Introduction to

Information Retrieval

CS276: Information Retrieval and Web Search
Christopher Manning and Pandu Nayak

Lecture 15: Distributed Word Representations for Information Retrieval
How can we more robustly match a user’s query intent?

- If user searches for [Dell notebook battery size], we would like to match documents discussing “Dell laptop battery capacity”
- If user searches for [Seattle motel], we would like to match documents containing “Seattle hotel”

- Problem is that our query and document vectors are orthogonal

- \[
\text{motel} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
\text{hotel} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = 0
\]
How can we more robustly match a user’s query intent?

- Use of anchor text may solve this by providing human authored synonyms, but not for new or less popular web pages, or non-hyperlinked collections.
- Relevance feedback could allow us to capture this if we get near enough to matching documents with these words.
- We can also fix this with information on word similarities:
  - A manual thesaurus of synonyms
  - A measure of word similarity
    - Calculated from a big document collection
    - Calculated by query log mining (common on the web)
Example of manual thesaurus
Thesaurus-based query expansion

- For each term, $t$, in a query, expand the query with synonyms and related words of $t$ from the thesaurus
  - feline $\rightarrow$ feline cat
- May weight added terms less than original query terms.
- Generally increases recall
- Widely used in many science/engineering fields
- May significantly decrease precision, particularly with ambiguous terms.
  - “interest rate” $\rightarrow$ “interest rate fascinate evaluate”
- There is a high cost of manually producing a thesaurus
  - And for updating it for scientific changes
Search log query expansion

- Context-free query expansion ends up problematic
  - \([\text{light hair}] \approx [\text{fair hair}]\)
    - So expand \([\text{light}] \Rightarrow [\text{light fair}]\)
  - But \([\text{bed light price}] \neq [\text{bed fair price}]\)

- You can learn query context-specific rewrites from search logs by attempting to identify the same user making a second attempt at the same user need
  - \([\text{Hinton word vector}]\)
  - \([\text{Hinton word embedding}]\)

- In this context, \([\text{vector}] \approx [\text{embedding}]\)
  - But not when talking about a \textit{disease vector} or C++!
Automatic Thesaurus Generation

- Attempt to generate a thesaurus automatically by analyzing the collection of documents
- Fundamental notion: similarity between two words
- Definition 1: Two words are similar if they co-occur with similar words.
- Definition 2: Two words are similar if they occur in a given grammatical relation with the same words.
- You can harvest, peel, eat, prepare, etc. apples and pears, so apples and pears must be similar.
- Co-occurrence based is more robust, grammatical relations are more accurate.

Why?
**Co-occurrence Thesaurus**

- Simplest way to compute one is based on term-term similarities in $C = AA^T$ where $A$ is term-document matrix.
- $w_{i,j} = $ (normalized) weight for $(t_i,d_j)$
- For each $t_i$, pick terms with high values in $C$

What does $C$ contain if $A$ is a term-doc incidence (0/1) matrix?
## Automatic thesaurus generation example

<table>
<thead>
<tr>
<th>Word</th>
<th>Nearest neighbors</th>
</tr>
</thead>
<tbody>
<tr>
<td>absolutely</td>
<td>absurd, whatsoever, totally, exactly, nothing</td>
</tr>
<tr>
<td>bottomed</td>
<td>dip, copper, drops, topped, slide, trimmed</td>
</tr>
<tr>
<td>captivating</td>
<td>shimmer, stunningly, superbly, plucky, witty</td>
</tr>
<tr>
<td>doghouse</td>
<td>dog, porch, crawling, beside, downstairs</td>
</tr>
<tr>
<td>makeup</td>
<td>repellent, lotion, glossy, sunscreen, skin, gel</td>
</tr>
<tr>
<td>mediating</td>
<td>reconciliation, negotiate, cease, conciliation</td>
</tr>
<tr>
<td>keeping</td>
<td>hoping, bring, wiping, could, some, would</td>
</tr>
<tr>
<td>lithographs</td>
<td>drawings, Picasso, Dali, sculptures, Gauguin</td>
</tr>
<tr>
<td>pathogens</td>
<td>toxins, bacteria, organisms, bacterial, parasites</td>
</tr>
<tr>
<td>senses</td>
<td>grasp, psyche, truly, clumsy, naïve, innate</td>
</tr>
</tbody>
</table>
Automatic Thesaurus Generation Issues

