

CS256/Winter 2009 Lecture #15

Zohar Manna

Particle Tableau

Particle Tableau: Motivation

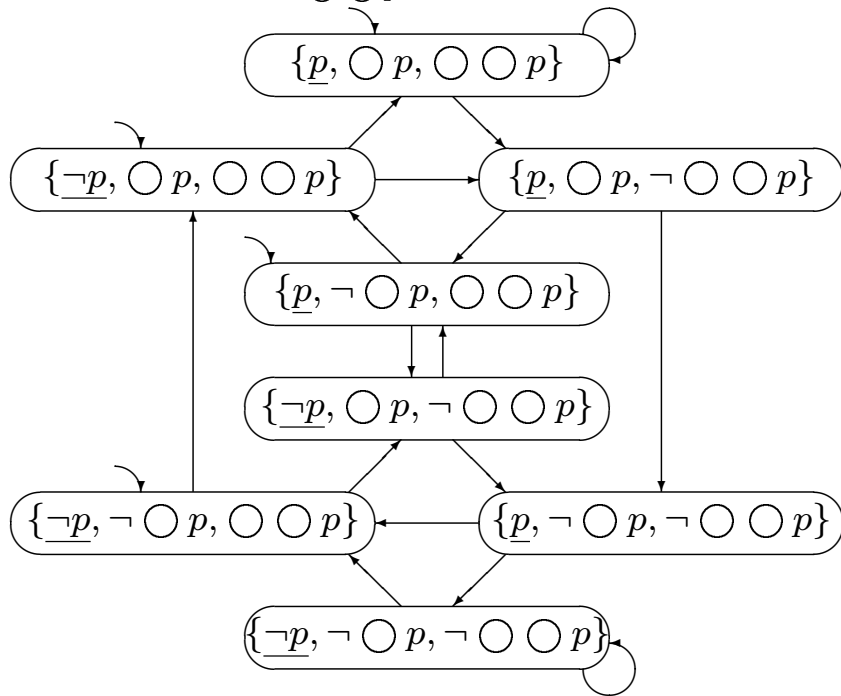
Consider $\boxed{\varphi : \bigcirc \bigcirc p}$

The closure Φ_φ has three basic formulas:

$p, \bigcirc p, \bigcirc \bigcirc p.$

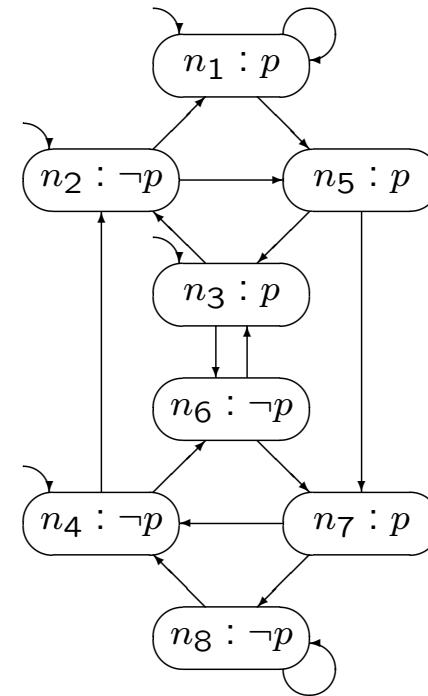
Thus, it has eight atoms.

The atom tableau $T_{\bigcirc \bigcirc p}$ is



Particle Tableau: Motivation

The ω -automaton $A_{\bigcirc \bigcirc p}$:



$$\mathcal{F}_M = \{ \text{all SCS's} \}$$

$$\mathcal{F}_S = \{ \}$$

Note: No promising formulas.

Particle Tableau: Motivation (Cont'd)

