Six-DOF Haptic Rendering I
Outline

- Motivation
- Direct rendering
- Proxy-based rendering
  - Theory
  - Taxonomy
Motivation

3-DOF avatar
The Holy Grail?
Tool-Mediated Interaction

How many degrees of freedom do we need?
One Caveat
6-DOF Interaction

3-DOF
Position/Translation

6-DOF
+ Orientation/Rotation
Avatars for 6-DOF Haptics

3-DoF
Position/Translation
Render Force

6-DoF
+ Orientation/Rotation
+ Render Torque
Impedance-Controlled Device

position, orientation

force, torque
Direct Rendering

- Analogue to force field rendering
- Must consider multiple contacts in different positions for 6-DOF rendering
Forces on a Body

\[ M_2 = r_2 \times F_2 \]

\[ M_1 = r_1 \times F_1 \]

Output to Device:

\[ F = \sum_i F_i \quad \tau = \sum_i M_i \]
Contact Model

For each contact, you will need

- The contact position on the tool,
- and one of
  - a force vector (magnitude + direction), or
  - a contact normal and penetration depth

\[ F = k_p d \hat{n} \]
Properties of Direct Rendering

What are the advantages and disadvantages?

[From B. Heidelberger et al., Vision Modeling and Visualization, 2004.]
Direct Rendering Summary

- **Advantages**
  - Easy to implement
  - Free space feels like free space

- **Limitations**
  - Object interpenetration
  - Pop-through
  - Force discontinuities
  - Unbounded stiffness!
Proxy-Based Rendering
This use of force is needed. More sophisticated treatment is required for large, precise selection of impedances to achieve high impedances, as shown in Figure 4.

Consider now the haptic display used during the peg insertion. The apparent stiffness felt at the tip of the virtual peg depends on the geometry of the tool, whereas our simulation parameters have considered only high impedances, not the very low impedances to which the tool might be sensitive due to haptic display damping. To handle this, additional physical damping allows higher impedances.

While the system quite nonlinear, the approach proposed here has the advantage of being straightforward and intuitive. It is designed to incorporate physical dampers, and even in the absence of physical damping, the system can be made passive, which ensures discrete time passivity.

A discrete time passive system is one in which ensuring sampled data passivity guarantees how the system behaves when sampled, as per the fundamental result that a system consisting of implicit equations, poor accuracy, there is an approach that does this directly.

6-DOF Virtual Coupling

- Translational and rotational spring/damper coupling
  - Force proportional to displacement
  - Torque proportional to orientation difference

- Virtual walls again!

[From W.A. McNeely et al., Proc. SIGGRAPH, 1999.]
Proxy Simulation in 3-DOF
Proxy Simulation in 6-DOF
Proxy Simulation

surface

F

T

?
Soft Constraints

\[ F_1 = k \Delta x_1 \]

\[ F_2 = k \Delta x_2 \]

\[ F_{\text{net}} = \sum_{i}^{n} F_i + F_{\text{vc}} \]
Proxy Motion

- Numerically integrate the ODE over time to obtain $\mathbf{x}$, the position of the avatar:

$$m\ddot{\mathbf{x}} = \mathbf{F}_{\text{net}}$$

- Do the same with moments to obtain orientation

$$\mathbf{F}_{\text{net}} = \sum_{i}^{n} \mathbf{F}_i + \mathbf{F}_{\text{vc}}$$
Potential Problems?

\[ F_{vc} = k_{vc} \Delta x \]
Quasi-Static Equilibrium

surface

avatar

$F_c$

$F_{net}$

$F_{vc}$
Quasi-Static Equilibrium

\[ F_c \]

\[ F_{\text{net}} \]

\[ F_{vc} \]
Quasi-Static Equilibrium

\[ \mathbf{F}_{\text{net}} = 0 \]

\[ \mathbf{F}_c \]

\[ \mathbf{F}_v \]

surface
Quasi-Static Proxy Motion

- Solve directly for the position $\mathbf{x}$ for which the net force acting on the proxy is zero:
  \[ \sum_{i} k \Delta x_i + k_{vc} \Delta x_{vc} = 0 \]

- Do the same with orientation to obtain net moment of zero

\[ F_{net} = \sum_{i} F_i + F_{vc} \]
Still Problems?

avatar
Hard Constraints

Generalized acceleration: \( \mathbf{a} \equiv (\ddot{\mathbf{a}}, \dddot{\mathbf{a}}) \)

Non-penetration constraint: \( \ddot{\mathbf{a}} \cdot \hat{\mathbf{n}} + \dddot{\mathbf{a}} \cdot (\mathbf{r} \times \hat{\mathbf{n}}) \geq 0 \)
Proxy Simulation
Solve for Contact Forces

Find $f_i$ which satisfy: 

$$a_i = \vec{a} \cdot \hat{n}_i + \vec{a} \cdot (r_i \times \hat{n}_i) \geq 0$$

With condition: 

$$f_i a_i = 0$$
Solve for Contact Forces

- Write motion of contact points as:
  \[ a = Af + b \]
- Express conditions in matrix form:
  \[ Af + b \geq 0, \quad f \geq 0 \quad \text{and} \quad f^T(Af + b) = 0 \]
- Solve linear complementarity problem for \( f \)
- Integrate ODE to obtain position as before

[From D. Baraff, Proc. SIGGRAPH, 1994.]
Solve Directly for Motion

\[ F, \tau \]

Surface

Device
Gauss’ Principle

- The proxy’s *constrained* motion is that which minimizes the acceleration energy:

\[
a_c = \arg \min_a \frac{1}{2} (F - Ma)^T M^{-1} (F - Ma)
\]

- Subject to the contact constraints:

\[
J_c a \geq 0
\]

- Solution can be obtained via quadratic programming or point projection

Solve Directly for Motion
Taxonomy

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<th>Soft Constraints</th>
<th>Hard Constraints</th>
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<td>Quasi-Static Equilibrium</td>
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[Adapted from M. A. Otaduy et al., Proceedings of the IEEE, 2013.]
Soft vs. Hard Constraints

\[ F_{\text{net}} = \sum_{i} F_i + F_{\text{vc}} \]

\[ \hat{a} \cdot \hat{n} + \tilde{a} \cdot (\mathbf{r} \times \hat{n}) \geq 0 \]
Proxy With vs. Without Mass

\[ m \ddot{x} = F_{net} \]

\[ F_{vc} = k_{vc} \Delta x \]

\[ \sum_{i}^{n} k \Delta x_i + k_{vc} \Delta x_{vc} = 0 \]
Demo
Summary

- Motivation for 6-DOF haptic rendering
- Direct rendering
  - Like force fields: not very good!
- Proxy-based rendering
  - Taxonomy of proxy-based methods
- On Thursday:
  - Study examples of 6-DOF rendering methods