CS265, Fall 2020

Class 11: Agenda, Questions, and Links

1 Announcements

- HW5 due Friday!

2 Questions?

Any questions from the minilectures and/or the quiz? (Second moment method and LLL)

- Go into small groups and ask each other your questions.
- Go to https://pollev.com/cs265 and ask your questions/comments there, or else upvote others' questions.

3 Practice with the LLL

Recall the *k-SAT* problem. There are n variables $x_1, \ldots, x_n$. We consider clauses that looks like $(x_{i_1} \vee x_{i_2} \vee \overline{x_{i_3}} \vee \cdots \vee x_{i_k})$; that is, a cluase is the OR of k literals. **For today, assume that each clause has k distinct variables that appear in it.** We have a formula φ that is the AND of m clauses. We would like to know: is φ satisfiable? That is, is there a way to assign values to the variables $x_1, x_2, \ldots$ so that φ evaluates to TRUE?

Group Work

Important: as you make progress on the question(s), one person in each room should record your progress on http://PollEv.com/cs265.

Suppose that each variable x_i is in at most t clauses, for some parameter t that will depend on k and that you'll work out in this problem. Apply the LLL to get a statement like the following:

> Suppose that each variable is in at most t clauses of φ. Then φ is satisfiable.

(You should try to get t to be as large as possible. It's not hard to see that the statement above is true if, say, $t = 1$, but you should get a value of t that grows with k.)

Hint: We set up this example in the minilecture video, we just didn't work out the conclusion. In the set-up of the video, we considered a uniformly random assignment to the variables $x_1, \ldots, x_n$, and we defined the bad event A_i to be the event that clause i is not satisfied.

Please fill out the polleverywhere to vote for your choice of t.

4 Practice with the second moment method

In a graph $G = (V, E)$, say that a vertex v is **isolated** if it has no neighboring vertices.

Group Work

Important: as you make progress on the question(s), one person in each room should record your progress on `http://PollEv.com/cs265`**.**

Let $G \sim G_{n,p}$ be a random graph where each edge is present independently with probability p, where $p = \frac{c \ln n}{n}$ for some constant $0 < c < 1$.

Use the Second Moment Method to show that, with probability at least $1 - o(1)$, there is some isolated vertex in G.

For this exercise, feel free to use the approximation $e^{-x} \approx 1 - x$ when x is small as an equality without worrying about it.

Hint: Consider the random variable X that is the number of isolated vertices in G, and recall that the second moment method says that $\Pr[X = 0] \leq \frac{\mathrm{Var}[X]}{(\mathbb{E}X)^2}$.

Hint: When computing the variance of X, you may want to consider the following question: given two distinct vertices u, v of G, what is the probability that *both* u and v are isolated?

Please fill out the polleverywhere to say what you think the $o(1)$ term should be.