Genetic Inheritance, Revisited

• Person has 2 genes for trait (eye color)
  ▪ Child receives 1 gene (equally likely) from each parent
  ▪ Child has brown eyes if either (or both) genes brown
  ▪ Child only has blue eyes if both genes blue
  ▪ Brown is “dominant” (d), Blue is “recessive” (r)
  ▪ Parents each have 1 brown and 1 blue gene

• 4 children, what is $P(3$ children with brown eyes$)$?
  ▪ Child has blue eyes: $p = (\frac{1}{2}) (\frac{1}{2}) = 14$ (2 blue genes)
  ▪ $P($child has brown eyes$) = 1 - (\frac{1}{4}) = 0.75$
  ▪ $X = \#$ of children with brown eyes. $X \sim Bin(4, 0.75)$
    \[
P(X = 3) = \binom{4}{3} (0.75)^3 (0.25)^1 \approx 0.4219\]
Error Correcting Codes

- Error correcting codes
  - Have original 4 bit string to send over network
  - Add 3 “parity” bits, and send 7 bits total
  - Each bit independently corrupted (flipped) in transition with probability 0.1
    - $X = \text{number of bits corrupted: } X \sim \text{Bin}(7, 0.1)$
    - But, parity bits allow us to correct at most 1 bit error
- $P(\text{a correctable message is received})$?
  - $P(X = 0) + P(X = 1)$
Error Correcting Codes (cont)

- Using error correcting codes: $X \sim \text{Bin}(7, 0.1)$
  \[
P(X = 0) = \binom{7}{0} (0.1)^0 (0.9)^7 \approx 0.4783
  \]
  \[
P(X = 1) = \binom{7}{1} (0.1)^1 (0.9)^6 \approx 0.3720
  \]
  \[
  P(X = 0) + P(X = 1) = 0.8503
  \]
- What if we didn’t use error correcting codes?
  \[
  X \sim \text{Bin}(4, 0.1)
  \]
  \[
P(\text{correct message received}) = P(X = 0)
  \]
  \[
P(X = 0) = \binom{4}{0} (0.1)^0 (0.9)^4 = 0.6561
  \]
- Using error correction improves reliability $\sim 30\%$!
Value of Choices

- Consider value you derive (from some choice)
  - Say, 2 choices, each with \( n \) consequences: \( c_1, c_2, \ldots, c_n \)
  - One of consequences \( c_i \) will occur with probability \( p_i \)
  - Each consequence has some value: \( V(c_i) \)
  - Which choice do you make?

- Example: Buy a $1 lottery ticket (for $1M prize)?
  - Probability of winning is \( 1/10^7 \)
  - \textbf{Buy}: \( c_1 = \text{win}, c_2 = \text{lose}, V(c_1) = 10^6 - 1, V(c_2) = -1 \)
  - \textbf{Don’t Buy}: \( c_1 = \text{lose}, V(c_1) = 0 \)
  - \( E(\text{buy}) = 1/10^7 (10^6 - 1) + (1 - 1/10^7) (-1) \approx -0.9 \)
  - \( E(\text{don’t buy}) = 1 (0) = 0 \)
  - “You can’t lose if you don’t play!”
Probability Tree

- Model outcomes of probabilistic events with tree
  - Also called “chance nodes”
    - Coin flip
      - Heads: $p$
      - Tails: $1 - p$
    - Buy ticket?
      - yes: $p(1000000 - 1) + (1 - p)(-1)$
      - no: $0$

- Useful for modeling decisions

- Expected payoff:
  - yes = $p(1000000 - 1) + (1 - p)(-1)$
  - no = 0
Utility

- Utility $U(x)$ is “value” you derive from $x$
  
  - Can be monetary, but often includes intangibles
    - E.g., quality of life, life expectancy, personal beliefs, etc.

