Query Execution 2 and Query Optimization

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Query Execution Overview

Query representation
(e.g. SQL)

Logical query plan
(e.g. relational algebra)

Optimized logical plan

Physical plan
(code/operators to run)
Example SQL Query

```
SELECT title 
FROM StarsIn 
WHERE starName IN ( 
    SELECT name 
    FROM MovieStar 
    WHERE birthdate LIKE ‘%1960’ 
) ;

(Find the movies with stars born in 1960)
```
Parse Tree

```
SELECT <SelList> FROM <FromList> WHERE <Condition>

<table>
<thead>
<tr>
<th>Attribute</th>
<th>RelName</th>
<th>Tuple</th>
<th>Query</th>
</tr>
</thead>
<tbody>
<tr>
<td>title</td>
<td>StarsIn</td>
<td></td>
<td></td>
</tr>
<tr>
<td>starName</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SELECT <SelList> FROM <FromList> WHERE <Condition>

<table>
<thead>
<tr>
<th>Attribute</th>
<th>RelName</th>
<th>Attribute</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>name</td>
<td>MovieStar</td>
<td>birthDate</td>
<td>'%1960'</td>
</tr>
</tbody>
</table>
```
Logical Query Plan

\[ \Pi_{\text{title}} \]
\[ \sigma_{\text{starName}=\text{name}} \]
\[ \times \]
\[ \Pi_{\text{name}} \]
\[ \sigma_{\text{birthdate LIKE} \%1960} \]
\[ \times \]
\[ \Pi_{\text{MovieStar}} \]
Improved Logical Query Plan

\[ \Pi_{\text{title}} \]
\[ \sigma_{\text{birthdate LIKE } '1960'} \]
\[ \Pi_{\text{name}} \]
\[ \Pi_{\text{starName=name}} \]
\[ \text{StarsIn} \]
\[ \text{MovieStar} \]
Estimate Result Sizes

Need expected size

StarsIn

\(\Pi\)

\(\sigma\)

MovieStar
One Physical Plan

Hash join

Parameters: join order, memory size, project attributes, ...

Seq scan

StarsIn

H

Index scan

MovieStar

Parameters: select condition, ...
Another Physical Plan

Hash join

Index scan
StarsIn

Seq scan
MovieStar

Parameters: join order, memory size, project attributes, ...

Parameters: select condition, ...
Another Physical Plan

Sort-merge join

Seq scan
StarsIn

Seq scan
MovieStar
Estimating Plan Costs

Logical plan

\[ P_1 \quad P_2 \quad \ldots \quad P_n \]

Physical plan candidates

\[ C_1 \quad C_2 \quad \ldots \quad C_n \]

Pick best!
Execution Methods: Once We Have a Plan, How to Run it?

Several options that trade between complexity, performance and startup time
Example: Simple Query

```sql
SELECT quantity * price
FROM orders
WHERE productId = 75
```

\[\Pi_{\text{quantity*price}} (\sigma_{\text{productId}=75} (\text{orders}))\]
Method 1: Interpretation

interface Operator {
    Tuple next();
}

class TableScan: Operator {
    String tableName;
}

class Select: Operator {
    Operator parent;
    Expression condition;
}

class Project: Operator {
    Operator parent;
    Expression[] exprs;
}

interface Expression {
    Value compute(Tuple in);
}

class Attribute: Expression {
    String name;
}

class Times: Expression {
    Expression left, right;
}

class Equals: Expression {
    Expression left, right;
}
class Attribute: Expression {
    String name;
    
    Value compute(Tuple in) {
        return in.getField(name);
    }
}

class Times: Expression {
    Expression left, right;
    
    Value compute(Tuple in) {
        return left.compute(in) * right.compute(in);
    }
}
class TableScan: Operator {
    String tableName;

    Tuple next() {
        // read & return next record from file
        return new Tuple();
    }
}

class Project: Operator {
    Operator parent;
    Expression[] exprs;

