EE 252 Spring 2005 Handout #______
Len Tyler May 17, 2005

Instructions for Class Project

Performance of Partially-Filled Spiral Arrays

Due 3 PM, Friday, June 3, 2005

General

This project involves the design of a partially filled array with many potential applications, depending the final scale and wavelength selected. For the purpose of this project, though, we want to think big. Consider the "Very Large Array," located near Socorro, New Mexico. This system comprises 27 'elements,'each of which is a 25 m diameter dish antenna! The VLA, as it is known, was designed to provide a radio view of the sky with a resolution roughly equivalent to that of a 2.5 cm diameter optical telescope.

Here we wish to examine the use of a spiral pattern as the basis of the array geometry, in contrast with the "Y" of the VLA. (See http://www.vla.nrao.edu/ for the official NRAO website describing the VLA.) The question is, for a sparsely populated array, what are the characteristics of array patterns based on a spiral array configuration?

Technical requirements will be distributed the week of May 9, but you can begin thinking about this before then. We are particularly concerned with:

i. Design of a spiral array(s) with the same number of elements (27) and limiting size (27 km radius) as the VLA;

ii. Estimating the quality of the beam obtained for several distributions of elements;

iii. Comparing results for at least two of three spiral configurations: Archimedes, logarithmic, and exponential.

You are to work in teams of two. It may happen that the number of participants is odd, in which case there will be one team of three, whose report is expected to reflect the additional personnel resource available. Members of a team will receive the same grade on the project. **Provide the name of your team to the Instructor or TA no later than Monday, May 16.**

Alternative for EE144/245 Students. Students taking the laboratory course may elect to submit an *a priori* design for their construction project to EE252. This should also be done by a team of two working at the same general level of complexity and analysis as the assigned EE252 project. If you elect this option, **provide us with a project proposal by Monday, May 16.** We will evaluate the proposal and respond quickly as to its suitability.

For either the EE252 design or the EE144/245 option, evaluation of the design project will be based on the technical quality and completeness of the study report submitted.

A word to the wise... Unless you have independent computational resources you will need to rely to some extent on University computers to work out your solution to the design problem. Beware that the load on University machines increases dramatically near the end of a quarter, and especially near the end of the academic year! Attempts to finish a design at the last minute will run up against this problem.

Technical Background

The technical basis for this problem and its solution comes from the class discussion and related problem sets. There is very good information regarding array theory in *Stutzman & Thiele*, Chapter 3. You also may wish to consult some of the sources listed in the bibliography for this class.

Although most arrays described in introductory texts comprise elements located on a regularly spaced grid, approximating an aperture antenna, this is not necessary. For imaging purposes, one can trade the cost of array construction—expressed in terms of the number of elements placed within the array—against the observing time required to form the image. Radio astronomers routinely make this trade-off, in the VLA for example, by providing only a sufficient number of elements to sample the Fourier space of the image, from which a complete image can be constructed. In the radio astronomy application, the set of all possible pairwise spacing between the elements corresponds to the set of two-dimensional Fourier components in the image plane. So for the VLA, one thinks of all the lines that can be drawn between the 27 antennas of the array! Beyond this, for radio astronomers, the time invariance of most astronomical sources means that observations made at different times can be combined in the image construction. Given this opportunity, radio astronomers also make use of Earth's rotation to obtain additional sets of baselines by sampling the pairwise signals from the elements of an array at different times of day. The point being that the final beam of an imaging array is shaped by the element spacings over a range of baselines, and not the maximum length of a particular spacing, *per se*.

Traditional array design is based on regularly space grids of local array elements, for which there are rather elegant, well-developed procedures. From the point of view above, though, these are very inefficient in terms of the use of real estate. Here we are asking for an *ad hoc* design in which the array elements are located along a constrained path on a surface constrained to a circular area. Our design criteria will be the quality of the primary beam and the level of the side lobes. You are to find a combination of path parameters that optimize the beam structure for a specified set of pointing directions. This could require some sort of directed search to accomplish. Some early analysis should allow you—or at least help you—to decide on the neighborhood of a solution, if not a final solution, which can be followed by numerical determination of performance and a limited numerical search to optimize the result. You do not need to design the array elements, only the path and the location of the elements along the path.

Technical Requirements

Antennas can be located at any position along the spiral. Your are to examine carefully at least two types of spiral, which include an Archimedes, or linear spiral, and either a (*i*) logarithmic or (*ii*) exponential spiral, as described in the table below:

Spiral Type	Description	Inclusion
Archimedean	$\rho = a\,(\phi - \phi_0)$	Required
Logarithmic	$\rho = a \ln[(\phi - \phi_0) - 1]$	Team Choice
Exponential	$\rho = \exp[a(\phi - \phi_0)] - 1$	Team Choice

In the table, for each entry, the variable a is a "free" design parameter, radius ρ is constrained to the range $0 \leq \rho \leq 27$ km, and the offset ϕ_0 is chosen so that at the maximum value of ϕ, *e.g.*, ϕ such that $\rho = 27$ km, the spiral terminates at the horizontal, *i.e.*, x axis, where $\phi = 0$. The orientation of spiral end is specified in order to simplify the specification of the beam patterns.

