CS265/CME309, Winter 2026

Class 1: Agenda and in-class Questions

1 Welcome!

Welcome to CS265/CME309! All logistics info can be found on http://web.stanford.edu/class/cs265

2 Polynomial Identity Testing

2.1 Group work

Group Work

1. First, introduce yourselves to each other. What year/program are you in? What class/activity/etc are you most excited about for this quarter?
2. Read the following definition.

   > A multivariate polynomial $f(x_1, \ldots, x_m)$ is *identically zero* if all its coefficients are zero. For example, the polynomial $f(x_1, x_2) = (x_1+1)(x_2+1) - 1 - x_1x_2 - (x_1 + x_2)$ is identically zero because when you expand it out, all of the terms cancel.

   Now, work on the following questions with your group.
3. Which of the following two polynomials are identically zero?

$$f(x, y) = (x - y)^2 + 2xy + (x + y)^3 - y(3x^2 + y(3x + y + 1)) - (x + 1)x^2$$

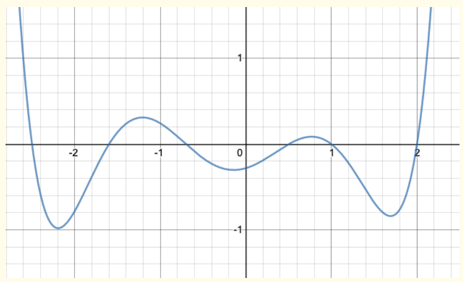
$$g(x, y) = (x - 2y)^2 + xy + (x + y)^3 - y(3x^2 + y(3x + y + 1)) - (x + 1)x^2$$

4. Let $f(x_1, x_2, \ldots, x_n)$ be a polynomial in n variables of total degree at most n. (For example, the *total degree* of $x_1^2x_2 + x_1x_2$ is 3, the maximum total degree of any monomial.[a]) Suppose that you are given an expression for f that has length $O(n)$. How long does it decide if f were identically zero in the worst case, **if** you use the straightforward way by expanding out every term?
   (a) $O(n)$
   (b) polynomial in n
   (c) $2^{\Omega(n)}$
5. At the end of this group work, we are going to challenge you with two polynomials f and g, and ask you which is identically zero. You're going to have **one minute** to answer. Think now about an efficient way to answer this challenge.

As part of your strategy, you may use a basic calculator (eg, `https://www.google.com/search?q=calculator`), but dumping the expression into Copilot or WolframAlpha or something to simplify or plot it is cheating.

Hint: *Remember that this is a class on randomized algorithms. Can you think of a randomized strategy?*

Hint: *You might take inspiration from the univariate case. Here's the graph of a polynomial $g(x)$ that is* not *identically zero. What is true about the values $g(x)$ for most choices of x?*



6. Would your strategy still work if Mary knew it ahead of time? That is, if I know your strategy — but not necessarily the outcome of any randomness in the strategy — could I come up with polynomials f, g that would foil your strategy?

   If your strategy would fail if I know it ahead of time, try to come up with another strategy that would (probably) succeed!

7. If you've finished all of the above, try to come up with an efficient *deterministic* strategy that would succeed at this task.

[a] More formally, the *total degree* of a monomial $x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$ is $\sum_{i=1}^n a_i$. The total degree of a polynomial is the largest total degree of any monomial that appears in it.

2.2 Challenge and Discussion!

- Challenge time!
- A bit of lecture on Polynomial Identity Testing and the Schwartz-Zippel Lemma.

3 What is a randomized algorithm?

- A bit of lecture about the basic framework for randomized algorithms, if time. (In case we don't get to this, there's also a YouTube video you can watch later – link on course website – and it's in the lecture notes).

4 Wrap Up

Before next time:

- Watch the two short videos for Class 2 on Canvas or on YouTube (link to YouTube on course website; to find it on Canvas, go to Panopto Videos→Pre-Lecture Videos and sort by name to find the ones that start with "Class 2.").
- Do the quiz on Gradescope.