    Tuple next() {
        tuple = parent.next();
        fields = [expr.compute(tuple) for expr in exprs];
        return new Tuple(fields);
    }
}
Running Our Query with Interpretation

ops = Project(
    expr = Times(Attr("quantity"), Attr("price")),
    parent = Select(
        expr = Equals(Attr("productId"), Literal(75)),
        parent = TableScan("orders")
    )
);

while(true) {
    Tuple t = ops.next();
    if (t != null) {
        out.write(t);
    } else {
        break;
    }
}
Method 2: Vectorization

Interpreting query plans one record at a time is simple, but it’s too slow

» Lots of virtual function calls and branches for each record (recall Jeff Dean’s numbers)

Keep recursive interpretation, but make Operators and Expressions run on batches
Implementing Vectorization

class TupleBatch {
    // Efficient storage, e.g.
    // schema + column arrays
}

interface Operator {
    TupleBatch next();
}

class Select: Operator {
    Operator parent;
    Expression condition;
}

...
Typical Implementation

Values stored in columnar arrays (e.g. int[]) with a separate bit array to mark nulls

Tuple batches fit in L1 or L2 cache

Operators use SIMD instructions to update both values and null fields without branching
Pros & Cons of Vectorization

+ Faster than record-at-a-time if the query processes many records

+ Relatively simple to implement

– Lots of nulls in batches if query is selective

– Data travels between CPU & cache a lot
Method 3: Compilation

Turn the query into executable code
Compilation Example

\[ \Pi_{\text{quantity} \times \text{price}} (\sigma_{\text{productId}=75} (\text{orders})) \]

generated class with the right field types for orders table

Can also theoretically generate vectorized code
Pros & Cons of Compilation

+ Potential to get fastest possible execution
+ Leverage existing work in compilers
  – Complex to implement
  – Compilation takes time
  – Generated code may not match hand-written
What’s Used Today?

Depends on context & other bottlenecks

**Transactional databases (e.g. MySQL):**
mostly record-at-a-time interpretation

**Analytical systems (Vertica, Spark SQL):**
vectorization, sometimes compilation

**ML libs (TensorFlow):** mostly vectorization (the records are vectors!), some compilation
Query Optimization
Outline

What can we optimize?

Rule-based optimization

Data statistics

Cost models

Cost-based plan selection
Outline

What can we optimize?

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Data statistics

Cost models

Cost-based plan selection
What Can We Optimize?

**Operator graph:** what operators do we run, and in what order?

**Operator implementation:** for operators with several impls (e.g. join), which one to use?

**Access paths:** how to read each table?
   » Index scan, table scan, C-store projections, …
Typical Challenge

There is an exponentially large set of possible query plans

Access paths for table 1 \(\times\) Access paths for table 2 \(\times\) Algorithms for join 1 \(\times\) Algorithms for join 2 \(\times\) …

Result: we’ll need techniques to prune the search space and complexity involved
Outline

What can we optimize?

Rule-based optimization

Data statistics

Cost models

Cost-based plan selection
What is a Rule?

Procedure to replace part of the query plan based on a pattern seen in the plan

Example: When I see `expr OR TRUE` for an expression `expr`, replace this with `TRUE`
Implementing Rules

Each rule is typically a function that walks through query plan to search for its pattern

```java
void replaceOrTrue(Plan plan) {
    for (node in plan.nodes) {
        if (node instanceof Or) {
            if (node.right == Literal(true)) {
                plan.replace(node, Literal(true));
                break;
            }
        }
    }
    // Similar code if node.left == Literal(true)
}
```
Implementing Rules

Rules are often grouped into *phases*
  » E.g. simplify Boolean expressions, pushdown selects, choose join algorithms, etc