Optimization Criteria

Case I For a random distribution of element locations along the array.

Case II For an optimum distribution of elements along the array that maximizes the ratio of the volume of the main beam to that of the side lobes within 10 beamwidths (BWs) of the peak of the main beam.

Required Products

Required products are as follows:

1. Test patterns with $\theta_{\text{peak}} = 0, \pi/4$, for a Case I test array constructed for an Archimedean spiral with parameters: $a = 1$, $r_{\text{max}} = 27\lambda$, and a uniformly spaced distribution of 27 elements along the spiral.
   - A geometrically accurate diagram showing the configuration of our array.
   - Patterns $P(\theta, \phi)$ for $\phi = 0, \pi/2$,
   - A contour plot of $P(\theta, \phi)$ for $\theta \leq \pi/4$.
2. Results for 27 km radius Archimedes spiral operating at $\lambda = 1$ m with a Case II element distribution:
   - A geometrically accurate diagram showing the configuration of our array.
   - Patterns in $\phi = 0, \pi/2$ planes for $\theta_{\text{peak}} = 0$.
   - Contour plot of $P(\theta, \phi)$ for $\theta \leq 10$ BW of main beam peak.

3. Results for 27 km radius 'ln' or 'exp' spiral also operating at $\lambda = 1$ m (except as noted) with a Case II element distribution:

   - A geometrically accurate diagram showing the configuration of our array.
   - Patterns in $\phi = 0, \pi/2$ planes for $\theta_{\text{peak}} = 0$.
   - Contour plot of $P(\theta, \phi)$ for $\theta \leq 10$ BW of main beam peak, also for $\theta_{\text{peak}} = 0$.
   - Contour plot of $P(\theta, \phi)$ for a spherical angle $\psi \leq 10$ BW of beam for array phased for maximum at $\theta_{\text{peak}} = \pi/4$, $\phi = 0$.
   - Contour plot of for geometry immediately above for a wavelength of 0.9 m.
   - Contour plot of $P(\theta, \phi)$ for spherical angle $\psi \leq 10$ BW of beam for array phased for peak at $\theta_{\text{peak}} = \pi/4$, $\phi = \pi/2$.

4. The parameters of your final spiral arrays:

   - Numerical values for the directivity of the primary beam, and the ratio of the volume of the main beam to that of the integrated side-lobe level near the primary beam.
   - The parameters of your final spiral configurations and the location of the elements along the spirals.

Instructions

Submit a written report on your project and the results on or before the date and time indicated above. Here are the ground rules, in no particular order:

1. Arrays as large as these probably will contain many grating-type lobes, although these may or may not have the usual regular repetitiveness in angle space exhibited by regular arrays. Note the restriction to examine the patterns only in the vicinity of the primary beam peak. In an actual array the grating lobes are mitigated by the choice of the individual elements—which are not addressed here—or by signal or image processing techniques in the data analysis.

2. Code your own solution(s).

3. Use MatLab and routines provided therewith for the needed numerical evaluations and plots of your results, as appropriate; for example, to show patterns as ϕ-cuts and contours.

4. Provide tables and diagrams as indicated in the list of required products, above.

5. Describe the method you use to obtain your final design, and provide evidence that at least a local convergence of design has been achieved, *i.e.*, that there is not a significantly better design in the 'space' near your solution.

6. In real life it sometimes happens that design specifications are in conflict—that is, some aspect(s) of the problem may over specified—if this occurs here, make clear the nature of the conflict and your assumptions in resolving it.

Your write-up need not be formal or longer than is needed for completeness. It needs, however, to be *both clear and literate.* The following items are required in your report:

1. A clear description of the method and/or a list of iteration steps by which you arrived at your design solutions; include a description of any preliminary analytical outline of the problem and your numerical analysis.

2. Plots and other required products in the lists above.

3. Include a copy of your code as an appendix to your report.

The project is due at the time indicated on page one, which is mid-afternoon on the day of the scheduled Final Exam. There will be a 10 percent penalty per day for late reports. If you are planning to graduate this quarter, be aware that the Registrar's hard deadline for submission of final grades for graduating students is 12 noon, June 9. Grades submitted after that time will not be included in calculations for satisfaction of degree requirements. If, however, you have to cope with an emergency near the end of the quarter let me know, and we will do what we can to help.

Corrections

May 17, 2005

1. Variables in equations describing spirals changed to be consistent with polar coordinates in the xy plane: $\rho \leftarrow r$, $\phi \leftarrow \theta$.

2. Notation added to specify direction of beam peak, θ_{peak}.

3. Wavelength specified as $\lambda = 1$ m in "Required Products," "2," and "3." Note exception for fifth entry under "3."

4. Entry "3" under"Instructions" clarified.

5. Other, minor and not so minor spelling and grammatical repairs!