CS256/Winter 2009 Lecture #10

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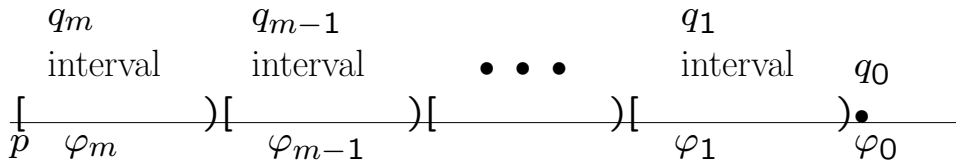
Nested Waiting-for Formulas



Rule nwait (nested waiting-for)

For assertions p, $q_0, q_1, \ldots, q_m$ and $\varphi_0, \varphi_1, \ldots, \varphi_m$

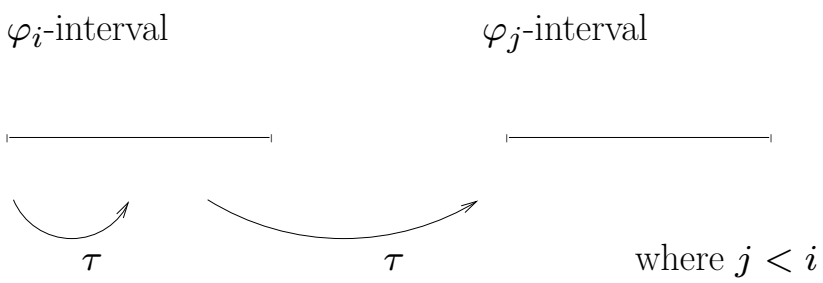
N1. $p \rightarrow \bigvee_{j=0}^{m} \varphi_j$

N2. $\varphi_i \rightarrow q_i$ for $i = 0, 1, \ldots, m$

N3. $\{\varphi_i\}\mathcal{T}\left\{\bigvee_{j\le i} \varphi_j\right\}$ for $i = 1, \ldots, m$

$$p \Rightarrow q_m \,\mathcal{W}\, q_{m-1} \cdots q_1 \,\mathcal{W}\, q_0$$

Nested Waiting-for Formulas (Cont'd)



Premise N3 states that for each assertion φ_i, each transition $\tau \in \mathcal{T}$ either preserves φ_i or
leads to some φ_j, with $j < i$.

Example: Program mux-pet1 (Fig. 3.4)

An example of a nested waiting-for formula is
1-bounded overtaking for MUX-PET1:

$$\underbrace{at\_\ell_3}_{p} \Rightarrow \underbrace{\neg at\_m_4}_{q_3} \; \mathcal{W} \; \underbrace{at\_m_4}_{q_2} \; \mathcal{W} \; \underbrace{\neg at\_m_4}_{q_1} \; \mathcal{W} \; \underbrace{at\_\ell_4}_{q_0}$$

It states that when process P_1 is at ℓ_3,
process P_2 can enter its critical section at most
once ahead of process P_1.

Example: Program mux-pet1 (Fig. 3.4)

(Peterson's Algorithm for mutual exclusion)

local y_1, y_2: **boolean** **where** $y_1 = \text{F}, y_2 = \text{F}$
s : **integer** **where** $s = 1$

$P_1 ::$
ℓ_0 : **loop forever do**
$$\left[\begin{array}{ll} \ell_1: & \textbf{noncritical} \\ \ell_2: & (y_1,\ s) := (\text{T},\ 1) \\ \ell_3: & \textbf{await } (\neg y_2) \vee (s \neq 1) \\ \ell_4: & \textbf{critical} \\ \ell_5: & y_1 := \text{F} \end{array}\right]$$

$\| \|$

$P_2 ::$
m_0 : **loop forever do**
$$\left[\begin{array}{ll} m_1: & \textbf{noncritical} \\ m_2: & (y_2,\ s) := (\text{T},\ 2) \\ m_3: & \textbf{await } (\neg y_1) \vee (s \neq 2) \\ m_4: & \textbf{critical} \\ m_5: & y_2 := \text{F} \end{array}\right]$$

With the following strengthenings all premises of rule NWAIT become state-valid.

p: $\underline{at\_\ell_3}$

φ_3: $at\_\ell_3 \wedge \underline{\neg at\_m_4} \wedge at\_m_3 \wedge s = 1$
"P_2 has priority over P_1"