- Quality of associations is usually a problem
- Sparsity

Term ambiguity may introduce irrelevant statistically correlated terms.
  - “planet earth facts” → “planet earth soil ground facts”

Since terms are highly correlated anyway, expansion may not retrieve many additional documents.
Co-occurrence Thesaurus

- Since terms are highly correlated anyway, expansion may not retrieve many additional documents
- Really want “second order” similarity: terms that appear in similar term contexts – perhaps local window, not whole doc
- Simplest way to compute one is a term-term matrix $D = CC^T$ based on term-term similarities in $C = AA^T$ where $A$ is term-document matrix. For each $t_i$, pick terms with high values in $D$

What does $D$ contain if $A$ was a term-doc incidence (0/1) matrix?
Can you directly learn term relations?

- Basic IR is scoring on $q^T d$
- No treatment of synonyms; no machine learning
- Can we learn parameters $W$ to rank via $q^T W d$
- Problem is again sparsity – $W$ is huge > $10^{10}$
Is there a better way?

- Idea:
  - Can we learn a low dimensional representation of a words in $\mathbb{R}^d$ such that dot products $u^Tv$ express word similarity?
  - We could still if we want to include a “translation” matrix between vocabularies (e.g., cross-language): $u^TWv$
    - But now $W$ is small!
  - Supervised Semantic Indexing (Bai et al. *Journal of Information Retrieval* 2009) shows successful use of learning $W$ for information retrieval
  
- But we’ll develop direct similarity in this class
Distributinal similarity based representations

- You can get a lot of value by representing a word by means of its neighbors
- “You shall know a word by the company it keeps”  
  - (J. R. Firth 1957: 11)
- One of the most successful ideas of modern statistical NLP

government debt problems turning into banking crises as has happened in saying that Europe needs unified banking regulation to replace the hodgepodge

️ These words will represent *banking* 🔄
Solution: Low dimensional vectors

- The number of topics that people talk about is small (in some sense)
  - Clothes, movies, politics, ...

- Idea: store “most” of the important information in a fixed, small number of dimensions: a dense vector

- Usually around 25 – 1000 dimensions

- How to reduce the dimensionality?
  - Go from big, sparse co-occurrence count vector to low dimensional “word embedding”
Traditional Way:
Singular Value Decomposition

For an $M \times N$ matrix $A$ of rank $r$ there exists a factorization (Singular Value Decomposition = SVD) as follows:

$$A = U \Sigma V^T$$

$M \times M$  $M \times N$  $V$ is $N \times N$

(Not proven here. See IIR chapter 18)
Singular Value Decomposition

\[ A = U \Sigma V^T \]

- \( AA^T = (U \Sigma V^T)(U \Sigma V^T)^T = (U \Sigma V^T)(V \Sigma U^T) = U \Sigma^2 U^T \)

The columns of \( U \) are orthogonal eigenvectors of \( AA^T \).

The columns of \( V \) are orthogonal eigenvectors of \( A^T A \).

Eigenvalues \( \lambda_1 \ldots \lambda_r \) of \( AA^T \) are the eigenvalues of \( A^T A \).

\[ \sigma_i = \sqrt{\lambda_i} \]

\( \Sigma = \text{diag}(\sigma_1 \ldots \sigma_r) \)
Low-rank Approximation

- SVD can be used to compute optimal low-rank approximations.
- Approximation problem: Find $A_k$ of rank $k$ such that

$$A_k = \min_{X:\text{rank}(X)=k} \|A - X\|_F$$

Frobenius norm

$$\|A\|_F = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2}.$$  

$A_k$ and $X$ are both $m \times n$ matrices. Typically, want $k \ll r$.  

Reduced SVD

- If we retain only $k$ singular values, and set the rest to 0, then we don’t need the matrix parts in color.
- Then $\Sigma$ is $k\times k$, $U$ is $M\times k$, $V^T$ is $k\times N$, and $A_k$ is $M\times N$.
- This is referred to as the reduced SVD.
- It is the convenient (space-saving) and usual form for computational applications.
- It’s what Matlab gives you.
Approximation error

- How good (bad) is this approximation?
- It’s the best possible, measured by the Frobenius norm of the error:

\[
\min_{X:\text{rank}(X)=k} \|A - X\|_F = \|A - A_k\|_F = \sigma_{k+1}
\]

where the \(\sigma_i\) are ordered such that \(\sigma_i \geq \sigma_{i+1}\).

