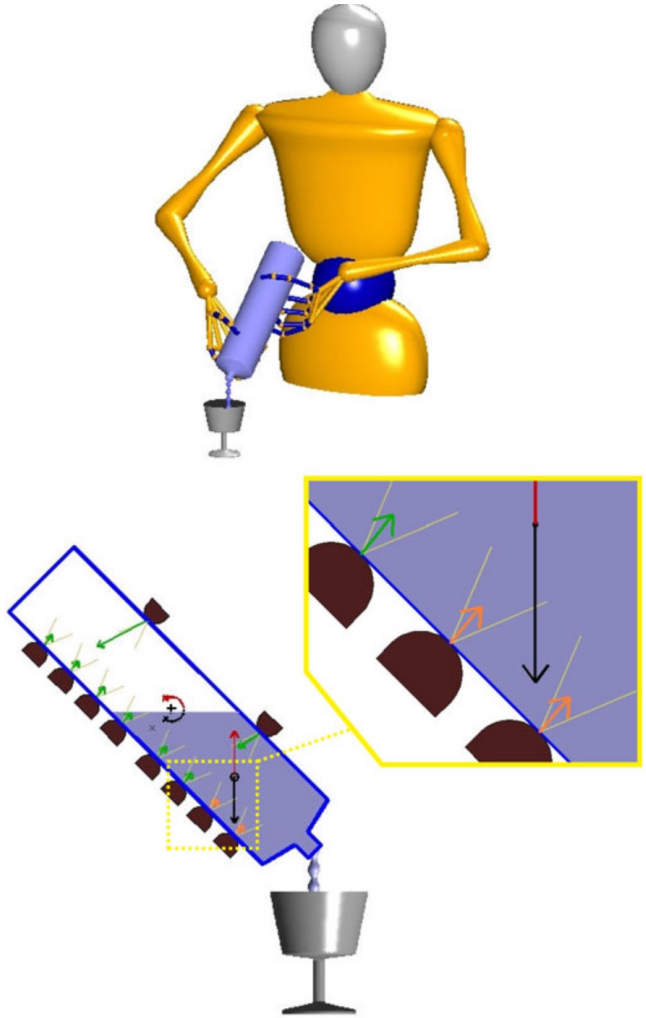
Motivating Example



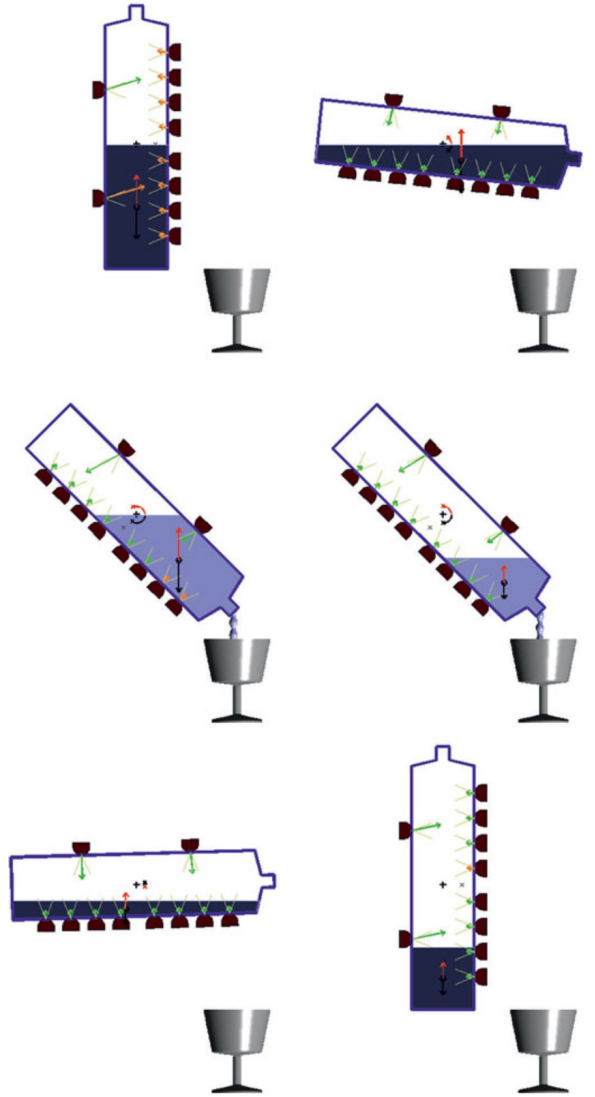


Fig. 3. Sequence of significant configurations of the bottle and of the forces during task execution with $n = 10$.