Each phase runs rules till they no longer apply

```java
plan = originalPlan;
while (true) {
  for (rule in rules) {
    rule.apply(plan);
  }
  if (plan was not changed by any rule) break;
}
```
Simple rules can work together to optimize complex query plans (if designed well):

\[
\text{SELECT * FROM users WHERE}
\]
\[
(age>=16 \&\& \text{loc}==\text{CA}) \text{ || (age>=16 \&\& \text{loc}==\text{NY}) || age>=18}
\]

\[
(age>=16) \&\& (\text{loc}==\text{CA} \text{ || loc}==\text{NY}) \text{ || age>=18}
\]

\[
(age>=16 \&\& \text{(loc IN (CA, NY)) || age>=18}
\]

\[
\text{age>=18 || (age>=16 \&\& \text{(loc IN (CA, NY))}}
\]
Example Extensible Optimizer

For Thursday, you’ll read about Spark SQL’s Catalyst optimizer

» Written in Scala using its pattern matching features to simplify writing rules
» >500 contributors worldwide, >1000 types of expressions, and hundreds of rules

We’ll also use Spark SQL in assignment 2
/*
 * Licensed to the Apache Software Foundation (ASF) under one or more
 * contributor license agreements. See the NOTICE file distributed with
 * this work for additional information regarding copyright ownership.
 * The ASF licenses this file to You under the Apache License, Version 2.0
 * (the "License"); you may not use this file except in compliance with
 * the License. You may obtain a copy of the License at
 * http://www.apache.org/licenses/LICENSE-2.0
 *
 * Unless required by applicable law or agreed to in writing, software
 * distributed under the License is distributed on an "AS IS" BASIS,
 * WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied.
 * See the License for the specific language governing permissions and
 * limitations under the License.
 */

package org.apache.spark.sql.catalyst.optimizer

import scala.collection.mutable

import org.apache.spark.sql.AnalysisException
import org.apache.spark.sql.catalyst.analysis._
import org.apache.spark.sql.catalyst.catalog.{InMemoryCatalog, SessionCatalog}
import org.apache.spark.sql.catalyst.expressions._
import org.apache.spark.sql.catalyst.expressions.aggregate._
abstract class Optimizer(sessionCatalog: SessionCatalog) {
    extends RuleExecutor[LogicalPlan] {

    def defaultBatches: Seq[Batch] = {
        val operatorOptimizationRuleSet = Seq(
            // Operator push down
            PushProjectionThroughUnion,
            ReorderJoin,
            EliminateOuterJoin,
            PushPredicateThroughJoin,
            PushDownPredicate,
            PushDownLeftSemiAntiJoin,
            PushLeftSemiLeftAntiThroughJoin,
            LimitPushDown,
            ColumnPruning,
            InferFiltersFromConstraints,
            // Operator combine
            CollapseRepartition,
            CollapseProject,
            CollapseWindow,
            CombineFilters,
            CombineLimits,
            CombineUnions,
            // Constant folding and strength reduction
            TransposeWindow,
            NullPropagation,
            ConstantPropagation,
            FoldablePropagation,
            OptimizeIn,
            ConstantFolding,
            ReorderAssociativeOperator,
            LikeSimplification,
            BooleanSimplification,
            SimplifyConditionals,
            RemoveDispensableExpressions,
            SimplifyBinaryComparison,
            ReplaceNullWithFalseInPredicate,
            PruneFilters,
Common Rule-Based Optimizations

Simplifying expressions in select, project, etc
» Boolean algebra, numeric expressions, string expressions, etc
» Many redundancies because queries are optimized for readability or generated by code

Simplifying relational operator graphs
» Select, project, join, etc

These relational optimizations have the most impact
Common Rule-Based Optimizations

Selecting access paths and operator implementations in simple cases
- Index column predicate ⇒ use index
- Small table ⇒ use hash join against it
- Aggregation on field with few values ⇒ use in-memory hash table

Rules also often used to do type checking and analysis (easy to write recursively)
Common Relational Rules

Push selects as far down the plan as possible

Recall:

\[ \sigma_p(R \bowtie S) = \sigma_p(R) \bowtie S \]  if p only references R

\[ \sigma_q(R \bowtie S) = R \bowtie \sigma_q(S) \]  if q only references S

\[ \sigma_{p \land q}(R \bowtie S) = \sigma_p(R) \bowtie \sigma_q(S) \]  if p on R, q on S

