Introduction to

Information Retrieval

Introducing Information Retrieval and Web Search
Information Retrieval

• Information Retrieval (IR) is finding material (usually documents) of an unstructured nature (usually text) that satisfies an information need from within large collections (usually stored on computers).

  – These days we frequently think first of web search, but there are many other cases:
    • E-mail search
    • Searching your laptop
    • Corporate knowledge bases
    • Legal information retrieval
Unstructured (text) vs. structured (database) data in the mid-nineties
Unstructured (text) vs. structured (database) data today
Basic assumptions of Information Retrieval

- **Collection**: A set of documents
  - Assume it is a static collection for the moment

- **Goal**: Retrieve documents with information that is relevant to the user’s information need and helps the user complete a task
The classic search model

User task

Info need

Query

Search engine

Results

Collection

Get rid of mice in a politically correct way

Info about removing mice without killing them

how trap mice alive

Misconception?

Misformulation?
How good are the retrieved docs?

- **Precision**: Fraction of retrieved docs that are relevant to the user’s information need
- **Recall**: Fraction of relevant docs in collection that are retrieved

- More precise definitions and measurements to follow later
Introduction to Information Retrieval

Term-document incidence matrices
Unstructured data in 1620

• Which plays of Shakespeare contain the words *Brutus AND Caesar* but *NOT Calpurnia*?

• One could *grep* all of Shakespeare’s plays for *Brutus* and *Caesar*, then strip out lines containing *Calpurnia*?

• Why is that not the answer?
  – Slow (for large corpora)
  – *NOT Calpurnia* is non-trivial
  – Other operations (e.g., find the word *Romans* near *countrymen*) not feasible
  – Ranked retrieval (best documents to return)
    • Later lectures
## Term-document incidence matrices

<table>
<thead>
<tr>
<th>Word</th>
<th>Antony and Cleopatra</th>
<th>Julius Caesar</th>
<th>The Tempest</th>
<th>Hamlet</th>
<th>Othello</th>
<th>Macbeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antony</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Brutus</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Caesar</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Calpurnia</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cleopatra</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>mercy</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>worser</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

1 if play contains word, 0 otherwise

**Brutus AND Caesar BUT NOT Calpurnia**
Incidence vectors

• So we have a 0/1 vector for each term.

• To answer query: take the vectors for Brutus, Caesar and Calpurnia (complemented) → bitwise AND.

  – 110100 AND
  – 110111 AND
  – 101111 =
  – 100100

<table>
<thead>
<tr>
<th></th>
<th>Antony and Cleopatra</th>
<th>Julius Caesar</th>
<th>The Tempest</th>
<th>Hamlet</th>
<th>Othello</th>
<th>Macbeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antony</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Brutus</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Caesar</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Calpurnia</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cleopatra</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>mercy</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>worser</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Answers to query

• Antony and Cleopatra, Act III, Scene ii

*Agrippa* [Aside to DOMITIUS ENOBARBUS]: Why, Enobarbus,
When Antony found Julius *Caesar* dead,
He cried almost to roaring; and he wept
When at Philippi he found *Brutus* slain.

• Hamlet, Act III, Scene ii

*Lord Polonius*: I did enact Julius *Caesar* I was killed i’ the Capitol; *Brutus* killed me.
Bigger collections

• Consider $N = 1$ million documents, each with about 1000 words.
• Avg 6 bytes/word including spaces/punctuation
  – 6GB of data in the documents.
• Say there are $M = 500K$ distinct terms among these.
Can’t build the matrix

• 500K x 1M matrix has half-a-trillion 0’s and 1’s.

• But it has no more than one billion 1’s.
  – matrix is extremely sparse.

• What’s a better representation?
  – We only record the 1 positions.
Introduction to Information Retrieval

The Inverted Index
The key data structure underlying modern IR
Inverted index

• For each term \( t \), we must store a list of all documents that contain \( t \).
  – Identify each doc by a \textbf{docID}, a document serial number

Can we use fixed-size arrays for this?

