EE179 Spring 2021-22 Pauly
Introduction to Communications

Problem Set #1

Due: Friday Oct 8, 2021 at 5 PM.

1. *Satellite communications.* Satellite orbits are classified by their distance from the Earth's surface: LEO (low earth orbit, 160–2000 km), MEO (medium earth orbit, 2000–20000 km), and GEO (geostationary earth orbit, 35786 km). Find the round-trip delay of data sent between a satellite and the earth for LEO, MEO, and GEO satellites, assuming the speed of light is 3×10^8 m/s. If the maximum acceptable delay for a voice system is 30ms, which of these satellite systems would be acceptable for two-way voice communication?

2. *Scaled Impulse Functions.* In EE 102A we never scaled impulses in time. This is often useful.

   (a) Show that
   $$\delta(at) = \frac{1}{|a|}\delta(t).$$
   Start by evaluating
   $$\int_{-\infty}^{\infty} f(t)\delta(at)dt$$
   and then argue the conclusion.

   (b) Show that
   $$\delta(\omega) = \frac{1}{2\pi}\delta(f)$$
   where $\omega = 2\pi f$

   (c) Show that the transforms for $\cos(\omega t)$ in ω and $\cos(2\pi f t)$ in $2\pi f$ are consistent.

3. *Capacity of AWGN channel.* The capacity in bits per second of an additive white Gaussian noise (AWGN) channel is
   $$C = B\log_2\left(1 + \frac{P}{N_0 B}\right)$$
   where P is the received signal power, B is the signal bandwidth, and $N_0/2$ is the noise power spectral density (PSD). (The total noise power is $N_0 B$.) Consider a wireless channel where received power falls off with distance d according to the formula $P(d) = P_t\left(\frac{d_0}{d}\right)^3$. Given $d_0 = 10$ m, transmitter power $P_t = 1$ W, noise PSD $N_0 = 10^{-9}$ W/Hz, and channel bandwidth $B = 30$ KHz, find the capacity of this channel for transmitter-receiver distances of 100 m and 1 km.

4. *Correlation* The cross correlation operation is used for matched filtering. If $x(t)$ is a signal we are looking for in another signal $y(t)$, we perform the operation

$$(x \star y)(t) = \int_{-\infty}^{\infty} x(\tau) y^*(\tau - t) dt = \int_{-\infty}^{\infty} x(\tau + t) y^*(\tau) dt$$

   Derive a simple Fourier transform theorem for the cross correlation $\mathcal{F}\{(x \star y)(t)\}$, and the autocorrelation $\mathcal{F}\{(x \star x)(t)\}$.

5. *Hilbert Transform* The signal $m(t)$ is convolved with a Hilbert transform filter to produce the signal

$$m_h(t) = m(t) * \frac{1}{\pi t}$$

   (a) Parseval's theorem states that the energy in the time domain is the same as the energy in the frequency domain. For $2\pi f$ Fourier transforms this means that

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

   for a signal $x(t)$ with a Fourier transform $X(f)$. Use this to show that the energy of $m(t)$ is the same as for $m_h(t)$.

   (b) Show that

$$m_h(t) * \frac{1}{\pi t} = -m(t).$$