Introduction to

Information Retrieval

CS276: Information Retrieval and Web Search

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Lecture 6: Scoring, Term Weighting and the Vector Space Model

This lecture; IIR Sections 6.2-6.4.3

- Ranked retrieval
- Scoring documents
- Term frequency
- Collection statistics
- Weighting schemes
- Vector space scoring

Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1: "*standard user dlink 650*" → 200,000 hits
- Query 2: "*standard user dlink 650 no card found*": 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval, the system returns an ordering over the (top) documents in the collection for a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval has normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the top *k* (≈ 10) results
  - We don't overwhelm the user
- Premise: the ranking algorithm works

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

Take 1: Jaccard coefficient

- A common measure of overlap of two sets *A* and *B*
- jaccard$(A,B) = |A \cap B| \,/\, |A \cup B|$
- jaccard$(A,A) = 1$
- jaccard$(A,B) = 0$ if $A \cap B = 0$
- *A* and *B* don't have to be the same size.
- Always assigns a number between 0 and 1.

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms. Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this.

Recall (Lecture 2): Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a $\boxed{\text{count vector}}$ in $\mathbb{N}^{|V|}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary* and *Mary is quicker than John* have the same vectors
- This is called the bag of words model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents.

Term frequency tf

- The term frequency $\text{tf}_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.
  - Note: Frequency means count in IR
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms *t* in both *q* and *d*:
- score $= \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$
- The score is 0 if none of the query terms is present in the document.

Rare terms are more informative

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*.

Collection vs. Document frequency

- Collection frequency of *t* is the number of occurrences of *t* in the collection
- Document frequency of t is the number of documents in which t occurs
- Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

- Which word is for better search (gets higher weight)

idf weight

- df_t is the document frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $\mathrm{df}_t \leq N$
- We define the idf (inverse document frequency) of t by

$$\mathrm{idf}_t = \log_{10}\left(N/\mathrm{df}_t\right)$$

  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

idf example, suppose N = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\mathrm{idf}_t = \log_{10}\left(N/\mathrm{df}_t\right)$$

There is one idf value for each term t in a collection.

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
- For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$\mathrm{w}_{t,d} = \log(1 + \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Score for a document given a query

$$\mathrm{Score}(q,d) = \sum_{t \in q \cap d} \mathrm{tf.idf}_{t,d}$$

- There are many variants
  - How “tf” is computed (with/without logs)
  - Whether the terms in the query are also weighted
  - …

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

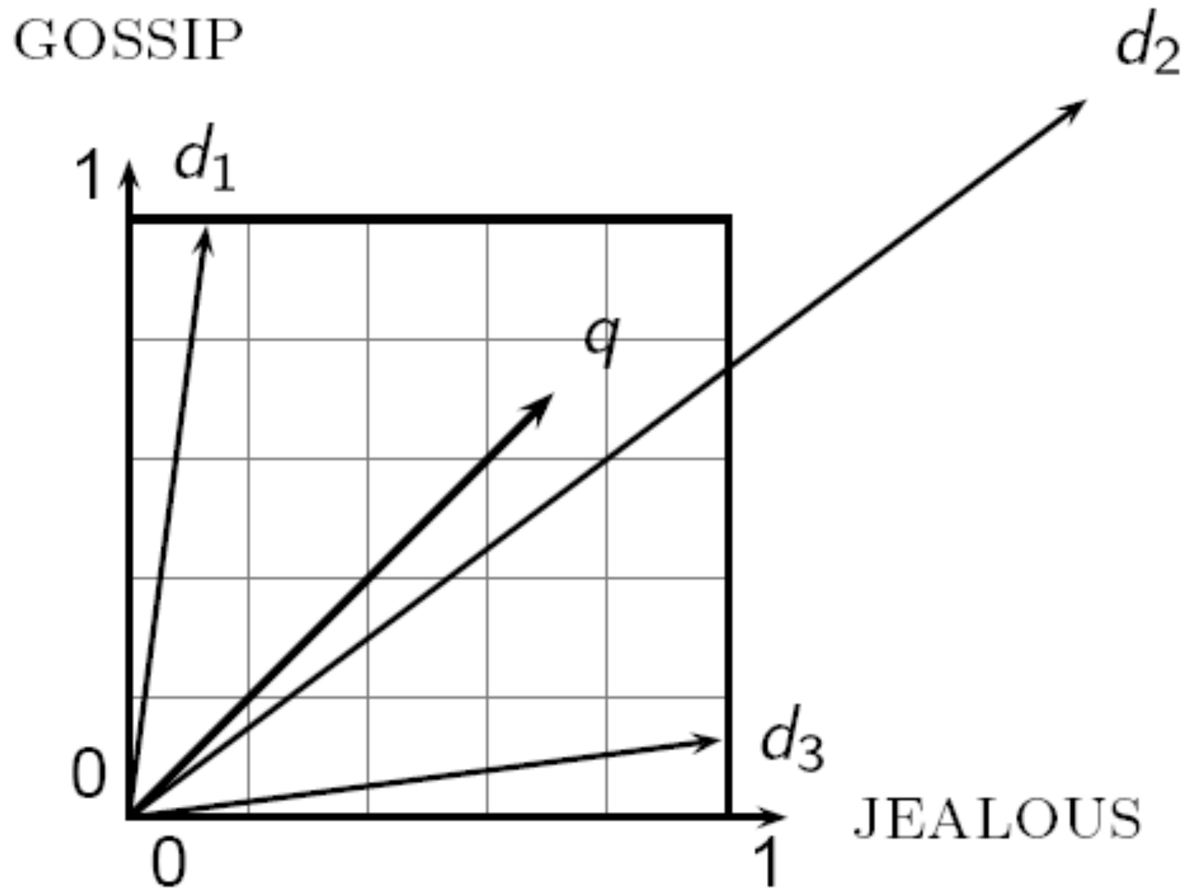
- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths.