φ_2: $at\_\ell_3 \wedge \underline{at\_m_4}$

φ_1: $at\_\ell_3 \wedge \underline{\neg at\_m_4} \wedge (at\_m_3 \rightarrow s = 2)$
"P_1 has priority over P_2"

$\varphi_0 = q_0$: $\underline{at\_\ell_4}$

or equivalently,

p: at_ℓ_3

φ_3: $at\_\ell_3 \ \wedge \ at\_m_3 \ \wedge \ s = 1$

φ_2: $at\_\ell_3 \ \wedge \ at\_m_4$

φ_1: $at\_\ell_3 \ \wedge \ (at\_m_{0..2,5} \ \vee \ (at\_m_3 \ \wedge \ s = 2))$

$\varphi_0 = q_0$: at_ℓ_4

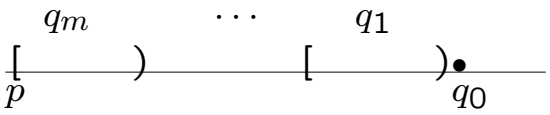
Concatenation of waiting-for formulas

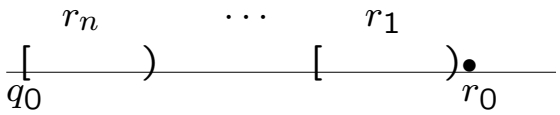
Rule CONC-W

$$p \;\Rightarrow\; q_m \,\mathcal{W}\, \cdots \,\mathcal{W}\, q_1 \,\mathcal{W}\, q_0 \qquad\qquad q_0 \;\Rightarrow\; r_n \,\mathcal{W}\, \cdots \,\mathcal{W}\, r_0$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$p \;\Rightarrow\; q_m \,\mathcal{W}\, \cdots \,\mathcal{W}\, q_1 \,\mathcal{W}\, r_n \,\mathcal{W}\, \cdots \,\mathcal{W}\, r_0$$





Collapsing of waiting-for formulas

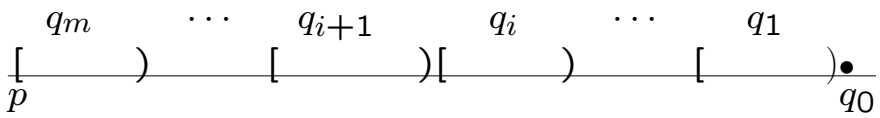
Rule COLL-W

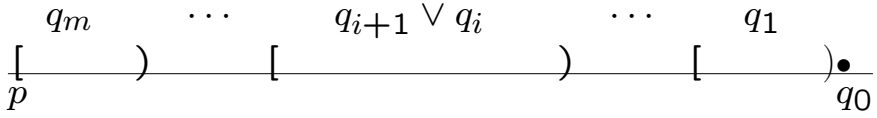
For $i > 0$

$$p \;\Rightarrow\; q_m \,\mathcal{W}\, \cdots \,\mathcal{W}\, q_{i+1} \,\mathcal{W}\, q_i \,\mathcal{W}\, \cdots \,\mathcal{W}\, q_0$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$p \;\Rightarrow\; q_m \,\mathcal{W}\, \cdots \,\mathcal{W}\, (q_{i+1} \vee q_i) \,\mathcal{W}\, \cdots \,\mathcal{W}\, q_0$$





Basic Verification Diagrams

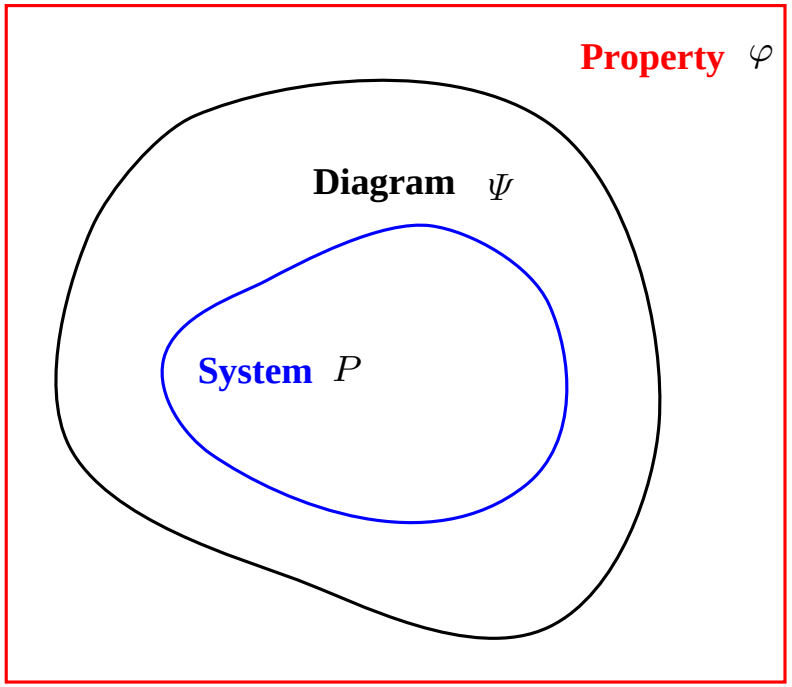
A visual summary of verification proofs

Verification Diagrams (VDs) allow a graphical representation of a proof of a temporal property.