- \textbf{Brutus}
- \textbf{Caesar}
- \textbf{Calpurnia}

What happens if the word \textbf{Caesar} is added to document 14?
Inverted index

• We need variable-size postings lists
  – On disk, a continuous run of postings is normal and best
  – In memory, can use linked lists or variable length arrays

- Some tradeoffs in size/ease of insertion

Sorted by docID (more later on why).
Inverted index construction

Documents to be indexed

Token stream

Linguistic modules

Modified tokens

Indexer

Inverted index

Friends, Romans, countrymen.

friend

roman

countryman

2

4

1

2

13

16

Sec. 1.2
Initial stages of text processing

• Tokenization
  – Cut character sequence into word tokens
    • Deal with “John’s”, a state-of-the-art solution

• Normalization
  – Map text and query term to same form
    • You want U.S.A. and USA to match

• Stemming
  – We may wish different forms of a root to match
    • authorize, authorization

• Stop words
  – We may omit very common words (or not)
    • the, a, to, of
Indexer steps: Token sequence

- Sequence of (Modified token, Document ID) pairs.

<table>
<thead>
<tr>
<th>Term</th>
<th>docID</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>did</td>
<td>1</td>
</tr>
<tr>
<td>enact</td>
<td>1</td>
</tr>
<tr>
<td>julius</td>
<td>1</td>
</tr>
<tr>
<td>caesar</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>was</td>
<td>1</td>
</tr>
<tr>
<td>killed</td>
<td>1</td>
</tr>
<tr>
<td>i'</td>
<td>1</td>
</tr>
<tr>
<td>the</td>
<td>1</td>
</tr>
<tr>
<td>capitol</td>
<td>1</td>
</tr>
<tr>
<td>brutus</td>
<td>1</td>
</tr>
<tr>
<td>killed</td>
<td>1</td>
</tr>
<tr>
<td>me</td>
<td>1</td>
</tr>
<tr>
<td>so</td>
<td>2</td>
</tr>
<tr>
<td>let</td>
<td>2</td>
</tr>
<tr>
<td>it</td>
<td>2</td>
</tr>
<tr>
<td>be</td>
<td>2</td>
</tr>
<tr>
<td>with</td>
<td>2</td>
</tr>
<tr>
<td>caesar</td>
<td>2</td>
</tr>
<tr>
<td>the</td>
<td>2</td>
</tr>
<tr>
<td>noble</td>
<td>2</td>
</tr>
<tr>
<td>brutus</td>
<td>2</td>
</tr>
<tr>
<td>hath</td>
<td>2</td>
</tr>
<tr>
<td>told</td>
<td>2</td>
</tr>
<tr>
<td>you</td>
<td>2</td>
</tr>
<tr>
<td>caesar</td>
<td>2</td>
</tr>
<tr>
<td>was</td>
<td>2</td>
</tr>
<tr>
<td>ambitious</td>
<td>2</td>
</tr>
</tbody>
</table>

Doc 1: I did enact Julius Caesar I was killed i’ the Capitol; Brutus killed me.

Doc 2: So let it be with Caesar. The noble Brutus hath told you Caesar was ambitious.
Indexer steps: Sort

