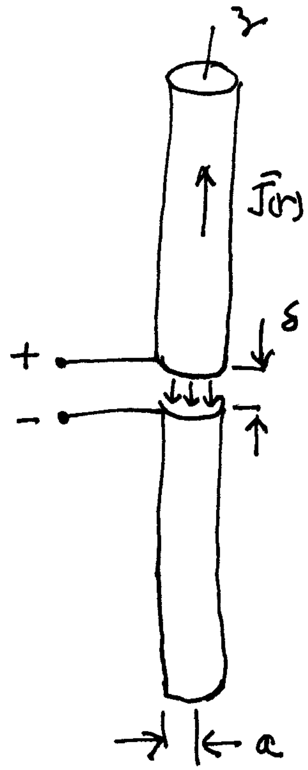
EE 252

Handout #30

Last time



$$\underbrace{\left(\nabla_r \nabla_r \cdot + k^2\right) \int_{vol} \frac{\bar{J}(\bar{r}')\, e^{-jkR}}{4\pi R}\, dv' = j\omega\epsilon_0 \bar{E}}$$

Pocklington's Eq.

$\downarrow$

$$\underbrace{\int_{-\ell/2}^{\ell/2} \frac{I(z')}{4\pi R}\, e^{-jkR}\, dz' = C \cos kz - j\frac{\omega\epsilon_0}{2k} \sin k|z| \qquad R^2 = (z-z')^2 + a^2 \;\; *}$$

Hallen's Equation

* current on z-axis, observation point on dipole, or v.v.

For solution to Hallen's Eq. we wrote

$$\int_0^{\ell/2} G(z,z')\, I(z')\, dz' = C \cos kz - j/_{2\eta} \sin k|z|$$

where $G(z,z') = \dfrac{e^{-jkr}}{4\pi r} + \dfrac{e^{-jkr'}}{4\pi r'}$

$r^2 = a^2 + (z-z')^2$; $r'^2 = a^2 + (z+z')^2$

Which we propose to solve approximately by method of Moments.

In general

$$\int_a^b G(z,z')\, f(z')\, dz' = g(z) \qquad \text{F. H., first kind}$$

$$f(z) \simeq \hat{f}(z) = c_1 f_1(z) + c_2 f_2(z) + \cdots + c_N f_N(z)$$

$$\sum_{n=1}^{N} c_n \underbrace{\int_a^b \int_a^b G(z,z')\, f_n(z')\, W_m(z)\, dz'\, dz}_{a_{mn}} = \underbrace{\int_a^b g(z)\, W_m(z)\, dz}_{b_m}$$

$$\sum_{n=1}^{N} c_n\, a_{mn} = b_m$$

/ . . .

Rewriting $[a_{mn}][c_n = [b_m$,

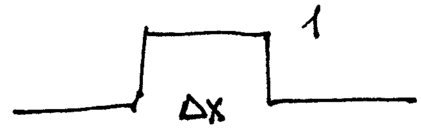
N equations in N unknowns

<u>Galerkin's Method</u> $w_m = f_m$

<u>Point-Matching Method</u> $w_m = \delta(z - z_m)$

Three Popular Basis Functions

Pulse



$$p_n(x) = \begin{cases} 1 & x \in \Delta x \\ 0 & x \notin \Delta x \end{cases}$$

Is the simplest and most common.

But there are alternatives that are more effective.

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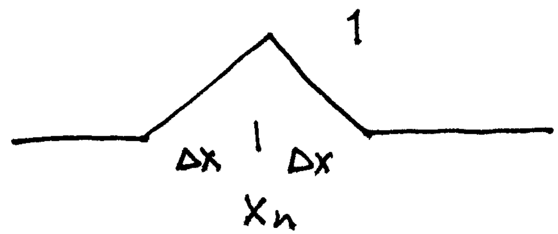
Triangle Basis Functions



