Problem Set #2
Due: Friday April 28, 2017 at 5 PM.

1. We have an input signal $m(t)$ that is band limited to $\pm B$ (its full bandwidth is 2B). We want to modulate it to a carrier frequency $f_c$ with the following system

\[
m(t) \xrightarrow{\text{Filter}} m(t) \cos(2\pi f_c t) \xrightarrow{\cos^3(2\pi f_o t)} \]

Design two different systems with different $f_o$'s that will produce the desired output. Specify $f_o$ and the filter for each. How large must $f_c$ be for this to work (this is seldom a problem!).

2. Another method for generating SSB signals is shown below

\[
m(t) \xrightarrow{\cos(2\pi (B/2) t)} \xrightarrow{-\pi/2} \xrightarrow{\cos(2\pi (f_c - B/2) t)} \xrightarrow{\pi/2} \xrightarrow{f} \xrightarrow{\cos(2\pi (B/2) t)} m_c(t) + m(t)
\]

where all of the signals are real. The $-\pi/2$ blocks simply take a $\cos()$ and output a $\sin()$ (a Hilbert transform).

(a) Draw a simpler version of this block diagram that uses complex signals. For example, the first two modulators on the left effectively take $m(t)$ and modulate it by $e^{j2\pi(B/2)t}$.
(b) Assume $m(t)$ has the spectrum

![Spectrum Diagram]

Sketch the spectrum after the first modulator, the lowpass filter.

(c) What operation does the last pair of modulators and the sum at the end perform? Sketch the final spectrum.

This approach is called the *Weaver Method*, and is frequently used in digital implementations. It is convenient in that only modulators and low pass filters are required. No Hilbert transform filter is needed.

3. The modulator described on slides 22 and 23 of Lecture 5 is called a *balanced mixer*. This exploits a non-linearity to modulate a signal to a carrier frequency. In class, we considered the case where there is a quadratic non-linearity. Assume that the nonlinearity also has a cubic term, so

$$y(t) = ax(t) + bx^2(t) + cx^3(t)$$

Again, assume the inputs are

$$x_1(t) = \cos(2\pi f_c t) + x(t)$$
$$x_2(t) = \cos(2\pi f_c t) - x(t)$$

What is the output $y_1(t) - y_2(t)$ of the balanced mixer? Is this usable as a mixer? What frequencies does it modulate to?