- Sort by terms
  - And then docID

Core indexing step

<table>
<thead>
<tr>
<th>Term</th>
<th>docID</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>did</td>
<td>1</td>
</tr>
<tr>
<td>enact</td>
<td>1</td>
</tr>
<tr>
<td>julius</td>
<td>1</td>
</tr>
<tr>
<td>caesar</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>was</td>
<td>1</td>
</tr>
<tr>
<td>killed</td>
<td>1</td>
</tr>
<tr>
<td>'</td>
<td>1</td>
</tr>
<tr>
<td>the</td>
<td>1</td>
</tr>
<tr>
<td>capitol</td>
<td>1</td>
</tr>
<tr>
<td>brutus</td>
<td>1</td>
</tr>
<tr>
<td>killed</td>
<td>1</td>
</tr>
<tr>
<td>me</td>
<td>1</td>
</tr>
<tr>
<td>so</td>
<td>2</td>
</tr>
<tr>
<td>let</td>
<td>2</td>
</tr>
<tr>
<td>it</td>
<td>2</td>
</tr>
<tr>
<td>be</td>
<td>2</td>
</tr>
<tr>
<td>with</td>
<td>2</td>
</tr>
<tr>
<td>caesar</td>
<td>2</td>
</tr>
<tr>
<td>the</td>
<td>2</td>
</tr>
<tr>
<td>noble</td>
<td>2</td>
</tr>
<tr>
<td>brutus</td>
<td>2</td>
</tr>
<tr>
<td>hath</td>
<td>2</td>
</tr>
<tr>
<td>told</td>
<td>2</td>
</tr>
<tr>
<td>you</td>
<td>2</td>
</tr>
<tr>
<td>caesar</td>
<td>2</td>
</tr>
<tr>
<td>was</td>
<td>2</td>
</tr>
<tr>
<td>ambitious</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>docID</th>
</tr>
</thead>
<tbody>
<tr>
<td>ambitious</td>
<td>2</td>
</tr>
<tr>
<td>be</td>
<td>2</td>
</tr>
<tr>
<td>brutus</td>
<td>1</td>
</tr>
<tr>
<td>brutus</td>
<td>2</td>
</tr>
<tr>
<td>capitol</td>
<td>1</td>
</tr>
<tr>
<td>caesar</td>
<td>1</td>
</tr>
<tr>
<td>caesar</td>
<td>2</td>
</tr>
<tr>
<td>caesar</td>
<td>2</td>
</tr>
<tr>
<td>did</td>
<td>1</td>
</tr>
<tr>
<td>enact</td>
<td>1</td>
</tr>
<tr>
<td>hath</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>'</td>
<td>1</td>
</tr>
<tr>
<td>it</td>
<td>2</td>
</tr>
<tr>
<td>julius</td>
<td>1</td>
</tr>
<tr>
<td>killed</td>
<td>1</td>
</tr>
<tr>
<td>killed</td>
<td>1</td>
</tr>
<tr>
<td>let</td>
<td>2</td>
</tr>
<tr>
<td>me</td>
<td>1</td>
</tr>
<tr>
<td>noble</td>
<td>2</td>
</tr>
<tr>
<td>so</td>
<td>2</td>
</tr>
<tr>
<td>the</td>
<td>1</td>
</tr>
<tr>
<td>the</td>
<td>2</td>
</tr>
<tr>
<td>told</td>
<td>2</td>
</tr>
<tr>
<td>you</td>
<td>2</td>
</tr>
<tr>
<td>was</td>
<td>1</td>
</tr>
<tr>
<td>was</td>
<td>2</td>
</tr>
<tr>
<td>with</td>
<td>2</td>
</tr>
</tbody>
</table>
Indexer steps: Dictionary & Postings

- Multiple term entries in a single document are merged.
- Split into Dictionary and Postings
- Doc. frequency information is added.

Why frequency?
Will discuss later.
Where do we pay in storage?

- How do we index efficiently?
- How much storage do we need?

IR system implementation
- How do we index efficiently?
- How much storage do we need?
Introduction to

Information Retrieval

Query processing with an inverted index
The index we just built

• How do we process a query?
  – Later - what kinds of queries can we process?
Query processing: AND

- Consider processing the query:
  
  *Brutus* \textit{AND} *Caesar*
  
  – Locate *Brutus* in the Dictionary;
    - Retrieve its postings.
  
  – Locate *Caesar* in the Dictionary;
    - Retrieve its postings.
  
  – “Merge” the two postings (intersect the document sets):

\begin{center}
  \begin{tabular}{cccccccc}
    1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 \\
    2 & 4 & 8 & 16 & 32 & 64 & 128 & \textbf{Brutus} \\
  \end{tabular}
\end{center}
The merge

- Walk through the two postings simultaneously, in time linear in the total number of postings entries

If the list lengths are $x$ and $y$, the merge takes $O(x+y)$ operations.

Crucial: postings sorted by docID.
Intersecting two postings lists (a “merge” algorithm)

\[
\text{INTERSECT}(p_1, p_2)
\]

1. \hspace{1em} \text{answer} \leftarrow \langle \rangle
2. \hspace{1em} \textbf{while } p_1 \neq \text{NIL} \textbf{ and } p_2 \neq \text{NIL}
3. \hspace{1em} \textbf{do if } \text{docId}(p_1) = \text{docId}(p_2)
4. \hspace{2em} \textbf{then } \text{ADD}(\text{answer, docId}(p_1))
5. \hspace{2em} p_1 \leftarrow \text{next}(p_1)
6. \hspace{1em} p_1 \leftarrow \text{next}(p_1)
7. \hspace{1em} \textbf{else if } \text{docId}(p_1) < \text{docId}(p_2)
8. \hspace{2em} \textbf{then } p_1 \leftarrow \text{next}(p_1)
9. \hspace{1em} \textbf{else } p_2 \leftarrow \text{next}(p_2)
10. \hspace{1em} \textbf{return } \text{answer}
Introduction to Information Retrieval

The Boolean Retrieval Model & Extended Boolean Models
Boolean queries: Exact match

• The **Boolean retrieval model** is being able to ask a query that is a Boolean expression:
  – Boolean Queries are queries using *AND*, *OR* and *NOT* to join query terms
    • Views each document as a set of words
    • Is precise: document matches condition or not.
  – Perhaps the simplest model to build an IR system on

• Primary commercial retrieval tool for 3 decades.