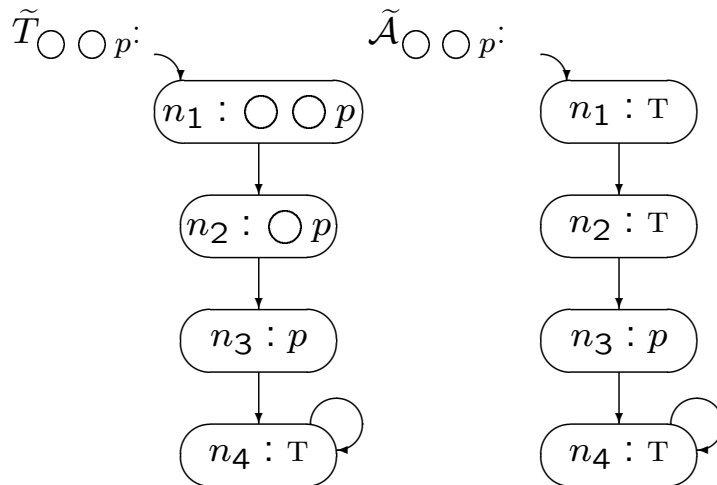
Particle Tableau: Motivation

Because of the atom construction rule:

for every $\psi \in \Phi_\varphi$,
 $\psi \in A$ iff $\neg\psi \notin A$,

every atom makes a commitment about every formula in the closure.

Clearly, some of these commitments are irrelevant in determining the satisfiability of the formula.



If we change the offending rule to

if $\psi \in A$ then $\neg\psi \notin A$

we get the particle tableau, which is usually considerably smaller than the atom tableau.

Particles

The idea of a particle is to assert what needs to be true, not what needs to be false, except for state formulas.

Thus, if $\psi \in A$, ψ needs to be true;
if $\psi \notin A$, ψ can be true or false.

Step 0: Push negations inside φ

We push all negations inside the formula such that negations only appear at the state level. This can be done with the help of the following congruences:

$$\begin{aligned} \neg \diamond p &\approx \square \neg p \\ \neg \bigcirc p &\approx \bigcirc \neg p \\ \neg \square p &\approx \diamond \neg p \\ \neg(p\mathcal{U}q) &\approx (\neg q)\mathcal{W}(\neg p \wedge \neg q) \\ \neg(p\mathcal{W}q) &\approx (\neg q)\mathcal{U}(\neg p \wedge \neg q) \end{aligned}$$

Thus, the closure only needs to contain positive formulas and the negation of state formulas.

Closure $\tilde{\Phi}_\varphi$

- $\varphi \in \tilde{\Phi}_\varphi$
- for every $\psi \in \tilde{\Phi}_\varphi$ and χ a subformula of ψ , $\chi \in \tilde{\Phi}_\varphi$
- for every ψ of the form

$$\square \psi_1, \diamond \psi_1, \psi_1\mathcal{U}\psi_2, \psi_1\mathcal{W}\psi_2,$$

if $\psi \in \tilde{\Phi}_\varphi$,
then $\bigcirc \psi \in \tilde{\Phi}_\varphi$

Particles: Definition

A particle of φ is any set $P \subseteq \tilde{\Phi}_\varphi$ that satisfies the following requirements:

- R_{sat} : $state(P)$ is satisfiable
- R_α : for every α -formula $\psi \in \tilde{\Phi}_\varphi$,
 $\psi \in P$ iff $\kappa(\psi) \in P$
- R_β : for every β -formula $\psi \in \tilde{\Phi}_\varphi$,
 $\psi \in P$ iff $\kappa_1(\psi) \in P$
or $\kappa_2(\psi) \subseteq P$ (or both)

Note: The empty set $\{\}$ is always a particle, denoted by P_\emptyset .

Examples:

$$\varphi : \diamond \square p$$

$$\tilde{\Phi}_\varphi : \{ \diamond \square p, \circ \diamond \square p, \square p, \circ \square p, p \}$$

$$\text{Particle: } \{ \diamond \square p, \circ \diamond \square p \}$$

$$\text{Atom: } \{ \diamond \square p, \circ \diamond \square p, \neg p, \neg \circ \square p, \neg \square p \}$$

$$\varphi : \circ \circ p$$

$$\tilde{\Phi}_\varphi : \{ \circ \circ p, \circ p, p \}$$

$$\text{Particle: } \{ \circ \circ p \}$$

$$\text{Atom: } \{ \circ \circ p, \circ p, \neg p \}$$

Cover of a Formula Set

Given a set of formulas $B \subseteq \tilde{\Phi}_\varphi$, we give a procedure for constructing the cover of B , a set of particles of φ that contain B .