$$t_n(x) = \frac{x - x_n + \Delta x}{\Delta x} \qquad x_n - \Delta x < x < x_n$$

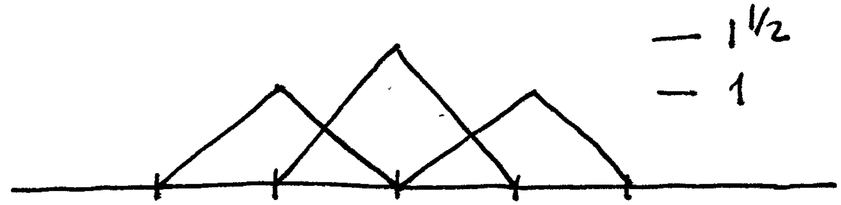
$$= -\frac{(x - x_n - \Delta x)}{\Delta x} \qquad x_n < x < x_n + \Delta x$$

$$= 0 \qquad \text{otherwise}$$

/...

Triangle basis functions provide a piecewise-linear approximation to the referenced function

E.g.



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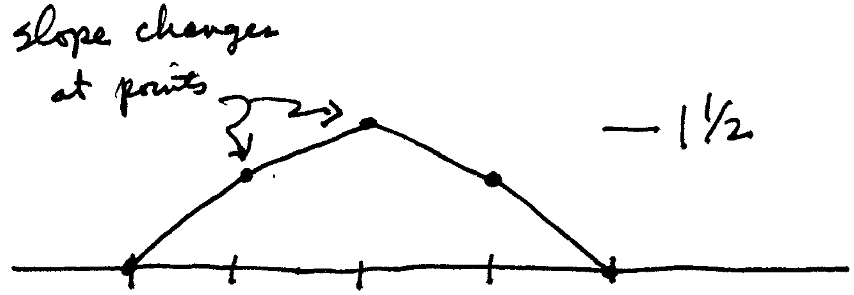


Similarly, a sinusoidal basis function is illustrated in S&T, Section 7.5 (pp 323-ff).

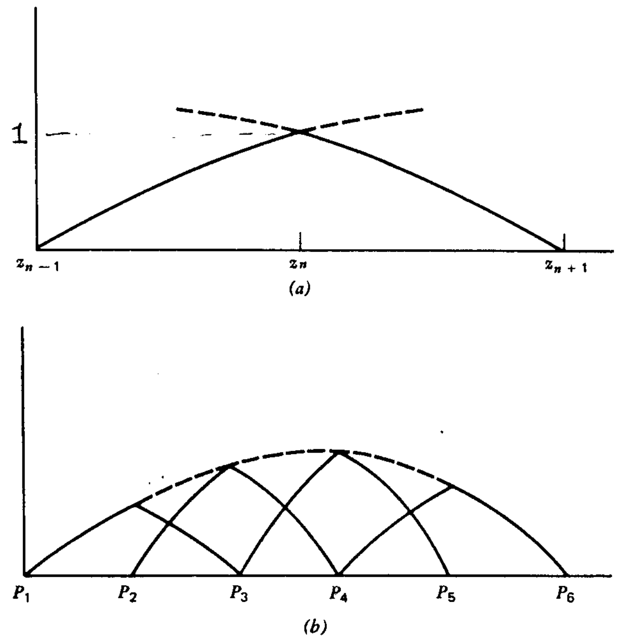


Figure 7-11 (*a*) Piecewise sinusoidal expansion function. (*b*) Set of overlapping piecewise sinusoidal expansion functions.

While the derivatives are still discontinuous at the "joins", the overall approximation to a general current distribution, e.g., is improved.