Idea: reduce # of records early to minimize work in later ops; enable index access paths
Common Relational Rules

Push projects as far down as possible

Recall:

\[ \Pi_x(\sigma_p(R)) = \Pi_x(\sigma_p(\Pi_{x \cup z}(R))) \quad \text{z = the fields in p} \]

\[ \Pi_{x \cup y}(R \bowtie S) = \Pi_{x \cup y}((\Pi_{x \cup z}(R)) \bowtie (\Pi_{y \cup z}(S))) \]

\[ \quad \text{x = fields in R, y = in S, z = in both} \]

Idea: don’t process fields you’ll just throw away
Project Rules Can Backfire!

Example:  
R has fields A, B, C, D, E  
p: A=3 ∧ B=“cat”  
x: {E}

\[ \Pi_x(\sigma_p(R)) \quad vs \quad \Pi_x(\sigma_p(\Pi_{\{A,B,E\}}(R))) \]
What if R has Indexes?

A = 3

B = “cat”

Intersect buckets to get pointers to matching tuples

In this case, should do $\sigma_p(R)$ first!
Many possible transformations aren’t always good for performance

Need more info to make good decisions

- **Data statistics**: properties about our input or intermediate data to be used in planning
- **Cost models**: how much time will an operator take given certain input data statistics?
Outline

What can we optimize?

Rule-based optimization

Data statistics

Cost models

Cost-based plan selection
What Are Data Statistics?

Information about the tuples in a relation that can be used to estimate size & cost

» Example: # of tuples, average size of tuples, # distinct values for each attribute, % of null values for each attribute

Typically maintained by the storage engine as tuples are added & removed in a relation

» File formats like Parquet can also have them
Some Statistics We’ll Use

For a relation R,

\[ T(R) = \# \text{ of tuples in } R \]
\[ S(R) = \text{average size of } R\text{'s tuples in bytes} \]
\[ B(R) = \# \text{ of blocks to hold all of } R\text{'s tuples} \]
\[ V(R, A) = \# \text{ distinct values of attribute } A \text{ in } R \]
# Example

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>cat</td>
<td>1</td>
<td>10</td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>cat</td>
<td>1</td>
<td>20</td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>dog</td>
<td>1</td>
<td>30</td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>dog</td>
<td>1</td>
<td>40</td>
<td>c</td>
<td></td>
</tr>
<tr>
<td>bat</td>
<td>1</td>
<td>50</td>
<td>d</td>
<td></td>
</tr>
</tbody>
</table>

A: 20 byte string
B: 4 byte integer
C: 8 byte date
D: 5 byte string
Example

\[
\begin{array}{|c|c|c|c|}
\hline
R: & A & B & C \\
\hline
\text{cat} & 1 & 10 & a \\
\text{cat} & 1 & 20 & b \\
\text{dog} & 1 & 30 & a \\
\text{dog} & 1 & 40 & c \\
\text{bat} & 1 & 50 & d \\
\hline
\end{array}
\]

T(R) = 5  
S(R) = 37  
V(R, A) = 3  
V(R, C) = 5  
V(R, B) = 1  
V(R, D) = 4

A: 20 byte string  
B: 4 byte integer  
C: 8 byte date  
D: 5 byte string
Challenge: Intermediate Tables

Keeping stats for tables on disk is easy, but what about intermediate tables that appear during a query plan?