Recursive function $cover_\varphi(B: \text{set of formulas})$:
set of particles

- if $state(B)$ is not consistent,
then return $\{\}$

- α -expansion
if for some α -formula $\psi \in \tilde{\Phi}_\varphi$,
 $\psi \in B$ but $\kappa(\psi) \not\subseteq B$,
then return
 $cover_\varphi(B \cup \kappa(\psi))$

- α^{-1} -expansion
if for some α -formula $\psi \in \tilde{\Phi}_\varphi$,
 $\kappa(\psi) \subseteq B$ but $\psi \notin B$,
then return
 $cover_\varphi(B \cup \{\psi\})$

- β -expansion
if for some β -formula $\psi \in \tilde{\Phi}_\varphi$,
 $\psi \in B$, but $\kappa_1(\psi) \notin B$ and $\kappa_2(\psi) \not\subseteq B$,
then return
 $cover_\varphi(B \cup \{\kappa_1(\psi)\})$
 \cup
 $cover_\varphi(B \cup \kappa_2(\psi))$

- β^{-1} -expansion
if for some β -formula $\psi \in \tilde{\Phi}_\varphi$,
 $\psi \notin B$, but $\kappa_1(\psi) \in B$ or $\kappa_2(\psi) \subseteq B$,
then return
 $cover_\varphi(B \cup \{\psi\})$

- return $\{B\}$

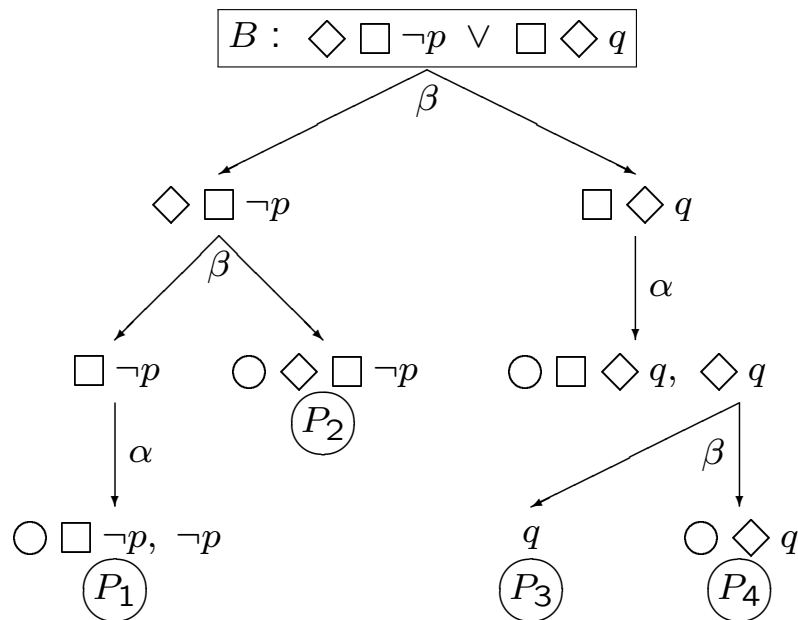
Note: $cover_\varphi(\underbrace{\{\}}_B) = \{P_\emptyset\}$

Tree Representation of the Procedure

Example: To find all particles covering

$$B = \varphi : \diamond \square \neg p \vee \square \diamond q$$

construct the tree:



Example (Cont'd): Particles

Thus,

$$cover_{\varphi}(\underbrace{\{\varphi\}}_B) = \{P_1, P_2, P_3, P_4\},$$

where

$$P_1 : \{ \varphi, \diamond \square \neg p, \square \neg p, \bigcirc \square \neg p, \neg p \}$$

$$P_2 : \{ \varphi, \diamond \square \neg p, \bigcirc \diamond \square \neg p \}$$

$$P_3 : \{ \varphi, \square \diamond q, \bigcirc \square \diamond q, \diamond q, q \}$$