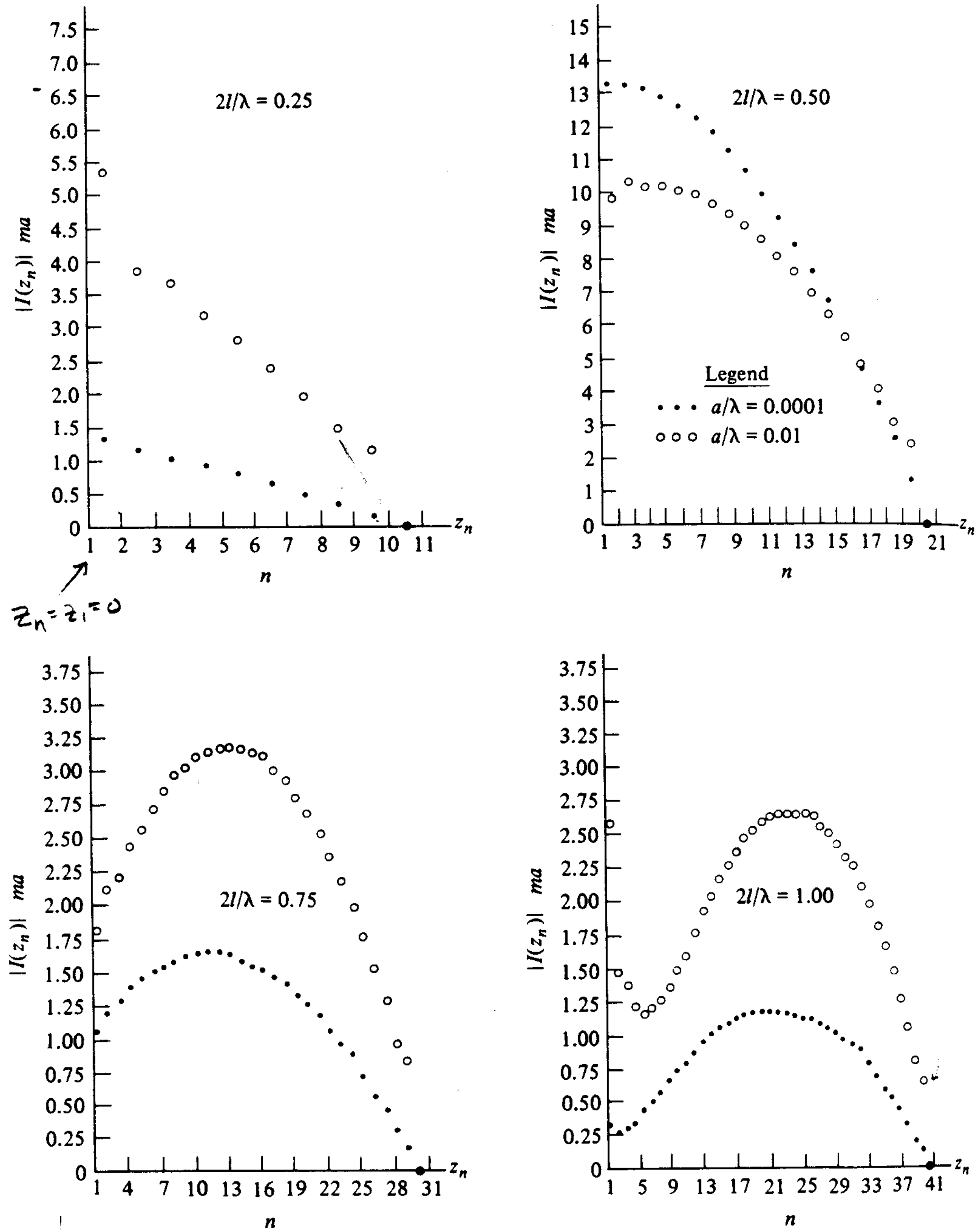
From S&T p. 324

Sample solutions for

$$\left(\frac{\lambda}{L}\right)^{-1} = 1/4,\ 1/2,\ 3/4,\ 1$$

$$a/\lambda = 0.0001,\ 0.01$$

shown for pulse function $f_n(z)$



$z_n = z_1 = 0$

Fig. 7.6 The Magnitude of $I(z)$ for Center-Fed Cylindrical Dipoles of Various Lengths and Diameters; Pulse Function Solution