Suggests why Frobenius error drops as \(k\) increases.
Latent Semantic Indexing via the SVD
Latent Semantic Indexing (LSI)

- Perform a low-rank approximation of document-term matrix (typical rank 100–300)
- General idea
  - Map documents (and terms) to a low-dimensional representation.
  - Design a mapping such that the low-dimensional space reflects semantic associations (latent semantic space).
  - Compute document similarity based on the inner product in this latent semantic space
LSA Example

- A simple example term-document matrix (binary)

<table>
<thead>
<tr>
<th></th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$d_5$</th>
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</table>
**LSA Example**

- Example of $C = U\Sigma V^T$: The matrix $U$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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LSA Example

- Example of $C = U\Sigma V^T$: The matrix $\Sigma$

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## LSA Example

- Example of $C = U\Sigma V^T$: The matrix $V^T$

$$
\begin{array}{c|ccccccc}
V^T & d_1 & d_2 & d_3 & d_4 & d_5 & d_6 \\
1 & -0.75 & -0.28 & -0.20 & -0.45 & -0.33 & -0.12 \\
2 & -0.29 & -0.53 & -0.19 & 0.63 & 0.22 & 0.41 \\
3 & 0.28 & -0.75 & 0.45 & -0.20 & 0.12 & -0.33 \\
4 & 0.00 & 0.00 & 0.58 & 0.00 & -0.58 & 0.58 \\
5 & -0.53 & 0.29 & 0.63 & 0.19 & 0.41 & -0.22 \\
\end{array}
$$
**LSA Example: Reducing the dimension**

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<tr>
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<th>$U$</th>
<th>1</th>
<th>2</th>
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Original matrix $C$ vs. reduced $C_2 = U\Sigma_2 V^T$

<table>
<thead>
<tr>
<th>$C$</th>
<th>$d_1$</th>
<th>$d_2$</th>
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<th>$d_4$</th>
<th>$d_5$</th>
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<td>-0.08</td>
<td>0.90</td>
<td>0.41</td>
<td>0.49</td>
</tr>
</tbody>
</table>
Performing the maps

- Each row and column of $A$ gets mapped into the $k$-dimensional LSI space, by the SVD.
- **Claim** – this is not only the mapping with the best (Frobenius error) approximation to $A$, but in fact *improves* retrieval.
- A query $q$ is also mapped into this space, by

  $$q_k = q^T U_k \Sigma_k^{-1}$$

- Query NOT a sparse vector.
“NEURAL EMBEDDINGS”
Idea: Directly learn low-dimensional word vectors

- Old idea. Relevant for this lecture & deep learning:
  - Learning representations by back-propagating errors. (Rumelhart et al., 1986)
  - A neural probabilistic language model (Bengio et al., 2003)
  - NLP (almost) from Scratch (Collobert & Weston, 2008)
  - A recent, even simpler and faster model: word2vec (Mikolov et al. 2013) → intro now
Interesting semantic patterns emerge in the vectors.

An Improved Model of Semantic Similarity Based on Lexical Co-Occurrence
Rohde et al. 2005
Interesting semantic patterns emerge in the vectors

An Improved Model of Semantic Similarity Based on Lexical Co-Occurrence
Rohde et al. 2005
Main Idea of word2vec

• Instead of capturing co-occurrence counts directly, predict surrounding words of every word

• Faster and can easily incorporate a new sentence/document or add a word to the vocabulary

• Two variants:
  • CBOW: Predict target from bag of words context
  • Skipgram: Predict context words from target (position-independent)
Details of 1 word context CBOW

- Objective function: Maximize the log probability of any target word given a context word

These matrices have word vectors!