Figure adapted from A Grasping Force Optimization Algorithm for Multiarm Robots With Multifingered Hands. Lipiello et al. Transactions on Robotics. 2013

Manipulation through Contact

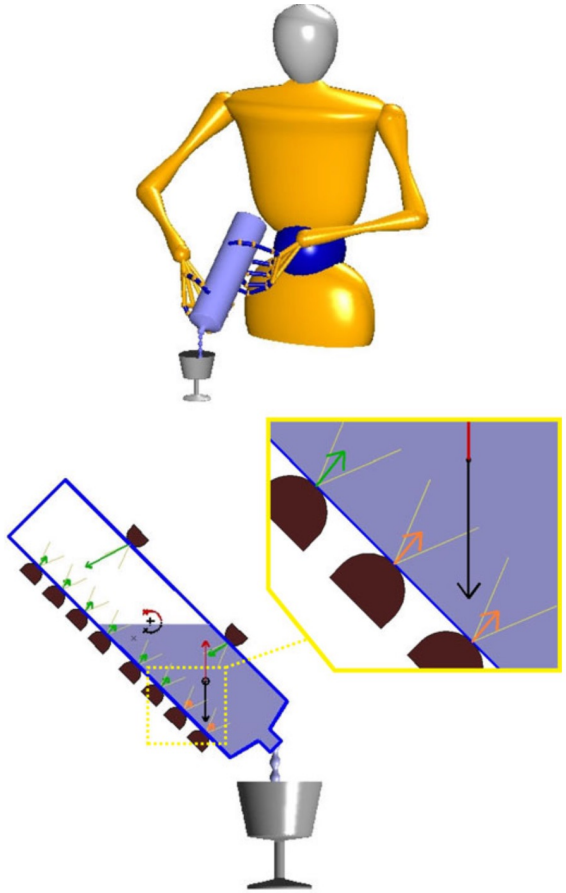
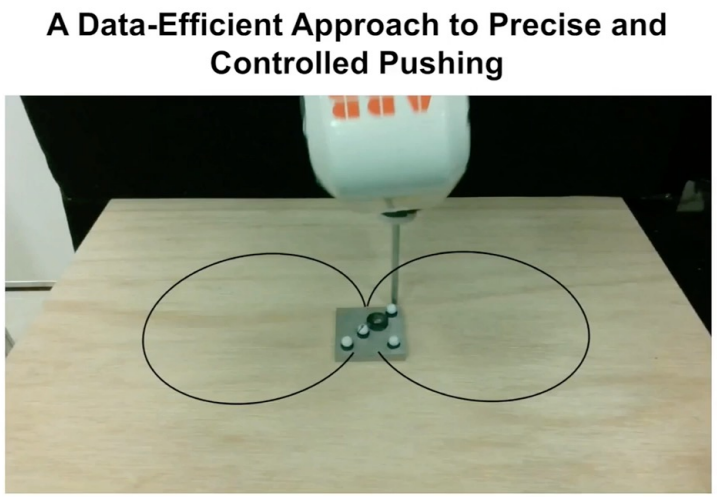


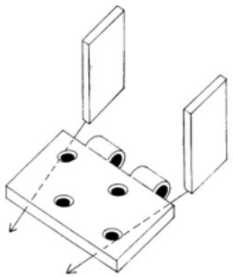
Figure adapted from A Grasping Force Optimization Algorithm for Multiarm Robots With Multifingered Hands. Lipiello et al. Transactions on Robotics. 2013

Learning Hierarchical Control for Robust In-Hand Manipulation. Li et al. ICRA 2020.

