CS265, Fall 2020

Class 17: Agenda, Questions, and Links

1 Announcements

- HW7 is out, due Friday (HW8, THE LAST ONE, will be released Friday)
- Mary's OH are canceled today.

2 Questions?

Any questions from the minilectures and/or the quiz and/or the warm-up? (Martingales, Azuma-Hoeffding)

- Go into small groups and ask each other your questions.
- Go to https://pollev.com/cs265 and ask your questions/comments there, or else upvote others' questions.

3 Chromatic numbers

In this exercise we'll practice using Azuma-Hoeffding

Group Work

Important: as you make progress on the question(s), one person in each room should record your progress on http://PollEv.com/cs265.
Let $G \sim G_{n,p}$ be a Erdos-Renyi random graph (so there are n vertices, and each edge is present independently with probability p). Let $A = \chi(G)$ be the chromatic number of G. That is, A is the minimum number of colors necessary to properly color G.

1. Consider the Doob *vertex exposure* martingale. That is:
   - For $i \in \{1, \ldots, n\}$, let X_i denote the the status of the edges between vertex i and vertices $\{i+1, \ldots, n\}$.
   - $Z_i = \mathbb{E}[A|X_1, \ldots, X_i]$

   Use the Azuma-Hoeffding inequality to show that

$$\Pr[|A - \mathbb{E}[A]| > c\sqrt{n}] \leq 2\exp(-c^2/2).$$

   (Notice that you may not know what $\mathbb{E}[A]$ is—that's okay!)

Hint: *To use Azuma-Hoeffding, you need to bound $|Z_i - Z_{i-1}|$. How much can your expectation of the chromatic color change if I tell you additional information about a single vertex?*

At this point, please fill out the PollEverywhere.

2. Repeat the same exercise with the *edge exposure* martingale:
   - Let X_i denote the the status of the i'th edge, for $i \in \{1, \ldots, \binom{n}{2}\}$.
   - $Z_i = \mathbb{E}[A|X_1, \ldots, X_i]$

   Do you get the same thing? Do you get something better? Worse?

3. (**CHALLENGING,** I don't expect anyone to get this; but something to think about if you finish early.) What can you say about $\mathbb{E}[A]$?

   Hint: *Check out* `https://arxiv.org/abs/0706.1725` *for a much stronger statement than just about* $\mathbb{E}[A]$.

4 Gambling

In this exercise, we'll get yet more practice applying Azuma-Hoeffding.

Group Work

Important: as you make progress on the question(s), one person in each room should record your progress on `http://PollEv.com/cs265`**.**
Consider the following gambling game:

- At time t, you can choose to bet *any* amount you like in $[0, B]$, where B is a house limit.
- A fair coin is flipped. If it's heads, you win the amount that you bet; if tails, you lose the amount that you bet.

You're allowed to be in debt; you don't stop when you run out of money.

1. Suppose that the amount you bet is a deterministic function of everything that's happened so far. Set up a martingale $\{Z_t\}$ (with respect some sequence $\{X_t\}$ that you have to define) so that Z_t is the amount of money you have at time t.
2. Use the Azuma-Hoeffding inequality to bound

$$\Pr[|Z_n| \geq cB\sqrt{n}].$$

   At this point, please fill out the PollEverywhere.

3. Now suppose that you can use *any* betting strategy you like, even a randomized one. Is your martingale from part 1 still a martingale? If not, repeat parts 1 and 2 when your betting strategy can be randomized.