Examples:

\[ \sigma_p(R) \]  \quad \text{We already have T(R), S(R), V(R, a), etc, but how to get these for tuples that pass p?}

\[ R \bowtie S \]  \quad \text{How many and what types of tuple pass the join condition?}

Should we do \((R \bowtie S) \bowtie T\) or \(R \bowtie (S \bowtie T)\) or \((R \bowtie T) \bowtie S\)?
Stat Estimation Methods

Algorithms to estimate subplan stats

An ideal algorithm would have:

1) Accurate estimates of stats
2) Low cost
3) Consistent estimates (e.g. different plans for a subtree give same estimated stats)

Can’t always get all this!
Size Estimates for $W = R_1 \times R_2$

$S(W) =$

$T(W) =$
Size Estimates for $W = R_1 \times R_2$

$S(W) = S(R_1) + S(R_2)$

$T(W) = T(R_1) \times T(R_2)$
Size Estimate for $W = \sigma_{A=a}(R)$

$S(W) =$

$T(W) =$
Size Estimate for $W = \sigma_{A=a}(R)$

$S(W) = S(R)$  \quad \text{Not true if some variable-length fields are correlated with value of } A$

$T(W) =$
Example

<table>
<thead>
<tr>
<th>R</th>
<th>A</th>
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<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>cat</td>
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</tr>
<tr>
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<td>b</td>
<td></td>
</tr>
<tr>
<td>dog</td>
<td>1</td>
<td>30</td>
<td>a</td>
<td></td>
</tr>
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</tr>
<tr>
<td>bat</td>
<td>1</td>
<td>50</td>
<td>d</td>
<td></td>
</tr>
</tbody>
</table>

V(R,A)=3  \quad V(R,B)=1  \quad V(R,C)=5  \quad V(R,D)=4

W = \sigma_{Z=\text{val}}(R)  \quad T(W) =
Example

\[ W = \sigma_{Z=\text{val}}(R) \]

\[ T(W) = \]

<table>
<thead>
<tr>
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<tr>
<td>bat</td>
<td>1</td>
<td>50</td>
<td>d</td>
<td></td>
</tr>
</tbody>
</table>

\[ V(R,A) = 3 \]
\[ V(R,B) = 1 \]
\[ V(R,C) = 5 \]
\[ V(R,D) = 4 \]

what is probability this tuple will be in answer?
Example

<table>
<thead>
<tr>
<th>R</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<td>d</td>
<td></td>
</tr>
</tbody>
</table>

$W = \sigma_{Z=\text{val}}(R)$

$T(W) = \frac{T(R)}{V(R,Z)}$

$V(R,A) = 3$

$V(R,B) = 1$

$V(R,C) = 5$

$V(R,D) = 4$
Assumption:

Values in select expression $Z=\text{val}$ are uniformly distributed over all $V(R, Z)$ values
Alternate Assumption:

Values in select expression $Z=\text{val}$ are uniformly distributed over a domain with $\text{DOM}(R, Z)$ values
**Example**

Alternate assumption

V(R,A)=3, DOM(R,A)=10
V(R,B)=1, DOM(R,B)=10
V(R,C)=5, DOM(R,C)=10
V(R,D)=4, DOM(R,D)=10

\[ W = \sigma_{Z=\text{val}}(R) \]

T(W) =
Example

\[ W = \sigma_{z=\text{val}}(R) \quad T(W) = \]

Alternate assumption
\[ V(R,A)=3, \text{ DOM}(R,A)=10 \]
\[ V(R,B)=1, \text{ DOM}(R,B)=10 \]
\[ V(R,C)=5, \text{ DOM}(R,C)=10 \]
\[ V(R,D)=4, \text{ DOM}(R,D)=10 \]

what is probability this tuple will be in answer?
Example

#### Alternate assumption

- $V(R,A) = 3$, $DOM(R,A) = 10$
- $V(R,B) = 1$, $DOM(R,B) = 10$
- $V(R,C) = 5$, $DOM(R,C) = 10$
- $V(R,D) = 4$, $DOM(R,D) = 10$

\[
W = \sigma_{Z=\text{val}}(R)
\]

\[
T(W) = \frac{T(R)}{DOM(R,Z)}
\]
Selection Cardinality

\[ SC(R, A) = \text{average # records that satisfy equality condition on R.A} \]

\[
SC(R,A) = \begin{cases} 
\frac{T(R)}{V(R,A)} \\
\frac{T(R)}{\text{DOM}(R,A)} 
\end{cases}
\]
What About $W = \sigma_{z \geq \text{val}(R)}$?