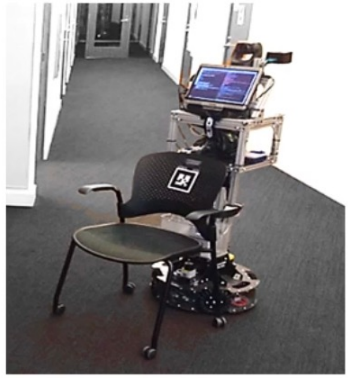


A Data-Efficient Approach to Precise and Controlled Pushing. Hogan et al. CORL 2018.

Case Study – Planar Pushing

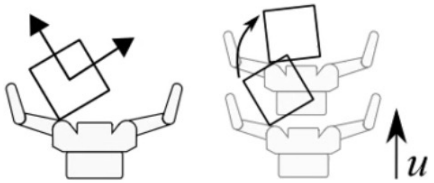


Reorient parts
- Mason 1986



Transport **large objects**
- Meriçli 2015

Push-grasp under **clutter**
- Dogar 2010



Track object **pose**
- Koval 2015

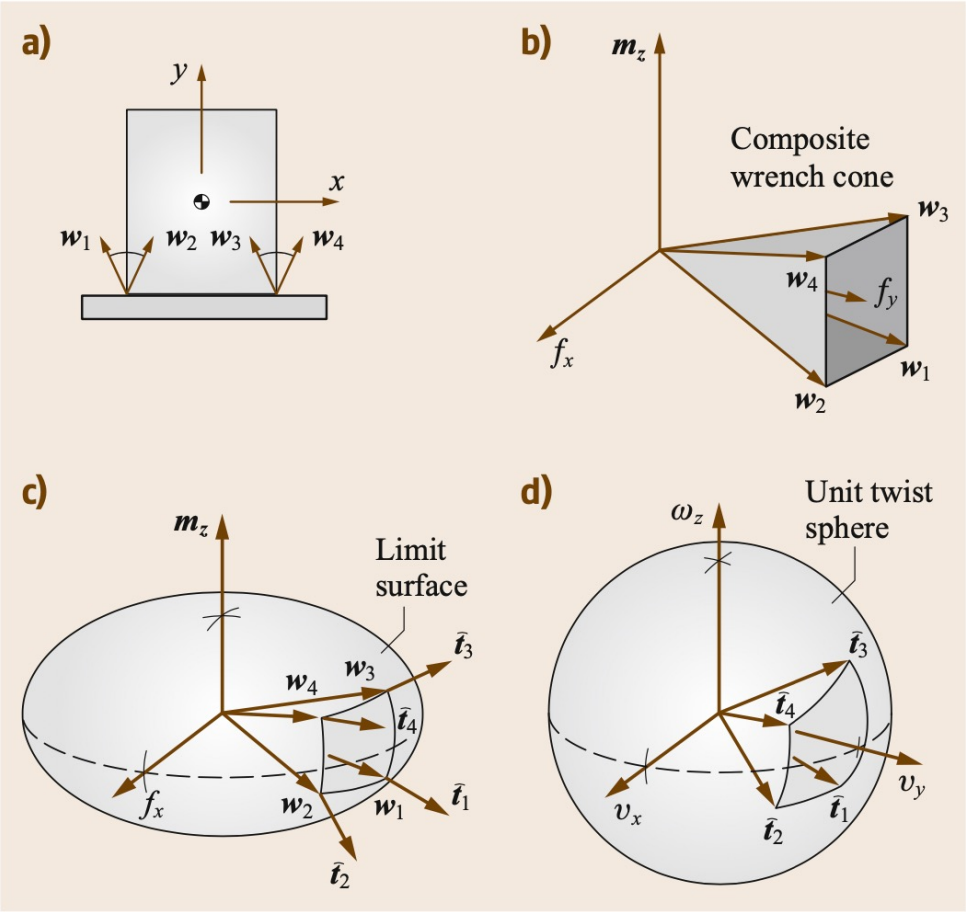
Modeling Planar Pushing

Friction limit surface: describes friction forces occurring when part slides over support.