from Elliott

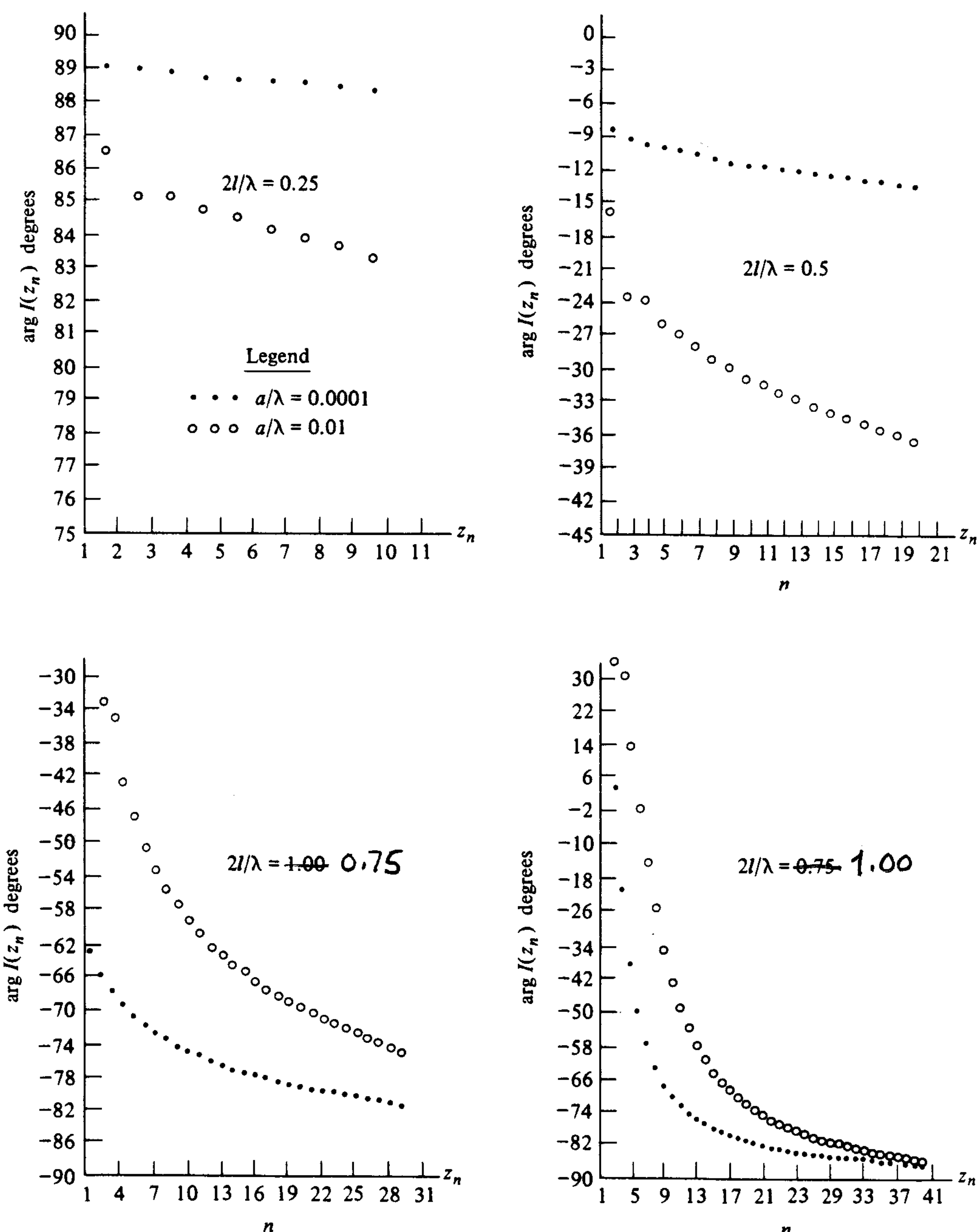


Fig. 7.7 The Phase of $I(z)$ for Center-Fed Cylindrical Dipoles of Various Lengths and Diameters; Pulse Function Solution