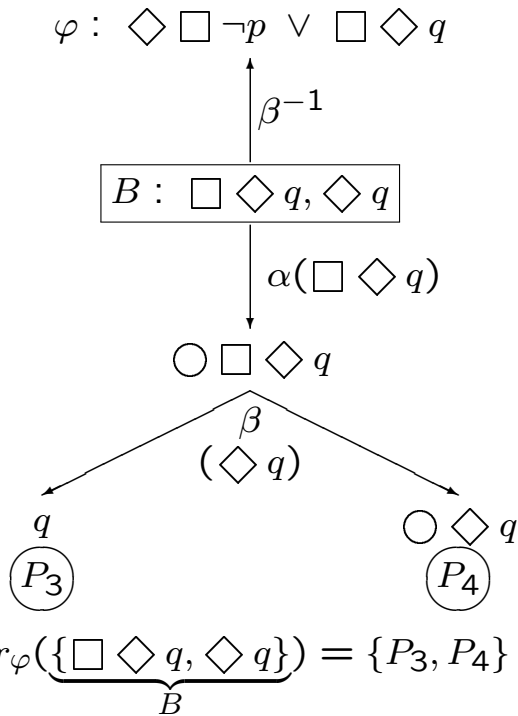
$$P_4 : \{ \varphi, \square \diamond q, \bigcirc \square \diamond q, \diamond q, \bigcirc \diamond q \}$$

Example: $\varphi : \diamond \square \neg p \vee \square \diamond q$

To find all particles of φ covering

$$B : \{\square \diamond q, \diamond q\}$$

construct the tree:



Incremental Particle Tableau Construction

Idea: Start with initial φ -particles and only construct particles that are reachable from previously constructed particles.

Implied successors $imps(P)$ of particle P :

if $\bigcirc \psi \in P$, then $\psi \in imps(P)$

Successors of particle P :

$$succ(P) = cover_\varphi(imps(P))$$

Algorithm for constructing \tilde{T}_φ :

- initially, $\tilde{T}_\varphi = cover_\varphi(\{\varphi\})$
these are the initial nodes.
- for each particle $P \in \tilde{T}_\varphi$,
let $S = succ(P)$
for each $Q \in S$,
if $Q \notin \tilde{T}_\varphi$, then add it
draw an edge from P to Q

Example: Construct \tilde{T}_φ for $\varphi : \diamond \square \neg p \vee \square \diamond q$

Particles:

$$P_1 : \{ \varphi, \diamond \square \neg p, \square \neg p, \underline{\square \neg p}, \neg p \}$$

$$P_2 : \{ \varphi, \diamond \square \neg p, \underline{\diamond \square \neg p} \}$$

$$P_3 : \{ \varphi, \square \diamond q, \underline{\square \diamond q}, \diamond q, q \}$$

$$P_4 : \{ \varphi, \square \diamond q, \underline{\square \diamond q}, \diamond q, \underline{\diamond q} \}$$

$$\text{imps}(P_1) = \{ \square \neg p \}$$

$$\text{succ}(P_1) = \text{cover}(\{ \square \neg p \}) = \{ P_1 \}$$

$$\text{imps}(P_2) = \{ \diamond \square \neg p \}$$

$$\text{succ}(P_2) = \text{cover}(\{ \diamond \square \neg p \}) = \{ P_1, P_2 \}$$

$$\text{imps}(P_3) = \{ \square \diamond q \}$$

$$\text{succ}(P_3) = \text{cover}(\{ \square \diamond q \}) = \{ P_3, P_4 \}$$

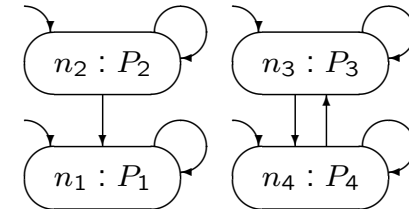
$$\text{imps}(P_4) = \{ \square \diamond q, \diamond q \}$$

$$\text{succ}(P_4) = \text{cover}(\{ \square \diamond q, \diamond q \}) = \{ P_3, P_4 \}$$

Example (Cont'd):

Particle	<i>imps</i>	<i>succ</i>
P_1	$\square \neg p$	P_1
P_2	$\diamond \square \neg p$	P_1, P_2
P_3	$\square \diamond q$	P_3, P_4
P_4	$\square \diamond q, \diamond q$	P_3, P_4

\tilde{T}_φ :



Fulfillment

A particle P fulfills formula $\psi \in \tilde{\Phi}_\varphi$, which promises r , if

$$\psi \notin P \text{ or } r \in P.$$

An SCS S is fulfilling if every promising formula $\psi \in \tilde{\Phi}_\varphi$ is fulfilled by some particle $P \in S$.

Proposition:

An LTL formula φ is satisfiable
iff

\tilde{T}_φ has a fulfilling SCS that is reachable from an initial node.