When pushed with a wrench within the limit surface: **no motion.**

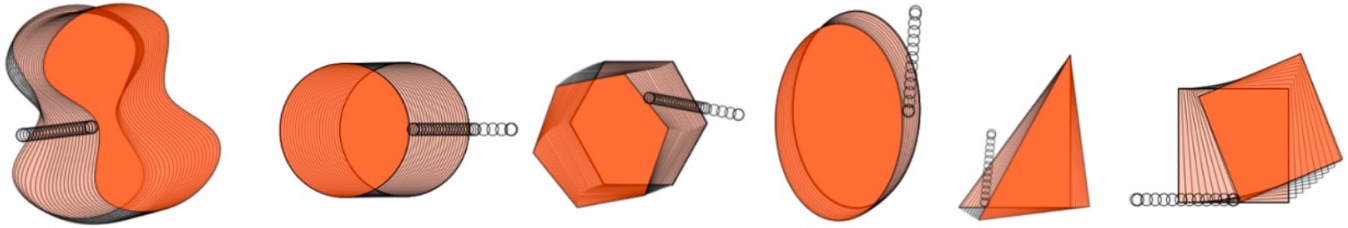
For **quasi-static pushing**: wrench on the limit surface; object twist normal to limit surface where **twist** = linear and angular velocity: $t_i = (v_x^i, v_y^i, \omega_z^i)$

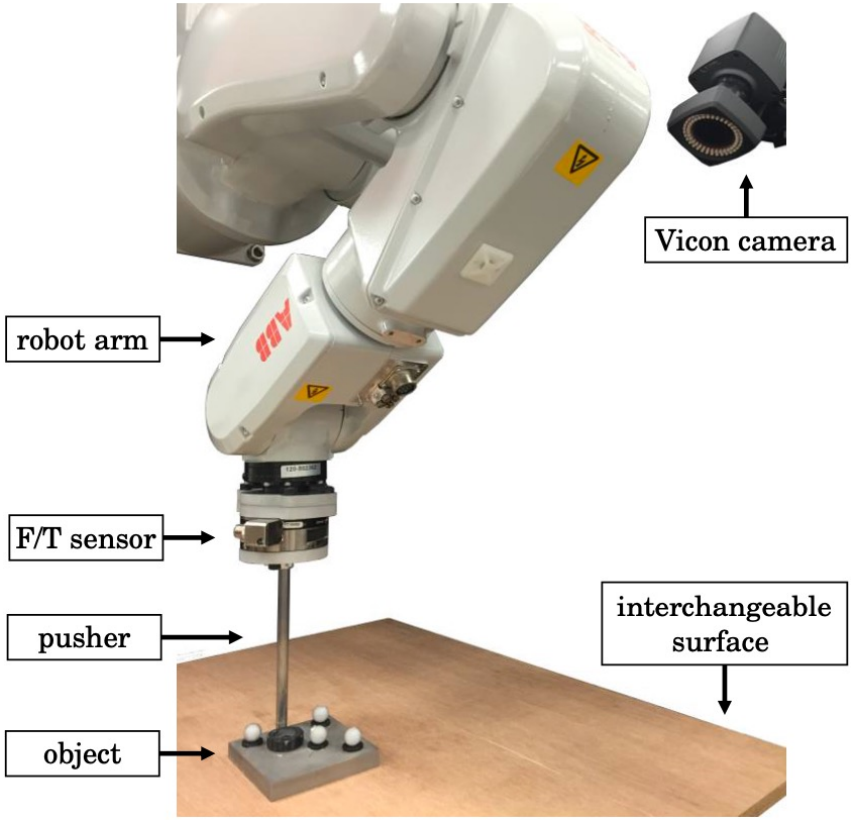
If **object translates without rotation** the friction force magnitude $\mu m g$ where μ = friction coefficient, m = object mass, g = gravitational acceleration



Relation between wrench cone, limit surface and unit twist sphere. Adopted from Chapter 37, Fig 37.10 in Springer Handbook of Robotics.

