Sample Final Examination Problems

1. **Choice of modulating signal.**
   a. Throughout the course, the modulating carrier signal for both AM and FM was chosen to be $\cos 2\pi ft$. Why is cosine preferable to sine?
   b. Suppose that we modulate with a nonsinusoidal periodic signal, such as a square wave or a sawtooth function. What changes should be made to the modulating system?

2. **Overmodulation.** A signal $m(t) = \sin 2\pi t$ is transmitted using AM modulation:
   $$\varphi_{AM}(t) = (1 + km(t)) \cos 20\pi t.$$  
   a. Does the bandwidth of $\varphi_{AM}(t)$ depend on the modulation index $k$?
   b. Sketch the envelope of $\varphi_{AM}(t)$ for $k = 1.2$.
   c. Can the signal $m(t)$ be recovered when $k > 1$? Explain briefly.

3. **Quadrature demodulation.** The main motivation for the in-phase and quadrature representation of modulated signals is to utilize the fact that sine and cosine are orthogonal signals, and thus can be used to carry independent information over a channel. In particular, a modulated signal $s(t) = s_I(t) \cos(2\pi ft) - s_Q(t) \sin(2\pi ft)$ modulated with different information signals $s_I(t)$ and $s_Q(t)$ can be demodulated such that both information signals can be detected in the receiver. This is especially useful for digital signals. Consider the quadrature demodulator system shown in the following figure. Assume $n(t) = 0$.

   ![Quadrature Demodulator Diagram]

   a. The phase offsets in the carriers are nonzero but known. Express the outputs of the demodulators in terms of the original signals $s_I(t)$ and $s_Q(t)$.
   b. Are there values of $\phi_1$ and $\phi_2$ such that $s_I(t)$ and $s_Q(t)$ cannot be recovered from the demodulator output?
   c. Suppose that $\phi_1$ and $\phi_2$ are small. Find formulas for $s_I(t)$ and $s_Q(t)$ in terms of $\tilde{s}_I(t)$ and $\tilde{s}_Q(t)$ that are linear in $\phi_1$ and $\phi_2$.

4. **Sine from sinc.** By the sampling theorem (page 306 in the textbook), $\sin t$ can be reconstructed from samples at $t = k\pi/2$, $k = 0, \pm 1, \pm 2, \ldots$ (twice the Nyquist rate).
   a. Write $\sin t$ in terms of the sample values and the function $\sin x/x$.
   b. Use part (a) to find a series for $\sin \pi/4 = \sqrt{2}/2$. 
5. **Ramp random process.**
   
a. Let $X(t)$ be a random process with $P\{X(t) = t\} = \frac{1}{2}$ and $P\{X(t) = 2 - at\} = \frac{1}{2}$ for every $t$. Find the mean and autocorrelation functions of $X(t)$.
   
b. For what value(s) of $a$ is $X(t)$ a WSS random process?

6. **Error control coding for AWGN channel.** Binary data is sent over a communication link with additive white Gaussian noise. The measured bit error rate is 1%.
   
a. Find the signal-to-noise ratio corresponding to bit error probability 0.01.
   
b. To reduce the error probability, each bit is transmitted three times, and the receiver uses majority vote to estimate the bit. Find the error probability for this coding method.
   
c. A more advanced receiver adds the analog values of the three received signals and decides based on $Y < 0$ or $Y > 0$:

   $$Y = Y_1 + Y_2 + Y_3 = (X_1 + Z_1) + (X_2 + Z_2) + (X_3 + Z_3),$$

   where $X_1, X_2, X_3$ are the three copies of the transmitted bit and $Z_1, Z_2, Z_3$ are three independent noise random variables. Find the signal-to-noise ratio and the corresponding bit error probability.

   A table and plot of the $Q(\cdot)$ function are on the next page.
Table and plot of $Q(x)$

Table of $Q(x)$ for $0 \leq x \leq 4$.

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<th>$Q(x)$</th>
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Plot of $Q(x)$ for $0 \leq x \leq 6$. 

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