Why distance is a bad idea

The Euclidean distance between $\vec{q}$ and $\vec{d_2}$ is large even though the distribution of terms in the query $\vec{q}$ and the distribution of terms in the document $\vec{d_2}$ are very similar.



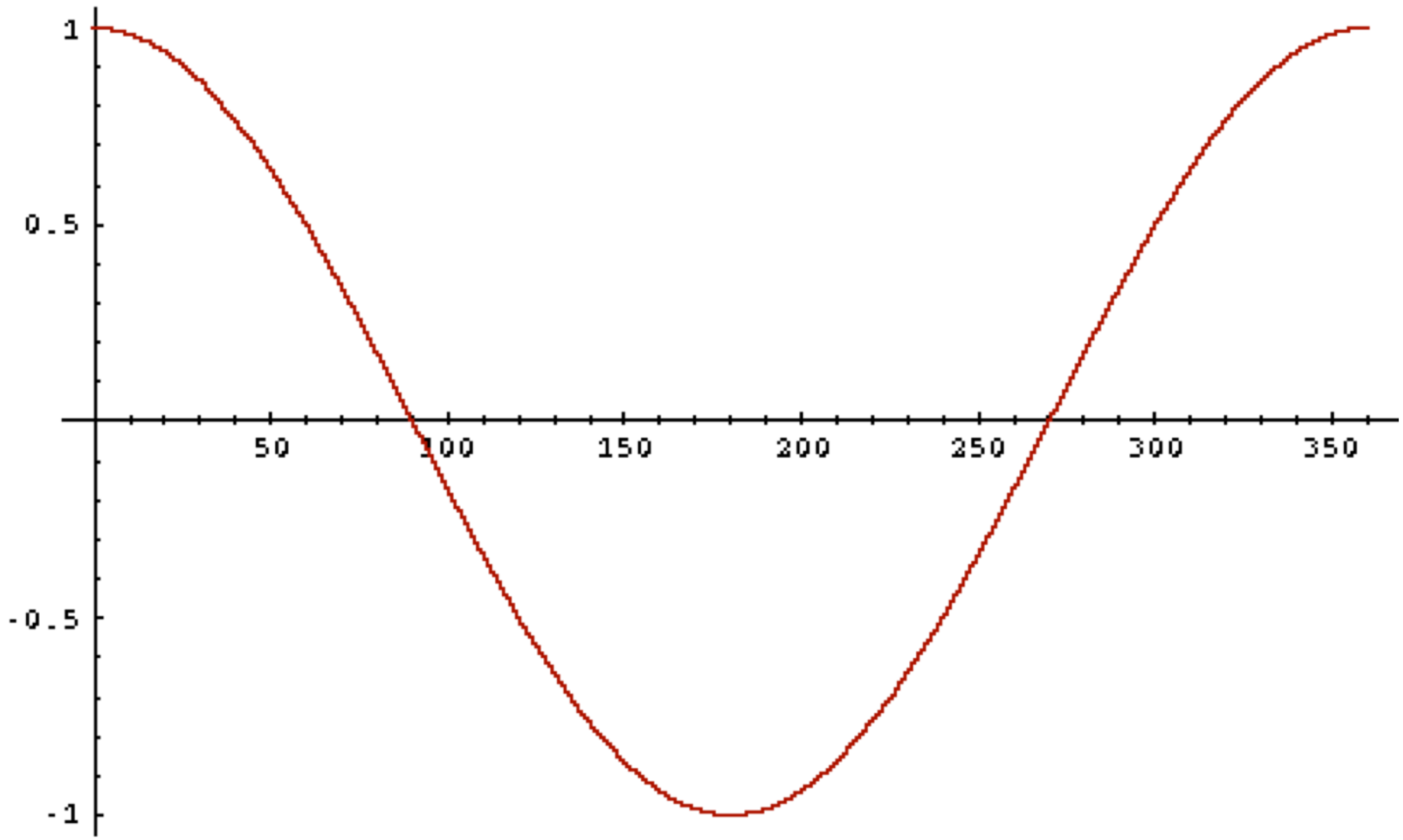
Use angle instead of distance

- Thought experiment: take a document *d* and append it to itself. Call this document *d*′.
- "Semantically" d and d′ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.
- Key idea: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent.
  - Rank documents in decreasing order of the angle between query and document
  - Rank documents in increasing order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0^{o}, 180^{o}]

From angles to cosines



- But how should we be computing cosines?

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\left\|\vec{x}\right\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
  - Long and short documents now have comparable weights

cosine(query,document)

$$\cos(\vec{q},\vec{d}) = \frac{\overbrace{\vec{q}\bullet\vec{d}}^{\text{Dot product}}}{|\vec{q}||\vec{d}|} = \underbrace{\frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|}}_{\text{Unit vectors}} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the weight of term i in the query
d_i is the weight of term i in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