To prove φ is P-valid, find diagram Ψ such that:

$$\mathcal{L}(P) \subseteq \mathcal{L}(\Psi) \subseteq \mathcal{L}(\varphi)$$

i.e., every P-computation σ is a Ψ-sequence and every Ψ-sequence σ is a model of φ (satisfies $\sigma \models \varphi$).



$\mathcal{L}(P) \subseteq \mathcal{L}(\Psi)$ proved by verification conditions.

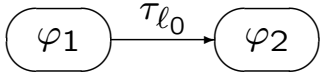
$\mathcal{L}(\Psi) \subseteq \mathcal{L}(\varphi)$ follows from well-formedness of diagram.

Verification Diagram (VD)

Directed labeled graph with

- Nodes – labeled by assertions

- Edges – labeled by names of transitions



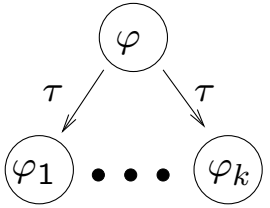
- Terminal Node ("goal") – no edges depart from it

Verification conditions (VCs)

VD provides a concise representation of sets of VCs:

- The verification condition associated with a node labeled by φ and a transition τ is

$$\Rightarrow \{\varphi\}\ \tau\ \{\varphi \vee \varphi_1 \vee \ldots \vee \varphi_k\}$$



  There is an implicit τ-edge connecting each φ-node to itself.

- Nonterminal node without outgoing edges

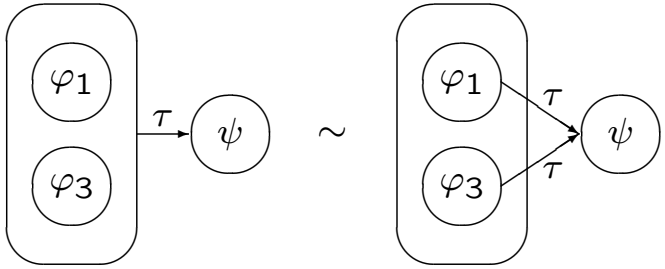
$$\Rightarrow \{\varphi\}\ \tau\ \{\varphi\}$$

  Note: No verification conditions for terminal node.

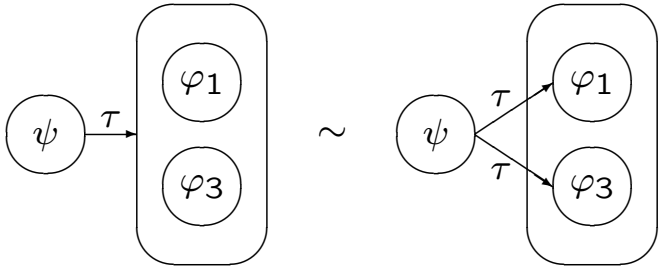
Definition: VD is P-valid iff all VCs associated with nodes in the diagram are P-state valid

Compound Nodes: Statecharts Conventions

- Departing edges

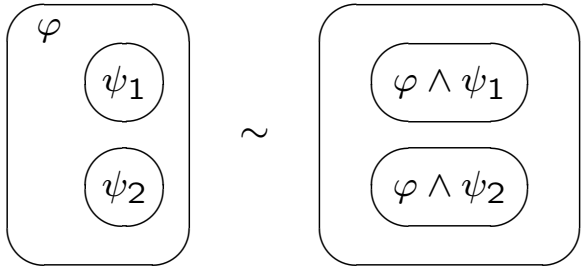


- Arriving edges



Compound Nodes: Statecharts Conventions

- Common factors



Classes of Diagrams

- Proofs of invariance properties

  $$\Box\, q$$

  are represented by INVARIANCE diagrams

- Proofs of precedence properties

  $$p \;\Rightarrow\; q_m \,\mathcal{W}\, q_{m-1} \;\cdots\; q_1 \,\mathcal{W}\, q_0$$

  are represented by WAIT diagrams

- Proofs of response properties

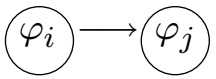
  $$p \Rightarrow \Diamond\, q$$

  are represented by CHAIN and
  RANK diagrams (Vol. III)

