EE178 Autumn 2017-18
Problem Session 9

11/30/2017

1. A professor gives tests that are hard, medium, or easy. If she gives a hard test, her next test will be either medium or easy with equal probability. However, if she gives a medium or easy test, there is a 0.5 probability that her next test will be of the same difficulty, and a 0.25 probability for each of the other two levels of difficulty. Construct an appropriate Markov chain and find the steady-state probabilities.

2. Consider a Markov chain with states $\{1, 2\}$ transition probability matrix

\[
\Pi = \begin{bmatrix}
0.7 & 0.3 \\
0.4 & 0.6
\end{bmatrix}
\]

We will show that for any initial distribution $V(0) = (V_1(0), V_2(0))$, the distribution at time $n$, $V(n)$ converges to the stationary distribution.

(a) For $V(0) = (0.1, 0.9)$, compute,

i. $P(X_2 = 1, X_1 = 1, X_0 = 1)$

ii. $P(X_2 = 1, X_0 = 1)$

iii. $P(X_2 = 1|X_0 = 1)$

(b) Compute the stationary distribution for this Markov chain.

(c) Use induction to show that

\[
\Pi^n = \begin{bmatrix}
\frac{4}{7} + \frac{2}{7}(0.3)^n & \frac{3}{7} - \frac{3}{7}(0.3)^n \\
\frac{4}{7} - \frac{4}{7}(0.3)^n & \frac{2}{7} + \frac{4}{7}(0.3)^n
\end{bmatrix}
\]

(d) Compute $V(n)$ in terms of $V(0)$ and $n$. Does it converge to the stationary distribution?

3. We consider a game where you start with 1 point. At each step you either lose a point or gain a point with probability 0.5 each. If you reach 0 points, you stop the game and stay at 0 points forever. Similarly, if you reach 3 points, you stop the game and stay at 3 points forever.

(a) Model this process as a Markov chain. Write the transition probability matrix.

(b) Compute the probability that the game ends with 0 points (assuming you start with 1 point).