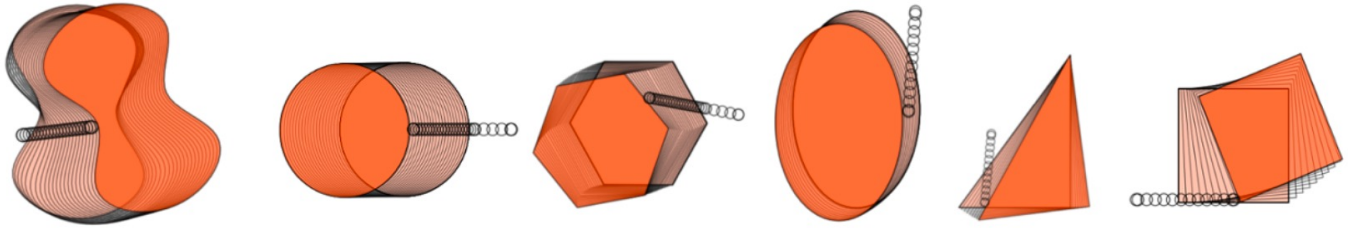
Validating Models for Planar Pushing

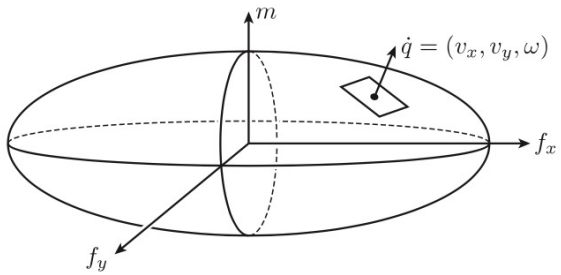




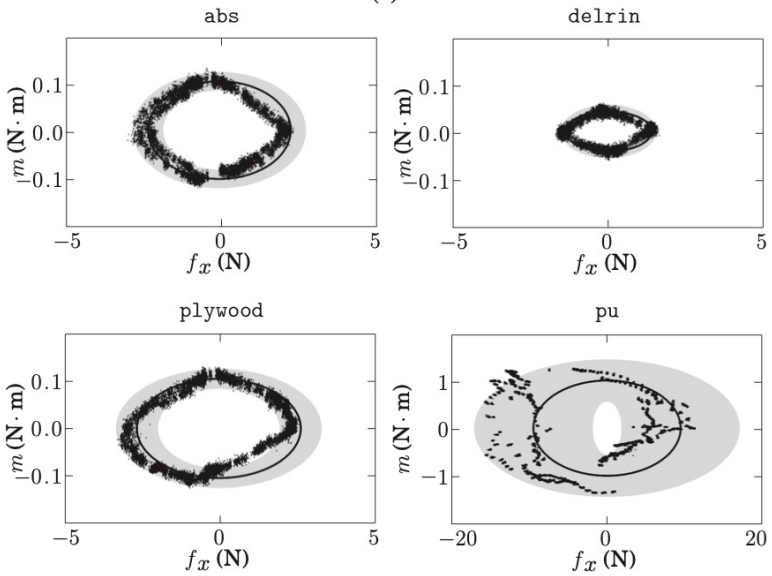
IROS 2016, "More than a Million Ways to Be Pushed: A High-Fidelity Experimental Dataset of Planar Pushing" by Peter Yu, Maria Bauza et al.

Validating Models for Planar Pushing





(a)



(b)

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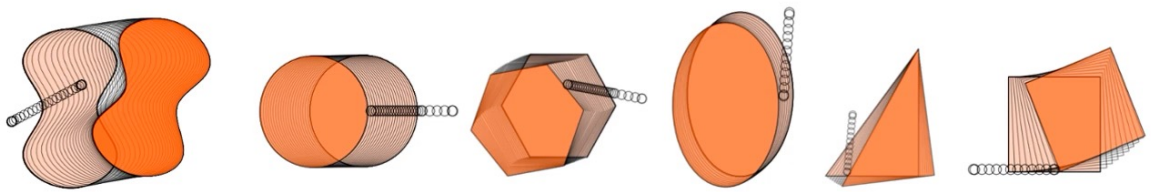
Validating Models for Planar Pushing

More than a Million Ways to Be Pushed.