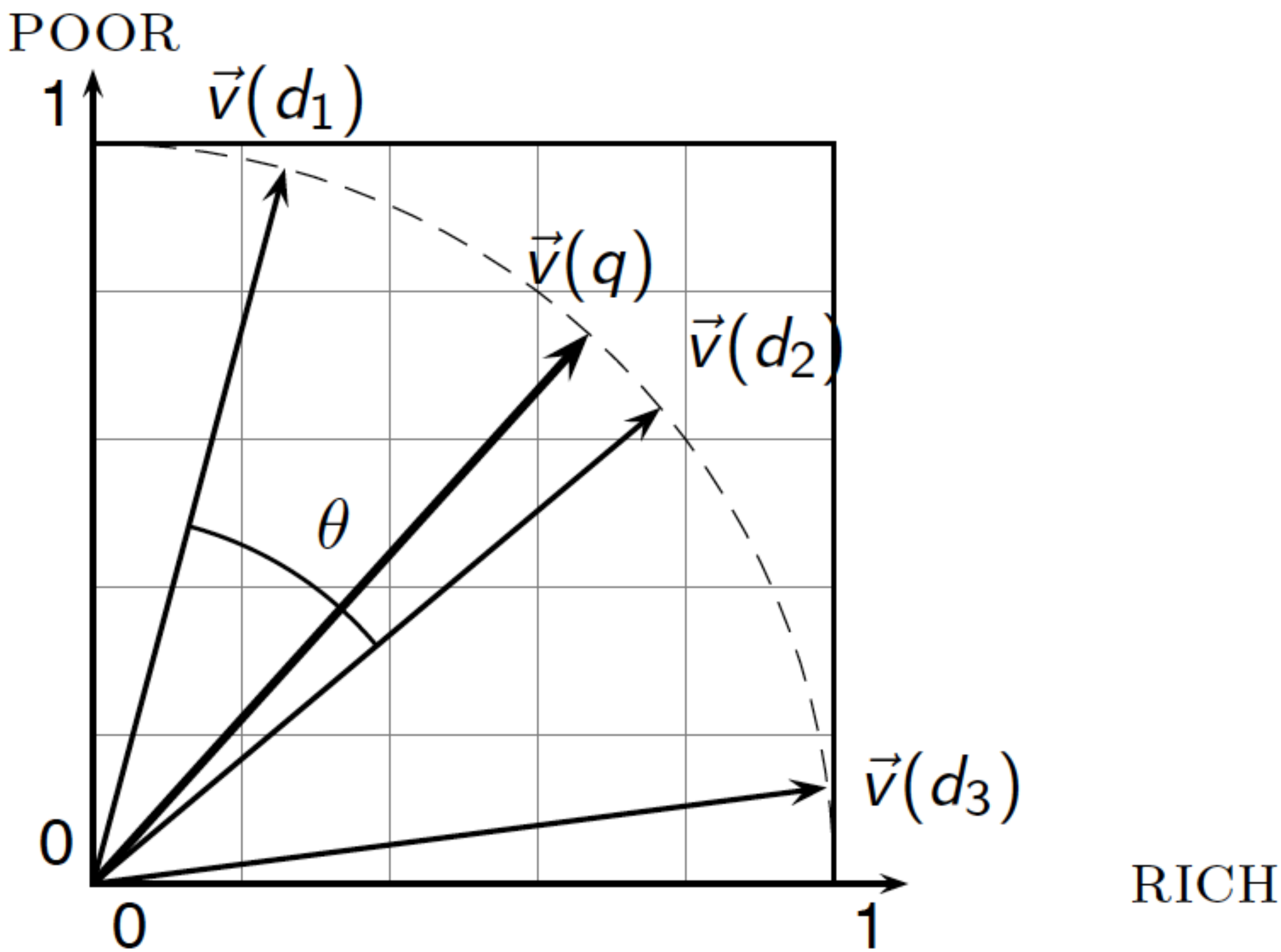
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized.

Cosine similarity illustrated



Cosine similarity amongst 3 documents

How similar are the novels

SaS: *Sense and Sensibility*

PaP: *Pride and Prejudice*, and

WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

dot(SaS,PaP) ≈ 12.1
dot(SaS,WH) ≈ 13.4
dot(PaP,WH) ≈ 10.1

cos(SaS,PaP) ≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

Computing cosine scores

```
CosineScore(q)
float Scores[N] = 0
float Length[N]
for each query term t
do calculate w_{t,q} and fetch postings list for t
    for each pair(d, tf_{t,d}) in postings list
    do Scores[d] += w_{t,d} × w_{t,q}
Read the array Length
for each d
do Scores[d] = Scores[d]/Length[d]
return Top K components of Scores[]
```

Computing cosine scores

- Previous algorithm scores term-at-a-time (TAAT)
- Algorithm can be adapted to scoring document-at-a-time (DAAT)
- Storing $w_{t,d}$ in each posting could be expensive
  - …because we'd have to store a floating point number
  - For tf-idf scoring, it suffices to store $tf_{t,d}$ in the posting and idf_t in the head of the postings list
- Extracting the top K items can be done with a priority queue (e.g., a heap)

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1+\log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5+\frac{0.5\times\mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0,\log\frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases}1 & \text{if } \mathrm{tf}_{t,d}>0\\ 0 & \text{otherwise}\end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha<1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t\in d}(\mathrm{tf}_{t,d}))}$				

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation *ddd.qqq,* using the acronyms from the previous table
- A very standard weighting scheme is: lnc.ltc
- Document: logarithmic tf (l as first character), no idf and cosine normalization
- Query: logarithmic tf (l in leftmost column), idf (t in second column), cosine normalization …

tf-idf example: lnc.ltc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Doc length $= \sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., $K = 10$) to the user

Resources for today's lecture

- IIR 6.2 – 6.4.3

- http://www.miislita.com/information-retrieval-tutorial/cosine-similarity-tutorial.html
  - Term weighting and cosine similarity tutorial for SEO folk!