Attempts to predict one-hot vector
CBOW model (one context word)

\[ h = x^T W = W_{(k, \cdot)} := v_{w_I}, \quad \text{Word score} \]

\[ u_j = v'_{w_{j}}^T \cdot h \]

\[ p(w_j | w_I) = y_j = \frac{\exp(u_j)}{\sum_{j'=1}^{V} \exp(u_{j'})} = \frac{\exp \left( v'_{w_{O}}^T v_{w_I} \right)}{\sum_{j'=1}^{V} \exp \left( v'_{w_{j'}}^T v_{w_I} \right)} \]
CBOW model

Want to maximize

\[ p(w_O|w_I) = \max y_j^* \]

\[ = \max \log y_j^* \]

\[ = u_j^* - \log \sum_{j'=1}^V \exp(u_{j'}) := -E \]

- Do this by differentiating wrt each variable and walking downhill to minimize \( E \). Remember:

\[ \frac{\partial x^T a}{\partial x} = \frac{\partial a^T x}{\partial x} = a \]

- Chain rule: If \( y = f(u) \) and \( u = g(x) \), i.e. \( y = f(g(x)) \), then:

\[ \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} \]
CBOW model

Want to maximize

\[
p(w_O|w_I) = \max y_j^* \\
= \max \log y_j^* \\
= u_j^* - \log \sum_{j'=1}^V \exp(u_{j'}) := -E
\]

\[
\frac{\partial E}{\partial u_j} = y_j - t_j := e_j \quad \text{where } t_j = \mathbb{1}(j = j^*)
\]

\[
\frac{\partial E}{\partial w'_{ij}} = \frac{\partial E}{\partial u_j} \cdot \frac{u_j}{\partial w'_{ij}} = e_j \cdot h_i
\]
CBOBW model:
Stochastic gradient descent updates

\[ \frac{\partial E}{\partial w'_{ij}} = \frac{\partial E}{\partial u_j} \cdot \frac{u_j}{\partial w'_{ij}} = e_j \cdot h_i \]

\[
\begin{align*}
    w'_{ij}^{(\text{new})} &= w'_{ij}^{(\text{old})} - \eta \cdot e_j \cdot h_i \\
    \text{where } \eta > 0 \text{ is the learning rate}
\end{align*}
\]

\[
\begin{align*}
    v'_{wj}^{(\text{new})} &= v'_{wj}^{(\text{old})} - \eta \cdot e_j \cdot h
\end{align*}
\]
CBO W model: W matrix

\[
\frac{\partial E}{\partial h_i} = \sum_{j=1}^{V} \frac{\partial E}{\partial u_j} \cdot \frac{\partial u_j}{\partial h_i} = \sum_{j=1}^{V} e_j \cdot w_{ij} := EH_i
\]

\[
h_i = \sum_{k=1}^{V} x_k \cdot w_{ki}
\]

\[
\frac{\partial E}{\partial w_{ki}} = \frac{\partial E}{\partial h_i} \cdot \frac{\partial h_i}{\partial w_{ki}} = EH_i \cdot x_k
\]

\[
\frac{\partial E}{\partial W} = x \cdot EH \quad v_{wI}^{(new)} = v_{wI}^{(old)} - \eta \cdot EH
\]
Training regime

- Start with small, random vectors for words
- Iteratively go through millions of words in contexts
  - Work out prediction, work out error
  - Backpropagate error to update word vectors
  - Repeat
- Result is dense vectors for all words

\[
\text{linguistics} = \begin{pmatrix}
0.286 \\
0.792 \\
-0.177 \\
-0.107 \\
0.109 \\
-0.542 \\
0.349 \\
0.271
\end{pmatrix}
\]
Word similarity in word2vec
Linear Relationships in word2vec

These representations are very good at encoding similarity and dimensions of similarity!

- Analogies testing dimensions of similarity can be solved quite well just by doing vector subtraction in the embedding space

  Syntactically
  - $x_{apple} - x_{apples} \approx x_{car} - x_{cars} \approx x_{family} - x_{families}$
  - Similarly for verb and adjective morphological forms

  Semantically (Semeval 2012 task 2)
  - $x_{shirt} - x_{clothing} \approx x_{chair} - x_{furniture}$
  - $x_{king} - x_{man} \approx x_{queen} - x_{woman}$
Word Analogies

Test for linear relationships, examined by Mikolov et al.

\[
d = \arg \max_x \frac{(w_b - w_a + w_c)^T w_x}{\|w_b - w_a + w_c\|}
\]

\[a:b :: c:?\]

man:woman :: king:? 

