Many people have exponential (risk averse) curves

\[ U(x) = 1 - e^{-x/R} \]

- R is your “risk tolerance”
- Larger R = less risk aversion
  - Makes utility function more “linear”
- \( R \approx \) highest value of Y for which you would play:

```
Play?  yes  no
       0.5  0.5

Y       0.5

-$Y/2  $0
```
Caveat I: Framing Effects

• How gambles are *framed* matters
  ▪ There are psychological effects that aren’t captured in model
  ▪ More on this later in class

• Zero illusion
  ▪ Gambles are usually not in a vacuum
    ○ Coin flip for $10 vs. -$5 is really (bank account + $10) vs. (bank account - $5)
  ▪ Utility function needs to reflect your assets, potential, etc. (which includes intangibles)
    ○ This can be really hard to determine

• Take care in determining and verifying utility functions
  ▪ Try assessing many points along your utility curve
  ▪ Adjust curve and repeat
Caveat II: Portfolio Effects

- The decisions you make may sometime need to be considered as part of portfolio
  - E.g., buying stock in a company you work for
    - If company does badly, stock goes down and you might lose your job
    - There is correlated risk here: losing job impacts utility function
- Sometimes there is negative correlation
  - E.g., working for a company that does well when the economy does poorly (e.g., vehicle repossession)
    - Job security is negatively correlated with value of other assets
- Need to be mindful of this in decision making
Thomas Bayes Needs a Volunteer

So good to see you again!
Two Envelopes

• I have two envelopes, will allow you to have one
  ▪ One contains $X$, the other contains $2X$
  ▪ Select an envelope
    ○ Before you open it, want to switch for other envelope?
  ▪ Open it. Would you like to switch for other envelope?
  ▪ To help you decide, compute $E[$ in other envelope$]$
    ○ Let $Y = $ in envelope you selected
      $$E[$ in other envelope$] = \frac{1}{2} \cdot \frac{Y}{2} + \frac{1}{2} \cdot 2Y = \frac{5}{4} Y$$
  ▪ Before opening envelope, think either equally good
  ▪ So, what happened by opening envelope?
    ○ And does it really make sense to switch?
Discuss!
Two Envelopes Solution

• The “two envelopes” problem set-up
  ▪ Two envelopes: one contains $X$, other contains $2X$
  ▪ You select an envelope and open it
    o Let $Y = \$ in envelope you selected
    o Let $Z = \$ in other envelope
      \[ E[Z | Y] = \frac{1}{2} \cdot \frac{Y}{2} + \frac{1}{2} \cdot 2Y = \frac{5}{4} Y \]
  ▪ Before opening envelope, think either equally good
    o So, what happened by opening envelope?
  ▪ $E[Z | Y]$ above assumes all values $X$ (where $0 < X < \infty$) are equally likely
    o Note: there are infinitely many possible values of $X$
    o Can’t have equal (non-zero) probabilities over infinitely many possibilities (total probability of all outcomes won’t sum to 1)
Subjectivity of Probability

• Belief about contents of envelopes
  - Since implied probability over X is not a true probability distribution, what is our probability distribution over X?
    - Frequentist: play game infinitely many times and see how often different values come up.
    - Problem: I only allow you to play the game once
  - Bayesian probability
    - Have prior belief of probability for X (or anything for that matter)
    - Prior belief is a subjective probability
      - By extension, all probabilities are subjective
    - Allows us to answer question when we have no/limited data
      - E.g., probability a coin you’ve never flipped lands on heads
The Envelope, Please

• **Bayesian**: have prior probability over X, P(X)
  - Let Y = $ in envelope you selected
  - Let Z = $ in other envelope
  - Open your envelope to determine Y
  - If Y > E[Z | Y], keep your envelope, otherwise switch
    - No inconsistency!
  - Opening envelope provides data to compute P(X | Y) and thereby compute E[Z | Y]
  - Of course, there’s the issue of how you determined your prior distribution over X…
    - Bayesian: Doesn’t matter how you determined prior, but you must have one (whatever it is)
    - Imagine if envelope you opened contained $20.01
The Dreaded Half Cent

![Image of a coin with 'LIBERTY' on one side and 'HALF CENT' on the other side, with the year 1847.]
Probability Gets Weird

• Consider that we have three spinners:
  
  ▪ Each spinner has probability of getting some number
  ▪ You and opponent each pick a distinct spinner
  ▪ Person who spins highest number wins
    ○ You get to choose first!
Probability Gets Weird

- Consider that we have three spinners:

- If you are only choosing between A and B, what is pick?
  - A has 0.56 chance of winning

- If you are only choosing between A and C, what is pick?
  - A has 0.51 chance of winning

- If you are only choosing between B and C, what is pick?
  - B has \((0.56 + 0.22) \times 0.51 + 0.22 \times 1 = 0.6178\) chance of winning

- A dominant and C dominated with two players
Probability Gets Weird

Consider that we have three spinners:

- A has $0.56 \times 0.51 = 0.2856$ chance of winning
- B has $0.22 \times 0.51 + 0.22 \times 1 = 0.3322$ chance of winning
- C has $0.49 \times 0.78 = 0.3822$ chance of winning

C is best choice with three players
A fares the worst with three players

What if spinners represent efficacy of three different medicines?

This is known as “Blythe’s Paradox”