Amplitude Modulations, III

- Power of AM signals
- Single sideband modulation (SSB)
  - Frequency domain
  - Modes
  - Time domain; Hilbert transform
- Vestigial sideband modulation (VSB)
  - VSB spectrum
  - Modulator and demodulator
  - NTSC TV signals
- Quadrature modulation
  - Spectral efficiency
  - Modulator and demodulator
Power of AM Signals

The power of an AM signal is the sum of the power of two components.

$$\varphi_{AM}(t) = (A + m(t)) \cos \omega_c t = A \cos \omega_c t + m(t) \cos \omega_c t$$

Carrier power is

$$P_c = A^2 \frac{1}{2\pi\omega_c} \int_0^{1/2\pi\omega_c} \cos^2 \omega_c \, dt = \frac{1}{2} A^2$$

Signal power is

$$P_s = \frac{1}{2} P_m,$$

where message power is average power over an “appropriate” interval:

$$P_m = \overline{m^2(t)} = \frac{1}{T} \int_{t_0}^{t_0+T} m^2(t) \, dt$$

Example: the power of a tone $\cos \omega_m t$ is $\frac{1}{2}$. 
Power of AM Signals (cont.)

The carrier tone simplifies demodulation but carries no information.

The power efficiency is defined by

\[ \eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{m^2(t)}{A^2 + m^2(t)} \]

Examples: tone modulation \( m(t) = \mu A \cos \omega_c t \) where \( 0 < \mu \leq 1 \).

\[ \eta = \frac{\frac{1}{2}(\mu A)^2/2}{A^2/2 + \frac{1}{2}(\mu A)^2/2} = \frac{\mu^2}{2 + \mu^2} \]

The efficiency increases with \( \mu \); the maximum value is \( 1/3 \).

Efficiency falls off rapidly as \( \mu \) decreases. For \( \mu = 0.5 \),

\[ \eta = \frac{(0.5)^2}{2 + (0.5)^2} = \frac{1}{9} \]

(Don’t worry about efficiency; baseband would need much more power.)
Single Sideband (SSB) in Frequency Domain

(a) Baseband

(b) DSB

(c) USB

(d) LSB

(e) 

$M(f)$

- $B$
- $0$
- $B$

$f ightarrow$

$-f_c$

$0$

$f_c$

$-2f_c$

$0$

$2f_c$

$f ightarrow$
SSB Tone Modulation

The modulated sinusoid is a sinusoid at frequency $f_c \pm f_m$. 

(a)  

(b)  

(c)  

(d)  

---

EE 179, April 21, 2014  
Lecture 10, Page 5
SSB Modes

Shortwave radio uses upper MF and all of HF (1.8–30 mHz).

Transmitter and receiver must agree on use of LSB vs. USB.

International broadcast bands:

- 120m (2300-2495 kHz): LSB
- 90m (3200-3400 kHz): LSB
- 75m (3900-4000 kHz): USB
- 60m (4750-5060 kHz): LSB
- 49m (5900-6200 kHz): USB
- 41m (7200-7450 kHz): USB

NIST’s WWV broadcasts time from Fort Collins, CO, on frequencies 2.5, 5, 10, 15, and 20 MHz. Each frequency uses a different antenna tower whose length is half the wavelength (from 15m to 120m). Effective radiated power (ERP) is 2.4 kW for 2.5 and 20 MHz and 10 kW other frequencies.
SSB in Time Domain

The upper sideband is the output of filtering a modulated signal $m(t) \cos \omega_c t$ with an ideal bandpass filter:

$$H_{USB}(f) = \begin{cases} 
1 & f_c \leq |f| \leq f_c + B \\
0 & \text{otherwise} 
\end{cases}$$

This signal can be represented using the Hilbert transform of $m(t)$:

$$\phi_{USB}(t) = m(t) \cos \omega_c t \pm m_h(t) \sin \omega_c t$$

where

$$m_h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\alpha)}{t - \alpha} \, d\alpha = m(t) * \frac{1}{\pi t}$$

The transfer function of the Hilbert transform is

$$H(f) = -j \operatorname{sgn} f = \begin{cases} 
-j = 1 \cdot e^{-j\pi/2} & f > 0 \\
j = 1 \cdot e^{j\pi/2} & f < 0 
\end{cases}$$
SSB Modulation

The phase shifters are linear time-invariant circuits that can be implemented using convolution in the time domain.
Vestigial Sideband Modulation (VSB)

- VSB is similar to SSB but it retains a small portion (a vestige) of the unneeded sideband. This reduces DC distortion.