Wait Diagrams

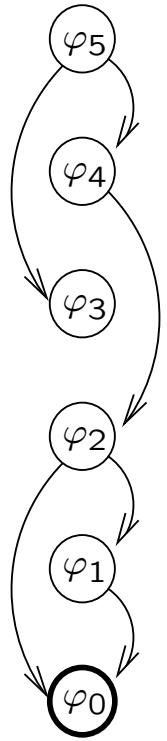
VDs with nodes $\varphi_m, \ldots, \varphi_0$ such that:

- weakly acyclic, i.e.,

  if 

  then $i \geq j$

- φ_0 is a terminal node



`Claim` (wait diagram):

A P-valid WAIT diagram establishes that

$$\bigvee_{j=0}^{m} \varphi_j \;\Rightarrow\; \varphi_m \,\mathcal{W}\, \varphi_{m-1} \;\cdots\; \varphi_1 \,\mathcal{W}\, \varphi_0$$

is P-valid.

If, in addition,

(N1) $\quad p \;\rightarrow\; \bigvee_{j=0}^{m} \varphi_j$

(N2) $\quad \varphi_i \;\rightarrow\; q_i \quad \text{for} \quad i = 0, 1, \ldots, m$

are P-state valid, then

$$\boxed{p \;\Rightarrow\; q_m \,\mathcal{W}\, q_{m-1} \;\cdots\; q_1 \,\mathcal{W}\, q_0}$$

is P-valid.

`Example:` Program MUX-PET1 (Fig 3.4)

1-bounded overtaking from ℓ_3

$$\psi\colon\ \underbrace{at\_\ell_3}_{p} \;\Rightarrow\; (\underbrace{\neg at\_m_4}_{q_3})\ \mathcal{W}\ \underbrace{at\_m_4}_{q_2}\ \mathcal{W}\ (\underbrace{\neg at\_m_4}_{q_1})\ \mathcal{W}\ \underbrace{at\_\ell_4}_{q_0}$$

Proof is summarized in WAIT diagram

(Fig 3.8)

Example: Program mux-pet1 (Fig. 3.4)
(Peterson's Algorithm for mutual exclusion)

local y_1, y_2: **boolean** **where** $y_1 = \text{F}, y_2 = \text{F}$
s : **integer** **where** $s = 1$

$P_1 ::$
ℓ_0 : **loop forever do**
$$\begin{bmatrix} \ell_1: & \textbf{noncritical} \\ \ell_2: & (y_1,\ s) := (\text{T},\ 1) \\ \ell_3: & \textbf{await } (\neg y_2) \vee (s \neq 1) \\ \ell_4: & \textbf{critical} \\ \ell_5: & y_1 := \text{F} \end{bmatrix}$$

||

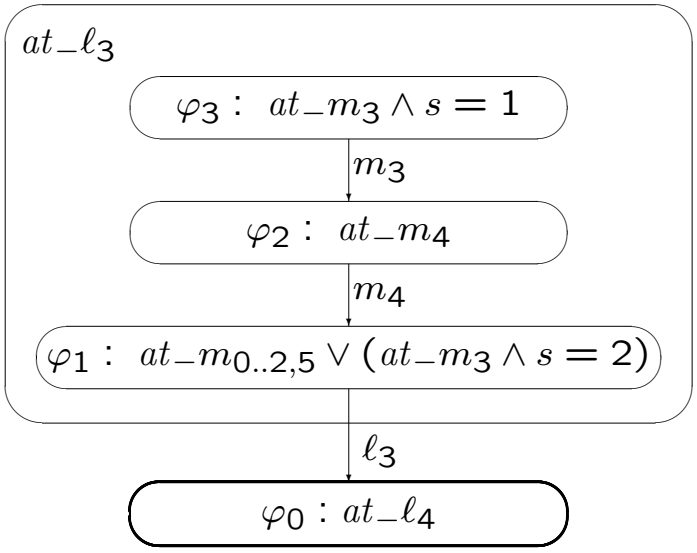
$P_2 ::$
m_0 : **loop forever do**
$$\begin{bmatrix} m_1: & \textbf{noncritical} \\ m_2: & (y_2,\ s) := (\text{T},\ 2) \\ m_3: & \textbf{await } (\neg y_1) \vee (s \neq 2) \\ m_4: & \textbf{critical} \\ m_5: & y_2 := \text{F} \end{bmatrix}$$