• Many search systems you still use are Boolean:
  – Email, library catalog, Mac OS X Spotlight
Example: WestLaw

- Largest commercial (paying subscribers) legal search service (started 1975; ranking added 1992; new federated search added 2010)
- Tens of terabytes of data; ~700,000 users
- Majority of users still use boolean queries
- Example query:
  - What is the statute of limitations in cases involving the federal tort claims act?
  - LIMIT! /3 STATUTE ACTION /S FEDERAL /2 TORT /3 CLAIM
    - /3 = within 3 words, /S = in same sentence
Example: WestLaw  http://www.westlaw.com/

• Another example query:
  – Requirements for disabled people to be able to access a workplace
  – disabl! /p access! /s work-site work-place (employment /3 place

• Note that SPACE is disjunction, not conjunction!
• Long, precise queries; proximity operators; incrementally developed; not like web search
• Many professional searchers still like Boolean search
  – You know exactly what you are getting
• But that doesn’t mean it actually works better…. 
Boolean queries:
More general merges

• **Exercise**: Adapt the merge for the queries:
  
  *Brutus AND NOT Caesar*
  
  *Brutus OR NOT Caesar*

• Can we still run through the merge in time $O(x+y)$? What can we achieve?
Merging

What about an arbitrary Boolean formula?

\((\text{Brutus OR Caesar}) \text{ AND NOT } (\text{Antony OR Cleopatra})\)

- Can we always merge in “linear” time?
  - Linear in what?

- Can we do better?
Query optimization

• What is the best order for query processing?
• Consider a query that is an AND of $n$ terms.
• For each of the $n$ terms, get its postings, then AND them together.

Query: *Brutus* AND *Calpurnia* AND *Caesar*
Query optimization example

- Process in order of increasing freq:
  - start with smallest set, then keep cutting further.

This is why we kept document freq. in dictionary

Execute the query as \((\text{Calpurnia AND Brutus}) \text{ AND } \text{Caesar}\).
More general optimization

- e.g., \((madding \ OR \ crowd) \ AND \ (ignoble \ OR \ strife)\)
- Get doc. freq.’s for all terms.
- Estimate the size of each OR by the sum of its doc. freq.’s (conservative).
- Process in increasing order of OR sizes.
Exercise

• Recommend a query processing order for

(tangerine OR trees) AND (marmalade OR skies) AND (kaleidoscope OR eyes)

• Which two terms should we process first?

<table>
<thead>
<tr>
<th>Term</th>
<th>Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>eyes</td>
<td>213312</td>
</tr>
<tr>
<td>kaleidoscope</td>
<td>87009</td>
</tr>
<tr>
<td>marmalade</td>
<td>107913</td>
</tr>
<tr>
<td>skies</td>
<td>271658</td>
</tr>
<tr>
<td>tangerine</td>
<td>46653</td>
</tr>
<tr>
<td>trees</td>
<td>316812</td>
</tr>
</tbody>
</table>
Query processing exercises

• Exercise: If the query is *friends AND romans AND (NOT countrymen)*, how could we use the freq of *countrymen*?

• Exercise: Extend the merge to an arbitrary Boolean query. Can we always guarantee execution in time linear in the total postings size?

• Hint: Begin with the case of a Boolean *formula* query: in this, each query term appears only once in the query.
Exercise

• Try the search feature at http://www.rhymezone.com/shakespeare/
• Write down five search features you think it could do better
Introduction to

Information Retrieval

Phrase queries and positional indexes
Phrase queries

• We want to be able to answer queries such as “stanford university” – as a phrase

• Thus the sentence “I went to university at Stanford” is not a match.
  – The concept of phrase queries has proven easily understood by users; one of the few “advanced search” ideas that works
  – Many more queries are implicit phrase queries

• For this, it no longer suffices to store only <term : docs> entries
A first attempt: Biword indexes

• Index every consecutive pair of terms in the text as a phrase
• For example the text “Friends, Romans, Countrymen” would generate the biwords
  – *friends romans*
  – *romans countrymen*
• Each of these biwords is now a dictionary term
• Two-word phrase query-processing is now immediate.
Longer phrase queries

- Longer phrases can be processed by breaking them down
- *stanford university palo alto* can be broken into the Boolean query on biwords:

  \[
  \text{stanford university AND university palo AND palo alto}
  \]

Without the docs, we cannot verify that the docs matching the above Boolean query do contain the phrase.