+ king [0.30 0.70]

- man [0.20 0.20]

+ woman [0.60 0.30]

queen [0.70 0.80]
COALS model (count-modified LSA)

[Rohde, Gonnerman & Plaut, ms., 2005]
Count based vs. direct prediction

**Count based methods**
- LSA, HAL (Lund & Burgess),
- COALS (Rohde et al),
- Hellinger-PCA (Lebret & Collobert)

  - Fast training
  - Efficient usage of statistics
  - Primarily used to capture word similarity
  - Disproportionate importance given to small counts

**Direct prediction methods**
- NNLM, HLBL, RNN, word2vec
- Skip-gram/CBOW, (Bengio et al; Collobert & Weston; Huang et al; Mnih & Hinton; Mikolov et al; Mnih & Kavukcuoglu)

  - Scales with corpus size
  - Inefficient usage of statistics
  - Generate improved performance on other tasks
  - Can capture complex patterns beyond word similarity
**Encoding meaning in vector differences**

[Pennington, Socher, and Manning, EMNLP 2014]

**Crucial insight:** Ratios of co-occurrence probabilities can encode meaning components

<table>
<thead>
<tr>
<th></th>
<th>$x = \text{solid}$</th>
<th>$x = \text{gas}$</th>
<th>$x = \text{water}$</th>
<th>$x = \text{random}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x</td>
<td>\text{ice})$</td>
<td>large</td>
<td>small</td>
<td>large</td>
</tr>
<tr>
<td>$P(x</td>
<td>\text{steam})$</td>
<td>small</td>
<td>large</td>
<td>large</td>
</tr>
<tr>
<td>$\frac{P(x</td>
<td>\text{ice})}{P(x</td>
<td>\text{steam})}$</td>
<td>large</td>
<td>small</td>
</tr>
</tbody>
</table>
## Encoding meaning in vector differences

[Pennington et al., EMNLP 2014]

**Crucial insight:** Ratios of co-occurrence probabilities can encode meaning components

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<th></th>
<th>$x = \text{solid}$</th>
<th>$x = \text{gas}$</th>
<th>$x = \text{water}$</th>
<th>$x = \text{fashion}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x</td>
<td>\text{ice})$</td>
<td>$1.9 \times 10^{-4}$</td>
<td>$6.6 \times 10^{-5}$</td>
<td>$3.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$P(x</td>
<td>\text{steam})$</td>
<td>$2.2 \times 10^{-5}$</td>
<td>$7.8 \times 10^{-4}$</td>
<td>$2.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\frac{P(x</td>
<td>\text{ice})}{P(x</td>
<td>\text{steam})}$</td>
<td>$8.9$</td>
<td>$8.5 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
GloVe: A new model for learning word representations
[Pennington et al., EMNLP 2014]

\[ w_i \cdot w_j = \log P(i|j) \]

\[ w_x \cdot (w_a - w_b) = \log \frac{P(x|a)}{P(x|b)} \]

\[ J = \sum_{i,j=1}^{V} f \left( X_{ij} \right) \left( w_i^T \hat{w}_j + b_i + \tilde{b}_j - \log X_{ij} \right)^2 \]
Nearest words to *frog*:

1. frogs
2. toad
3. litoria
4. leptodactylidae
5. rana
6. lizard
7. eleutherodactylus

http://nlp.stanford.edu/projects/glove/
### Word analogy task  
[Mikolov, Yih & Zweig 2013a]

<table>
<thead>
<tr>
<th>Model</th>
<th>Dimensions</th>
<th>Corpus size</th>
<th>Performance (Syn + Sem)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBOW (Mikolov et al. 2013b)</td>
<td>300</td>
<td>1.6 billion</td>
<td>36.1</td>
</tr>
</tbody>
</table>
Glove Visualizations

http://nlp.stanford.edu/projects/glove/
Glove Visualizations: Company - CEO
Glove Visualizations: Superlatives
Word embeddings

Word embeddings are currently the hot new technology

Lots of applications whenever knowing word similarity helps prediction:

- Synonym handling in search
- Ad serving
- Language models
- Machine translation
- Sentiment analysis
- ...

...