- VSB signals are generated using standard AM or DSB-SC modulation, then passing modulated signal through a sideband shaping filter.

- Demodulation uses either standard AM or DSB-SC demodulation, depending on whether a carrier tone is transmitted.

- VSB modulation with envelope detection are used to modulate image in analog TV signals. (The audio signal is modulated using FM.)
VSB Spectrum

In vestigial sideband, part of the lower sideband is retained.

A nonideal bandbass filter is used to cut off the lower sideband gradually.
VSB Modulator

The transmitted signal has spectrum

\[ \Phi_{\text{VSB}}(f) = (M(f + f_c) + M(f - f_c)) H_i(f) \]

where \( H_i(f) \) is the *shaping filter* for the VSB modulator.

Modulator and demodulator:
The intermediate demodulator signal

\[ e(t) = \phi_{VSB}(t) \cdot 2 \cos \omega_c t \]

has spectrum

\[ \Phi_{VSB}(f + f_c) + \Phi_{VSB}(f - f_c) \]

We can recover \( m(t) \) by using a filter \( H_o(f) \) defined by

\[ H_o(f) = \frac{1}{H_i(f + f_c) + H_i(f - f_c)} , \quad |f| \leq B \]

This is a low-pass filter that is the inverse of \( H_i(f) \) when \( |f| \leq B \).
VSB Demodulation Filter

Surprisingly, $H_o(f)$ is not large for small values of $f$. 

\begin{align*}
\text{(a)}
\end{align*}

\begin{align*}
\text{(b)}
\end{align*}

\begin{align*}
\text{(c)}
\end{align*}
VSB Example: US TV NTSC

Using VSB instead of DSB saves about 3 MHz and allows a carrier tone. The shaping filter satisfies $H_i(f - f_c) + H_i(f + f_c) = c$, so $H_o(f) = 1/c$. 

---

**Diagram**

(a) DSB spectrum

- Audio spectrum (FM) centered at $f_c$ with a bandwidth of 0.25 MHz.
- Spectrum ranges from 0 to $f_c + 4.5$ MHz.

(b) VSB spectrum

- Bandwidth of 0.75 MHz centered at $f_c$.
- Spectrum ranges from $f_c - 1.25$ MHz to $f_c + 4.5$ MHz.
NTSC TV Signals

An analog TV image (*frame*) consists of rows that represent horizontal slices of a snapshot. The displayed image is continuous time, continuous valued replication of the image captured by a camera.

The sync pulse tells the TV circuitry to rapidly move the CRT beam to the beginning of the next line. When not retracing, the location of the beam varies linearly in both vertical and horizontal directions.
NTSC Transmitter and Receiver

(a) Audio signal → Audio amplifier → FM modulator

TV camera → Video + synchronizing-and blanking-pulse generator

Vertical and horizontal sweep → Vertical and horizontal blanking → Synchronizing and blanking signals

Sweep-synchronizing-and blanking-pulse generator

AM mod → Partial vestigial filter

(b) Received TV carrier → Frequency converter

RF shaping filter & amplifier → IF amplifier → Envelope detector

Video amplifiers & synchronization separator → DC restorer

FM demodulator → Audio amplifier

Synchronization separator → Sweep generator

Picture tube → Speaker

EE 179, April 21, 2014
Lecture 10, Page 16
Quadrature Amplitude Modulation (QAM)

- DSB-SC modulates a signal with bandwidth $B$ to a transmitted signal with bandwidth $2B$
- SSB reduces the transmitted bandwidth to $B$, but
  - requires more complex modulator
  - reduces SNR (for a fixed carrier amplitude)
- Quadrature amplitude modulation uses the $2B$ transmitter bandwidth to send two independent signals:
  \[
  \varphi_{\text{QAM}} = m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t
  \]
- QAM has the same *spectral efficiency* as SSB but does not need sharp band-pass filters
- QAM is used in almost all digital communication methods, including telephone modems, cable TV, satellite TV
At the demodulator:

\[ x_1(t) = \varphi_{QAM}(t) \cdot 2 \cos \omega_c t \]
\[ = (m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t) \cdot 2 \cos \omega_c t \]
\[ = 2m_1(t) \cos^2 \omega_c t + 2m_2(t) \sin \omega_c t \cos \omega_c t \]
\[ = m_1(t) + m_1(t) \cos 2\omega_c t + m_2(t) \sin 2\omega_c t \]

We can recover \( m_1(t) \) by passing \( x_1(t) \) through a LPF.