Class 8 Exercises

CS250/EE387, Winter 2022

1. In the lecture videos/notes, we saw the "Kautz-Singleton" construction for group testing matrices, and we instantiated it using RS codes. Say that $N = 300$ and $d = 2$ and you want to build a group testing matrix like this. How will you choose parameters for q, k? What will your final group testing matrix look like? How many tests does it use? (Note: you may need to come up with a group testing matrix for $N' > N$ items, and then drop some items, since 300 is not a power of a prime).

2. In this problem we will adapt the Kautz-Singleton construction from the lecture videos/notes to deal with false negatives and false positives. The set-up is the same: we have N items, at most d of which are positive, and we wish to make T tests. However, now there may be up to E false negatives and E false positives. (Here, a "false positive" is a test that does not contain any positive items but comes up positive anyway; a "false negative" is a test that does contain a positive item but comes up negative).

   (a) Come up with a condition that is similar to d-disjunctness and prove a statement like "if a pooling matrix Φ satisfies [your condition], then Φ can identify up to d positive items, even with up to E false positives and E false negatives. Assume that the false negatives/positives are worst-case.

   (b) Adapt the Kautz-Singleton argument to show that RS-code-based group testing schemes can handle false positives/negatives. How do the parameters depend on E? (Note: you don't need to change the construction, just the parameters). Your final answer should be of the form "the number of tests T needs to be at least [some function of N, d, and E]."

3. **(Bonus – if you finish early, here's something else to work on!)** Can you come up with a way to set parameters in the Kautz-Singleton construction to get good results when, say, $d = N/100$? (Notice that the bound of $d^2 \log N$ isn't great in this parameter regime...) What's the best group testing scheme you can come up with in this setting? (Don't worry about false postives/negatives). What's a natural lower bound on the number of tests you would need?

4. **(Bonus – if you finish early, here's something else to work on!)** Say that a group testing matrix $\Phi \in \{0,1\}^{t \times N}$ is "d-good" if it can identify up to d defective items. More precisely, for $d < N$, $\Phi \in \{0,1\}^{t \times N}$ is d-good iff the map from sets $T \subset [N]$ with $|T| \leq d$ to outcomes in $\{0,1\}^t$ given by

$$T \mapsto \left(\bigvee_{i \in T} \Phi_{1,i}, \bigvee_{i \in T} \Phi_{2,i}, \ldots, \bigvee_{i \in T} \Phi_{t,i} \right)$$

   is injective.

   In class we proved that if $\Phi \in \{0,1\}^{t \times N}$ is *d-disjunct,* then it is d-good.

   (a) Show that for $d = 2$, there are matrices that are d-good but not d-disjunct. (It's okay if you show this by giving a somewhat silly example).

   (b) Show that any d-good matrix is $(d-1)$-disjunct.