$T(W) = ?$
What About $W = \sigma_{z \geq val(R)}$?

$T(W) = ?$

Solution 1: $T(W) = T(R) / 2$
What About $W = \sigma_{z \geq \text{val}(R)}$?

$T(W) = ?$

Solution 1: $T(W) = T(R) / 2$

Solution 2: $T(W) = T(R) / 3$
Solution 3: Estimate Fraction of Values in Range

Example:

<table>
<thead>
<tr>
<th>R</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min=1</td>
</tr>
<tr>
<td></td>
<td>Max=20</td>
</tr>
</tbody>
</table>

\[ f = \frac{20 - 15 + 1}{20 - 1 + 1} = \frac{6}{20} \]  
(fraction of range)

\[ T(W) = f \times T(R) \]
Solution 3: Estimate Fraction of Values in Range

Equivalently, if we know values in column:

\[ f = \text{fraction of distinct values} \geq \text{val} \]

\[ T(W) = f \times T(R) \]
What About More Complex Expressions?

E.g. estimate selectivity for

SELECT * FROM R
WHERE user_defined_func(a) > 10
else if (is_funcclause(clause))
{
    /*
    * This is not an operator, so we guess at the selectivity. THIS IS A
    * HACK TO GET V4 OUT THE DOOR. FUNCS SHOULD BE ABLE TO HAVE
    * SELECTIVITIES THEMSELVES.        -- JMH 7/9/92
    */
    s1 = (Selectivity) 0.3333333;
}
Size Estimate for $W = R_1 \bowtie R_2$

Let $X = \text{attributes of } R_1$

$\quad Y = \text{attributes of } R_2$

Case 1: $X \cap Y = \emptyset$:

Same as $R_1 \times R_2$
Case 2: $W = R_1 \bowtie R_2$, $X \cap Y = A$

<table>
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<th>C</th>
<th></th>
<th>A</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Case 2: $W = R_1 \bowtie R_2, X \cap Y = A$

<table>
<thead>
<tr>
<th>$R_1$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>$R_2$</th>
<th>A</th>
<th>D</th>
</tr>
</thead>
</table>

Assumption ("containment of value sets"): 

$V(R_1, A) \leq V(R_2, A) \implies \text{Every A value in } R_1 \text{ is in } R_2$

$V(R_2, A) \leq V(R_1, A) \implies \text{Every A value in } R_2 \text{ is in } R_1$
**Computing $T(W)$ when $V(R_1, A) \leq V(R_2, A)$**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tbody>
<tr>
<td>$R_1$</td>
<td></td>
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<thead>
<tr>
<th></th>
<th>A</th>
<th>D</th>
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<tr>
<td>$R_2$</td>
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</table>

Take 1 tuple

1 tuple matches with $\frac{T(R_2)}{V(R_2, A)}$ tuples...

so $T(W) = \frac{T(R_1) \times T(R_2)}{V(R_2, A)}$
\[ V(R_1, A) \leq V(R_2, A) \implies T(W) = \frac{T(R_1) \times T(R_2)}{V(R_2, A)} \]

\[ V(R_2, A) \leq V(R_1, A) \implies T(W) = \frac{T(R_1) \times T(R_2)}{V(R_1, A)} \]
In General for $W = R_1 \bowtie R_2$

$$T(W) = \frac{T(R_1) \times T(R_2)}{\max(V(R_1, A), V(R_2, A))}$$

Where $A$ is the common attribute set
Case 2 with Alternate Assumption

Values uniformly distributed over domain

\[
\begin{array}{ccc}
R_1 & A & B & C \\
& & & \\
R_2 & A & D \\
\end{array}
\]