Example: $\varphi : \diamond \square \neg p \vee \square \diamond q$

Promising formulas:

$$\begin{aligned} \diamond \square \neg p &\text{ promises } \square \neg p \\ \diamond q &\text{ promises } q \end{aligned}$$

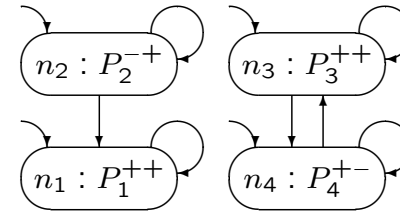
$$P_1^{++} : \{ \varphi, \diamond \square \neg p, \square \neg p, \bigcirc \square \neg p, \neg p \}$$

$$P_2^{-+} : \{ \varphi, \diamond \square \neg p, \bigcirc \diamond \square \neg p \}$$

$$P_3^{++} : \{ \varphi, \square \diamond q, \bigcirc \square \diamond q, \diamond q, q \}$$

$$P_4^{+-} : \{ \varphi, \square \diamond q, \bigcirc \square \diamond q, \diamond q, \bigcirc \diamond q \}$$

Particle Tableau \tilde{T}_φ :



Fulfilling SCS's : $\{n_1\}, \{n_3\}, \{n_3, n_4\}$

Hence, φ is satisfiable.

From Particle Tableau \tilde{T}_φ
to ω -Automaton \mathcal{A}_φ

For temporal formula φ , construct the ω -automaton

$$\mathcal{A}_\varphi : \underbrace{\langle N, N_0, E, \mu, \mathcal{F} \rangle}_{\substack{\text{Same as} \\ \tilde{T}_\varphi}}$$

where

- Node labeling μ :
For node $n \in N$ labeled by particle P in \tilde{T}_φ ,

$$\mu(n) = \text{state}(P).$$

- Acceptance condition \mathcal{F} :

Muller:

$$\mathcal{F} = \{ \text{SCS } S \mid S \text{ is fulfilling} \}$$

Street:

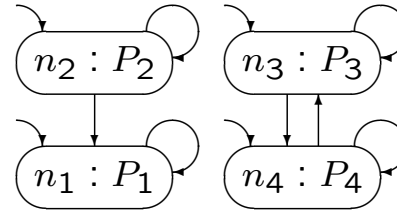
$$\mathcal{F} = \{ (P_\psi, R_\psi) \mid \psi \in \tilde{\Phi}_\varphi \text{ promises } r \},$$

where

$$\begin{aligned} P_\psi &= \{ P \mid \psi \notin P \} && \Leftarrow \\ R_\psi &= \{ P \mid r \in P \} \end{aligned}$$

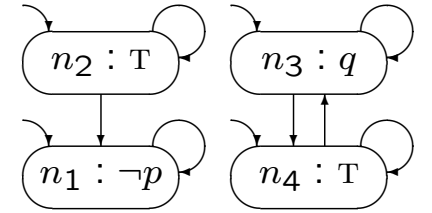
Example (Cont'd): $\varphi : \diamond \square \neg p \vee \square \diamond q$

Particle Tableau \tilde{T}_φ



with fulfilling SCS's
 $\{n_1\}, \{n_3\}, \{n_3, n_4\}$

Corresponding
 ω -Automaton \mathcal{A}_φ



$$\mathcal{F}_M = \{ \{n_1\}, \{n_3\}, \{n_3, n_4\} \}$$

$$\mathcal{F}_S = \{ (P_{\diamond \square \neg p}, R_{\diamond \square \neg p}), (P_{\diamond q}, R_{\diamond q}) \}$$

with

$$P_{\diamond \square \neg p} : \{n_3, n_4\}$$

$$R_{\diamond \square \neg p} : \{n_1\}$$

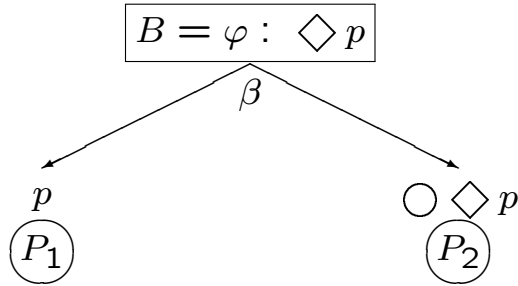
$$P_{\diamond q} : \{n_1, n_2\}$$

$$R_{\diamond q} : \{n_3\}$$

Example: To find all particles covering

$$\varphi : \diamond p$$

construct the tree:



Thus, $cover_{\varphi}(\underbrace{\{\varphi\}}_B) = \{P_1, P_2\}$, where

$$P_1 : \{ \varphi, p \}$$

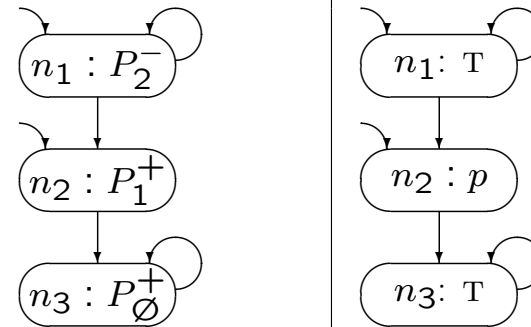
$$P_2 : \{ \varphi, \circ \diamond p \}$$

Example (Cont'd): $\varphi : \diamond p$

$$\begin{array}{lll} P_1 : \{ \varphi, p \} & \text{imps}(P_1) = \{ \} & \text{succ}(P_1) = \{ P_{\emptyset} \} \\ P_{\emptyset} & \text{imps}(P_{\emptyset}) = \{ \} & \text{succ}(\{ \}) = \{ P_{\emptyset} \} \\ P_2 : \{ \varphi, \circ \diamond p \} & \text{imps}(P_2) = \{ \varphi \} & \text{succ}(P_2) = \{ P_1, P_2 \} \end{array}$$

P_1^+, P_{\emptyset}^+ fulfilling P_2^- not fulfilling.

Particle Tableau \tilde{T}_{φ} Corresponding ω -Automaton \mathcal{A}_{φ}



with fulfilling SCS's $\{n_3\}$ $\mathcal{F}_M = \{ \{n_3\} \}$

$\mathcal{F}_S = \{ (\{n_3\}, \{n_2\}) \}$

Hence, φ is satisfiable.

Example: To find all particles covering

$$\varphi : \bigcirc \bigcirc p$$

construct the (trivial) tree:

$$\boxed{B = \varphi : \bigcirc \bigcirc p}$$

(only one node)

Thus,

$$\text{cover}_\varphi(\underbrace{\{\bigcirc \bigcirc p\}}_B) = \{P_1\},$$

where

$$P_1 : \{ \varphi \}.$$

Example (Cont'd): $\varphi : \bigcirc \bigcirc p$

$$P_1 : \{\varphi\} \quad \text{imps}(P_1) = \{\bigcirc p\} \quad \text{succ}(P_1) = \underbrace{\{\bigcirc p\}}_{P_2}$$

$$P_2 : \{\bigcirc p\} \quad \text{imps}(P_2) = \{p\} \quad \text{succ}(P_2) = \underbrace{\{p\}}_{P_3}$$

$$P_3 : \{p\} \quad \text{imps}(P_3) = \{\} \quad \text{succ}(P_3) = \{P_\emptyset\}$$

$$P_\emptyset \quad \text{imps}(P_\emptyset) = \{\} \quad \text{succ}(P_\emptyset) = \{P_\emptyset\}$$

No promising formulas

$$P_1^+ : \{\varphi\}$$

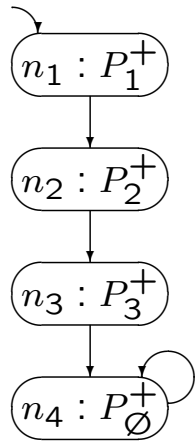
$$P_2^+ : \{\bigcirc p\}$$

$$P_3^+ : \{p\}$$

$$P_4^+ : P_\emptyset$$

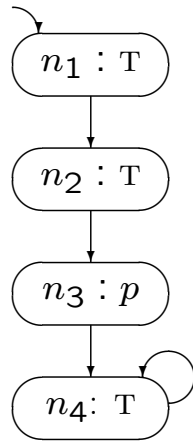
Example (Cont'd): $\varphi : \bigcirc \bigcirc p$

Particle Tableau \tilde{T}_φ



with fulfilling SCS
 $\{n_4\}$

Corresponding
 ω -Automaton \mathcal{A}_φ



$\mathcal{F}_M = \{\{n_4\}\}$

$\mathcal{F}_S = \{\}$

Hence, φ is satisfiable.