`Example:` Program MUX-PET1 (Con't)

WAIT diagram (Fig. 3.8)
(1-bounded overtaking from ℓ_3)

ψ: $\underbrace{at\_\ell_3}_{p} \Rightarrow$

$$(\underbrace{\neg at\_m_4}_{q_3})\ \mathcal{W}\ \underbrace{at\_m_4}_{q_2}\ \mathcal{W}\ (\underbrace{\neg at\_m_4}_{q_1})\ \mathcal{W}\ \underbrace{at\_\ell_4}_{q_0}$$



Example: Program MUX-PET1 (Con't)

Associated VCs

- From φ_3

  $\{\varphi_3\}\ m_3\ \{\varphi_3 \vee \varphi_2\}$

  $$\underbrace{\ldots}_{\varphi_3} \wedge \underbrace{\ldots \wedge at'\_m_4}_{\rho_{m_3}} \rightarrow \underbrace{\ldots}_{\varphi'_3} \vee \underbrace{at'\_m_4}_{\varphi'_2}$$

  $\{\varphi_3\}\ \overline{m_3}\ \{\varphi_3\}$

  for all non-m_3 transitions.
  But since we are at_ℓ_3, at_m_3, check only ℓ_3.

  $\{\varphi_3\}\ \ell_3\ \{\varphi_3\}$ holds, since

  $$\underbrace{at\_m_3 \wedge \ldots \wedge s = 1}_{\varphi_3} \wedge \underbrace{\ldots \wedge ((\neg y_2) \vee (s \neq 1))}_{\rho_{\ell_3}} \rightarrow \underbrace{\ldots}_{\varphi'_3}$$

  Recall that by χ_2, $at\_m_3 \rightarrow y_2$.

- From φ_2

  $\{\varphi_2\}\ m_4\ \{\varphi_2 \vee \varphi_1\}$

  $\{\varphi_2\}\ \overline{m_4}\ \{\varphi_2\}$

- From φ_1

  $\{\varphi_1\}\ \ell_3\ \{\varphi_1 \vee \varphi_0\}$

  $\{\varphi_1\}\ \overline{\ell_3}\ \{\varphi_1\}$

They are P-state valid
[not state-valid - require invariants $\chi_0, \ldots, \chi_4$]

Therefore,
WAIT diagram is valid over MUX-PET1

`Example:` Program MUX-PET1 (Con't)

Therefore,

$$\bigvee_{i=0}^{3} \varphi_i \Rightarrow \varphi_3 \,\mathcal{W}\, \varphi_2 \,\mathcal{W}\, \varphi_1 \,\mathcal{W}\, \varphi_0$$

is valid over MUX-PET1.

In addition,

$$\underbrace{at\_\ell_3}_{p} \rightarrow \bigvee_{j=0}^{3} \varphi_j$$

$$\varphi_0 \rightarrow \underbrace{at\_\ell_4}_{q_0} \qquad \varphi_1 \rightarrow \underbrace{\neg at\_m_4}_{q_1}$$

$$\varphi_2 \rightarrow \underbrace{at\_m_4}_{q_2} \qquad \varphi_3 \rightarrow \underbrace{\neg at\_m_4}_{q_3}$$

are P-state valid.

Therefore,

ψ: $at\_\ell_3 \Rightarrow$
$(\neg at\_m_4)\ \mathcal{W}\ at\_m_4\ \mathcal{W}\ (\neg at\_m_4)\ \mathcal{W}\ at\_\ell_4$

is valid over MUX-PET1

Invariance Diagrams

VDs with no terminal nodes (cycles OK)

`Claim (invariance diagram)`:

A P-valid INVARIANCE diagram establishes that

$$\bigvee_{j=1}^{m} \varphi_j \;\Rightarrow\; \Box(\bigvee_{j=1}^{m} \varphi_j)$$

is P-valid.