![Utility Diagram]

- Play?
  - yes
    - 0.5
    - $20,000
  - no
    - 0.5
    - $0

- $10,000
  - Play?
    - yes
      - 0.5
      - $20,000
    - no
      - 0.5
      - $0

  - Play?
    - yes
      - 0.5
      - $10,000
    - no
      - 0.5
      - $10,000
Micromort

- A **micromort** is 1 in 1,000,000 chance of death
  - How much would you need to be paid to take on the risk of a micromort?
  - How much would you pay to avoid a micromort?
    - $P(\text{die in plane crash}) \approx 1 \text{ in } 1,500,000$
    - $P(\text{killed by lightning}) \approx 1 \text{ in } 1,400,000$
  - How much would you need to be paid to take on a **decimort** (1 in 10 chance of death)?
  - If you think this is morbid, companies actually do this
    - Car manufacturers
    - Insurance companies
Non-Linear Utility of Money

- These two choices are different for most people
• Utility curve determines your “risk preference”
  ▪ Can be different in different parts of the curve
- First $50 is worth the same to you as “next” $50
First $50 is worth more to you than “next” $50
• First $50 is worth less to you than “next” $50
Risk Profiles

• Most people are risk averse
  ▪ Beyond some (reasonably small) amount
  ▪ Consider the notion of “necessities” vs. “luxuries”

• But there are some cases where people show risk seeking behavior
  ▪ Small cost, high potential payoff (with very low probability)
    ▪ E.g., playing the lottery
  ▪ Sometime “risk seeking” aspect is downplayed by giving utility to the “fun of playing”
    ▪ Total utility = expected payoff of the game + fun of playing the game
    ▪ E.g., gambling

• Utility functions change over time
  ▪ Tend to become less risk averse as economic viability increases
Utility Function Properties

- Increasing function
  - More money is preferred to less

- Continuous (smooth) function
  - Does not change “drastically”
  - A small change in input to function should not change output of function significantly

- Only the ordinal rankings of utility function matter for making a choice
  - Actual utility value may not be meaningful
  - Sometimes the unit of measurement is called “utils”
Maximizing Expected Utility

- Say your utility function is: $U(x) = \sqrt{x}$

- Consider the following “gambles”

  \[
  \begin{align*}
  \text{Gamble A: } & (0.5)U($100) + (0.5)U($0) = (0.5)10 + (0.5)0 = 5 \\
  \text{Gamble B: } & (1.0)U($36) = (1.0)6 = 6 \\
  \end{align*}
  \]

- Compute expect utility
  - Gamble A: $(0.5)U($100) + (0.5)U($0) = (0.5)10 + (0.5)0 = 5$
  - Gamble B: $(1.0)U($36) = (1.0)6 = 6$

- Select gamble that has maximal expected utility
  - Would choose Gamble B here
Compound Gamble

- I will flip a fair coin.
  - If “heads”, you win $5.
  - If “tails”, I roll a 6-sided die, you win $X where X is number rolled.

![Diagram of compound and reduced gambles]

**Compound gamble**

- 1/2 chance of winning $5
- 1/6 chance of winning $1
- 1/6 chance of winning $2
- 1/6 chance of winning $3
- 1/6 chance of winning $4
- 1/6 chance of winning $5
- 1/6 chance of winning $6

**Reduced (simple) gamble**

- 1/12 chance of winning $1
- 1/12 chance of winning $2
- 1/12 chance of winning $3
- 1/12 chance of winning $4
- 7/12 chance of winning $5
- 1/12 chance of winning $6
Justifying Expected Utility Maximization

- Subscribing to these properties $\rightarrow$ maximize expected utility
  - You are indifferent between *compound* gamble and *simple* gamble to which it reduces using probability theory
  - For two gambles A and B, you are willing to say $A \geq B$ or $B \geq A$
  - If $A \geq B$ and $B \geq C$, then $A \geq C$ (Transitivity)
  - If $A > B$ and $B > C$, then $E > B > D$, where

<table>
<thead>
<tr>
<th>Gamble D (“almost C”)</th>
<th>Gamble E (“almost A”)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - p$</td>
<td>$1 - p$</td>
</tr>
<tr>
<td>$p$</td>
<td>$p$</td>
</tr>
<tr>
<td>Gamble A</td>
<td>Gamble C</td>
</tr>
<tr>
<td>Gamble C</td>
<td>Gamble A</td>
</tr>
</tbody>
</table>

- If $A > B$, then $D > E$ where for any $p > 0$