A High-Fidelity Experimental Dataset of Planar Pushing



Kuan-Ting Yu, Maria Bauza, Nima Fazeli, and Alberto Rodriguez

Computer Science and Artificial Intelligence Lab &
Mechanical Engineering Department, MIT

MCube Lab

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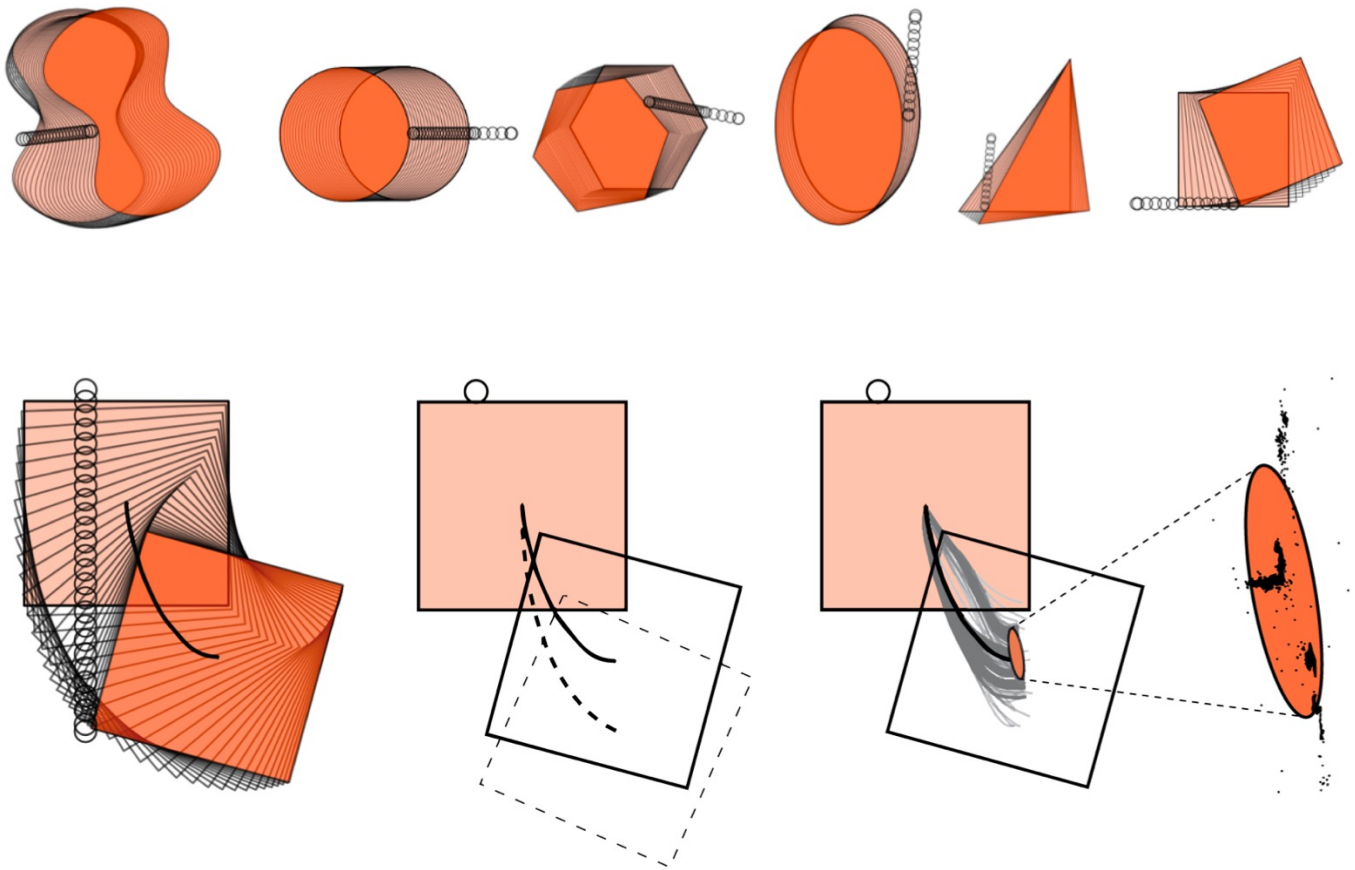

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Suggested Reading

- *Fast Computation of Optimal Contact Forces* by Boyd and Wegbreit. TRO 2007

Next time

- Learning-based approaches to Grasping and Manipulation