CS265, Fall 2020

Class 12: Agenda, Questions, and Links

1 Announcements

- HW5 due Friday! HW6 released Friday!

2 Questions?

Any questions from the minilectures and/or the quiz? (Constructive LLL)

- Go into small groups and ask each other your questions.
- Go to https://pollev.com/cs265 and ask your questions/comments there, or else upvote others' questions.

3 You prove the constructive LLL for another problem!

Consider the following problem (which has featured on a quiz). You are coloring the integers $\{1, \ldots, n\}$ either blue or red. You are given as input a collection of sets $S_1, S_2, \ldots, S_m \subseteq \{1, \ldots, n\}$, so that:

- Each set S_i has size at most k.
- Each set S_i intersects at most d other sets S_j, for some $d > 1$.

Our goal is to color the points $\{1, \ldots, n\}$ so that there is no monochromatic set S_i. (A set S_i is *monochromatic* if every element of it is either red or blue).

Group Work

Important: as you make progress on the question(s), one person in each room should record your progress on http://PollEv.com/cs265.

1. Mimic the proof of the constructive LLL that we saw for k-SAT to give a randomized algorithm that does the following.

   Suppose that $k \geq \log_2 d + 10000$. (Here, 10000 is a stand-in for "some big enough constant.") Then there is a randomized algorithm that proceeds by re-randomizing the sets S_i, (that is, it will iteratively look at different sets S_i and randomly re-color all of the points in that set), so that:

   - If the algorithm terminates, then all of the numbers $\{1, \ldots, n\}$ will be

colored so that there is no monochromatic set S_j.
- The expected number of times that the algorithm re-randomizes a set S_j is poly(m).

Don't worry about giving a complete proof with all the details, just work it out with enough detail that you believe it. As we did in the minilecture video, you may use the (informal) fact that "there is no function $f : \{0,1\}^X \to \{0,1\}^Y$ so that (a) $Y \ll X$ and (b) with high probability over a uniformly random $x \in \{0,1\}^X$, it is possible to recover x given $f(x)$."

Hint: To map this problem onto k-SAT, think of the S_j's as standing in for clauses, and the numbers $\{1, \ldots, n\}$ as standing in for variables.

Hint: It's not quite as straightforward as applying the mapping in the previous hint and calling it a day. In particular, can you still work backwards from the "print" statements in the k-SAT version to figure out the original random bits?

At this point, please note the differences between your proof and the proof we saw for k-SAT on polleverywhere. Once enough people have filled that out, we'll take a break to go over that.

2. What happens to your proof if the number of possible colors grows from two (blue and red) to some number t? In particular, can you get the same guarantee as above, but under a weaker guarantee (eg, $k \geq$ [something smaller than $\log d + 10000$]).

3. How does the answer that you got in the previous part compare to what Corollary 3 in the lecture notes would give you for this problem?

   At this point, if it's after the previous break, please note your responses on polleverywhere.

4. [**This question is open-ended and may be difficult—think about it after you finish the others if you still have time.**] What happens to your proof if the sets can have variable size? (e.g., if all but a few of them have size k, and a few can be really small? Or if they have average size k? Or....?)