Can have false positives!
Issues for biword indexes

• False positives, as noted before
• Index blowup due to bigger dictionary
  – Infeasible for more than biwords, big even for them

• Biword indexes are not the standard solution (for all biwords) but can be part of a compound strategy
Solution 2: Positional indexes

• In the postings, store, for each term the position(s) in which tokens of it appear:

<term, number of docs containing term;
doc1: position1, position2 ... ;
doc2: position1, position2 ... ;
etc.>
Positional index example

<be: 993427;
1: 7, 18, 33, 72, 86, 231;
2: 3, 149;
4: 17, 191, 291, 430, 434;
5: 363, 367, ...

Which of docs 1, 2, 4, 5 could contain “to be or not to be”?

• For phrase queries, we use a merge algorithm recursively at the document level
• But we now need to deal with more than just equality
Processing a phrase query

• Extract inverted index entries for each distinct term: \textit{to, be, or, not}.

• Merge their \textit{doc:position} lists to enumerate all positions with \textit{“to be or not to be”}.

  – \textit{to}:

    • 2:1,17,74,222,551; 4:8,16,190,429,433; 7:13,23,191; ...

  – \textit{be}:

    • 1:17,19; 4:17,191,291,430,434; 5:14,19,101; ...

• Same general method for proximity searches
Proximity queries

• **LIMIT! /3 STATUTE /3 FEDERAL /2 TORT**
  – Again, here, /k means “within k words of”.

• Clearly, positional indexes can be used for such queries; biword indexes cannot.

• Exercise: Adapt the linear merge of postings to handle proximity queries. Can you make it work for any value of k?
  – This is a little tricky to do correctly and efficiently
  – See Figure 2.12 of IIR
Positional index size

• A positional index expands postings storage substantially
  – Even though indices can be compressed

• Nevertheless, a positional index is now standardly used because of the power and usefulness of phrase and proximity queries ... whether used explicitly or implicitly in a ranking retrieval system.
Positional index size

• Need an entry for each occurrence, not just once per document

• Index size depends on average document size
  – Average web page has <1000 terms
  – SEC filings, books, even some epic poems ... easily 100,000 terms

• Consider a term with frequency 0.1%

<table>
<thead>
<tr>
<th>Document size</th>
<th>Postings</th>
<th>Positional postings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>100,000</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>
Rules of thumb

• A positional index is 2–4 as large as a non-positional index

• Positional index size 35–50% of volume of original text

  – Caveat: all of this holds for “English-like” languages
Combination schemes

• These two approaches can be profitably combined
  – For particular phrases ("Michael Jackson", "Britney Spears") it is inefficient to keep on merging positional postings lists
    • Even more so for phrases like "The Who"
• Williams et al. (2004) evaluate a more sophisticated mixed indexing scheme
  – A typical web query mixture was executed in ¼ of the time of using just a positional index
  – It required 26% more space than having a positional index alone
IR vs. databases: Structured vs unstructured data

- Structured data tends to refer to information in “tables”

<table>
<thead>
<tr>
<th>Employee</th>
<th>Manager</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith</td>
<td>Jones</td>
<td>50000</td>
</tr>
<tr>
<td>Chang</td>
<td>Smith</td>
<td>60000</td>
</tr>
<tr>
<td>Ivy</td>
<td>Smith</td>
<td>50000</td>
</tr>
</tbody>
</table>

Typically allows numerical range and exact match (for text) queries, e.g.,

\[ \text{Salary} < 60000 \ \text{AND Manager} = \text{Smith}. \]
Unstructured data

• Typically refers to free text

• Allows
  – Keyword queries including operators
  – More sophisticated “concept” queries e.g.,
    • find all web pages dealing with drug abuse

• Classic model for searching text documents
Semi-structured data

• In fact almost no data is “unstructured”
• E.g., this slide has distinctly identified zones such as the *Title* and *Bullets*
  • ... to say nothing of linguistic structure
• Facilitates “semi-structured” search such as
  – *Title* contains *data* AND *Bullets* contain *search*
• Or even
  – *Title* is about *Object Oriented Programming* AND *Author* something like *stro*rup
  – where * is the wild-card operator