If, in addition,

$$\text{(I1)} \quad \Theta \rightarrow \bigvee_{j=1}^{m} \varphi_j$$

$$\text{(I2)} \quad \bigvee_{j=1}^{m} \varphi_j \rightarrow q$$

are P-state valid, then

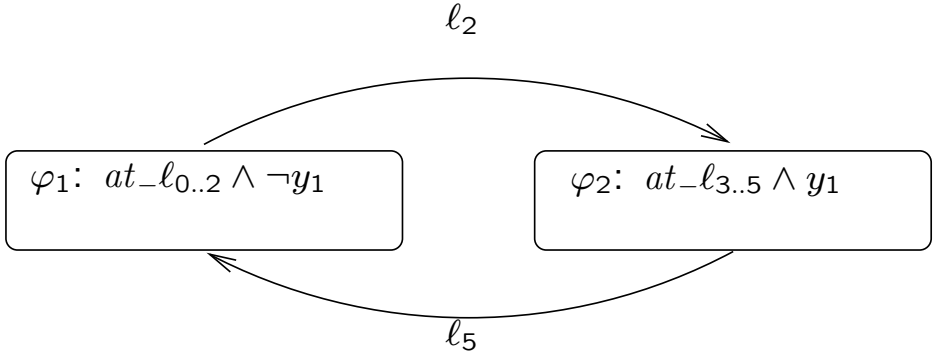
$$\boxed{\Box\, q}$$

is P-valid

Example: Program MUX-PET1 (Fig 3.4)

Establish $\boxed{\Box \underbrace{(y_1 \leftrightarrow at\_\ell_{3..5})}_{q}}$

INVARIANCE diagram
valid for program MUX-PET1



because

$\{\varphi_1\}\, \ell_2\, \{\varphi_1 \vee \varphi_2\}$ $\qquad$ $\{\varphi_1\}\, \overline{\ell_2}\, \{\varphi_1\}$

$\{\varphi_2\}\, \ell_5\, \{\varphi_2 \vee \varphi_1\}$ $\qquad$ $\{\varphi_2\}\, \overline{\ell_5}\, \{\varphi_2\}$

Thus

$$\varphi_1 \vee \varphi_2 \Rightarrow \Box(\varphi_1 \vee \varphi_2)$$

Also,

(I1) $\underbrace{at\_\ell_0 \wedge \neg y_1 \wedge \cdots}_{\Theta} \rightarrow$

$$\underbrace{at\_\ell_{0..2} \wedge \neg y_1}_{\varphi_1} \vee \underbrace{\cdots}_{\varphi_2}$$

(I2) $\underbrace{at\_\ell_{0..2} \wedge \neg y_1}_{\varphi_1} \vee \underbrace{at\_\ell_{3..5} \wedge y_1}_{\varphi_2} \rightarrow$

$$\underbrace{y_1 \leftrightarrow at\_\ell_{3..5}}_{q}$$

are state-valid

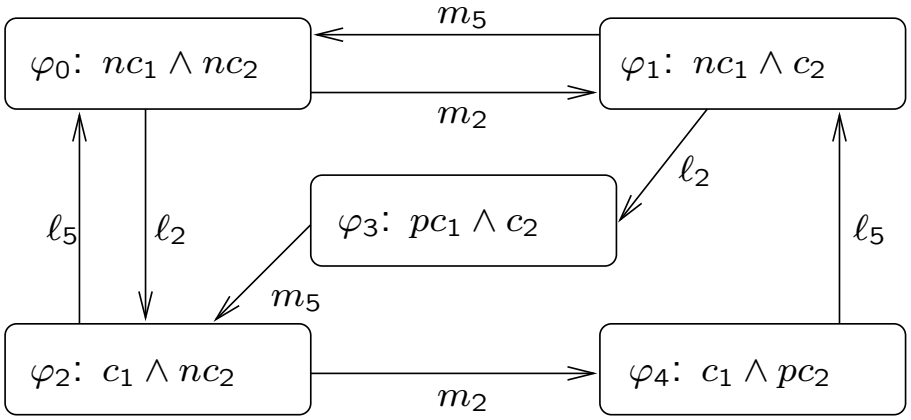
Therefore

$$\boxed{\Box \underbrace{(y_1 \leftrightarrow at\_\ell_{3..5})}_{q}}$$

is P-valid.

Example: Program MUX-PET1 (Fig. 3.4)

Establish $\boxed{\Box\,\neg(at\_\ell_4 \wedge at\_m_4)}$



non-critical: nc_1: $at\_\ell_{0..2}$

nc_2: $at\_m_{0..2}$

critical: c_1: $at\_\ell_{3..5} \wedge \neg y_2$

c_2: $at\_m_{3..5} \wedge \neg y_1$

pre-critical: pc_1: $at\_\ell_3 \wedge s = 1 \wedge y_2$

pc_2: $at\_m_3 \wedge s = 2 \wedge y_1$