This tuple matches \( T(R_2) / \text{DOM}(R_2, A) \), so

\[
T(W) = \frac{T(R_1) \cdot T(R_2)}{\text{DOM}(R_2, A)} = \frac{T(R_1) \cdot T(R_2)}{\text{DOM}(R_1, A)}
\]

Assume these are the same
Tuple Size after Join

In all cases:

\[ S(W) = S(R_1) + S(R_2) - S(A) \]

size of attribute A
Using Similar Ideas, Can Estimate Sizes of:

$$\Pi_{AB}(R)$$

$$\sigma_{A=a \land B=b}(R)$$

$$R \bowtie S$$ with common attributes A, B, C

Set union, intersection, difference, …
For Complex Expressions, Need Intermediate $T$, $S$, $V$ Results

E.g.  $W = \sigma_{A=a}(R_1) \bowtie R_2$

Treat as relation $U$

$T(U) = T(R_1) / V(R_1, A)$  \hspace{2cm}  $S(U) = S(R_1)$

Also need $V(U, *)$!!
To Estimate V

E.g., $U = \sigma_{A=a}(R_1)$

Say $R_1$ has attributes A, B, C, D

$V(U, A) =$

$V(U, B) =$

$V(U, C) =$

$V(U, D) =$
Example

\[ \begin{array}{c|cccc}
A & B & C & D \\
\hline
\text{cat} & 1 & 10 & 10 \\
\text{cat} & 1 & 20 & 20 \\
\text{dog} & 1 & 30 & 10 \\
\text{dog} & 1 & 40 & 30 \\
\text{bat} & 1 & 50 & 10 \\
\end{array} \]

\[ V(R_1, A) = 3 \]
\[ V(R_1, B) = 1 \]
\[ V(R_1, C) = 5 \]
\[ V(R_1, D) = 3 \]

\[ U = \sigma_{A=a}(R_1) \]
### Example

$$V(R_1, A) = 3$$
$$V(R_1, B) = 1$$
$$V(R_1, C) = 5$$
$$V(R_1, D) = 3$$

$$U = \sigma_{A=a}(R_1)$$

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$$V(U, A) = 1 \quad V(U, B) = 1 \quad V(U, C) = \frac{T(R1)}{V(R1,A)}$$

$$V(U, D) = \text{somewhere in between}...$$
Possible Guess in $U = \sigma_{A \geq a}(R)$

$V(U, A) = \frac{V(R, A)}{2}$

$V(U, B) = V(R, B)$
For Joins: $U = R_1(A, B) \bowtie R_2(A, C)$

We’ll use the following estimates:

$V(U, A) = \min(V(R_1, A), V(R_2, A))$

$V(U, B) = V(R_1, B)$

$V(U, C) = V(R_2, C)$

Called “preservation of value sets”
Example:

\[ Z = R_1(A,B) \Join R_2(B,C) \Join R_3(C,D) \]

| R_1 | T(R_1) = 1000  V(R_1,A)=50  V(R_1,B)=100 |
| R_2 | T(R_2) = 2000  V(R_2,B)=200  V(R_2,C)=300 |
| R_3 | T(R_3) = 3000  V(R_3,C)=90   V(R_3,D)=500 |
Partial Result: $U = R_1 \bowtie R_2$

$T(U) = \frac{1000 \times 2000}{200} = \frac{2000000}{200} = 10000$  

$V(U,A) = 50$  

$V(U,B) = 100$  

$V(U,C) = 300$
End Result: \( Z = U \bowtie R_3 \)

\[
T(Z) = \frac{1000 \times 2000 \times 3000}{200 \times 300}
\]

\[
V(Z,A) = 50
\]
\[
V(Z,B) = 100
\]
\[
V(Z,C) = 90
\]
\[
V(Z,D) = 500
\]
Another Statistic: Histograms

number of tuples in R with A value in a given range

$\sigma_{A \geq a}(R) = ?$

$\sigma_{A = a}(R) = ?$

Requires some care to set bucket boundaries
Outline

What can we optimize?

Rule-based optimization

Data statistics

Cost models

Cost-based plan selection