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Elliott

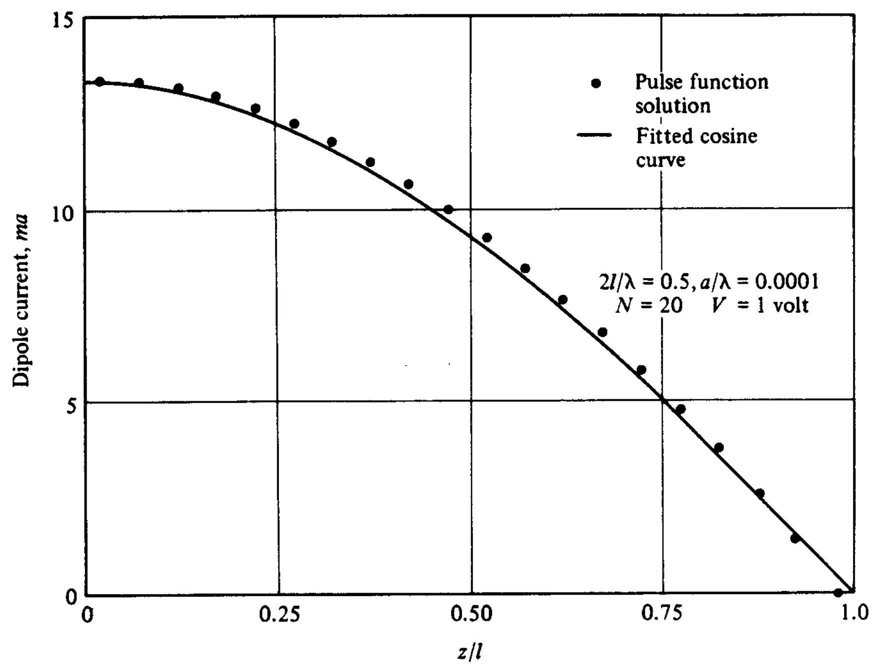

Fig. 7.8 The Current Distribution on a Half-Wavelength Long Center-Fed Cylindrical Dipole

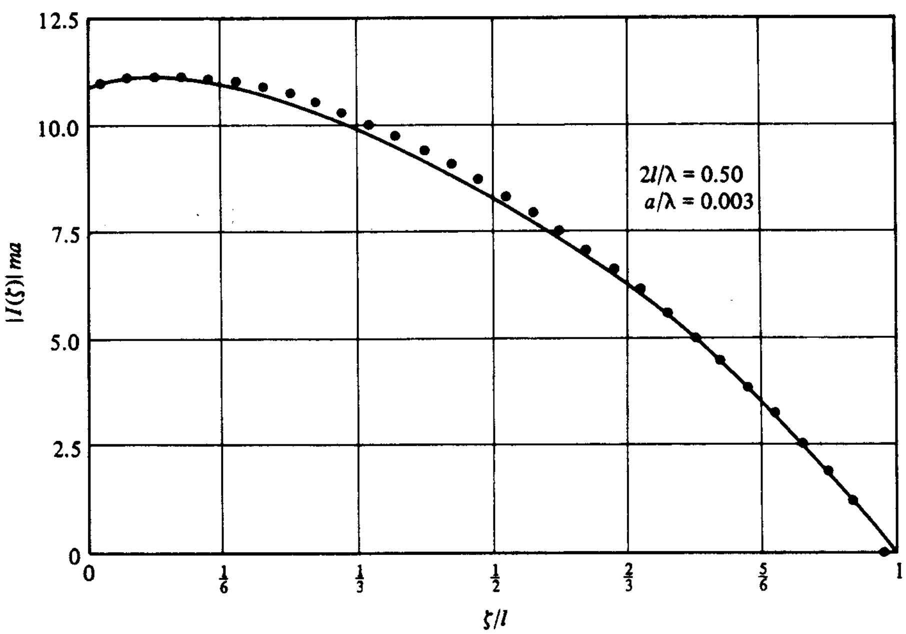

Fig. 7.9 Comparison of Theory and Experiment: Current Distribution on a Half-Wavelength Center-Fed Cylindrical Dipole (Solid Curve Experimental Results of T. Morita, *Proc. IRE*, vol. 38, pp. 898–904. © 1950 IEEE. Dots Computer Results Using Pulse Functions.)

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Obtained w/ sinusoidal basis functions

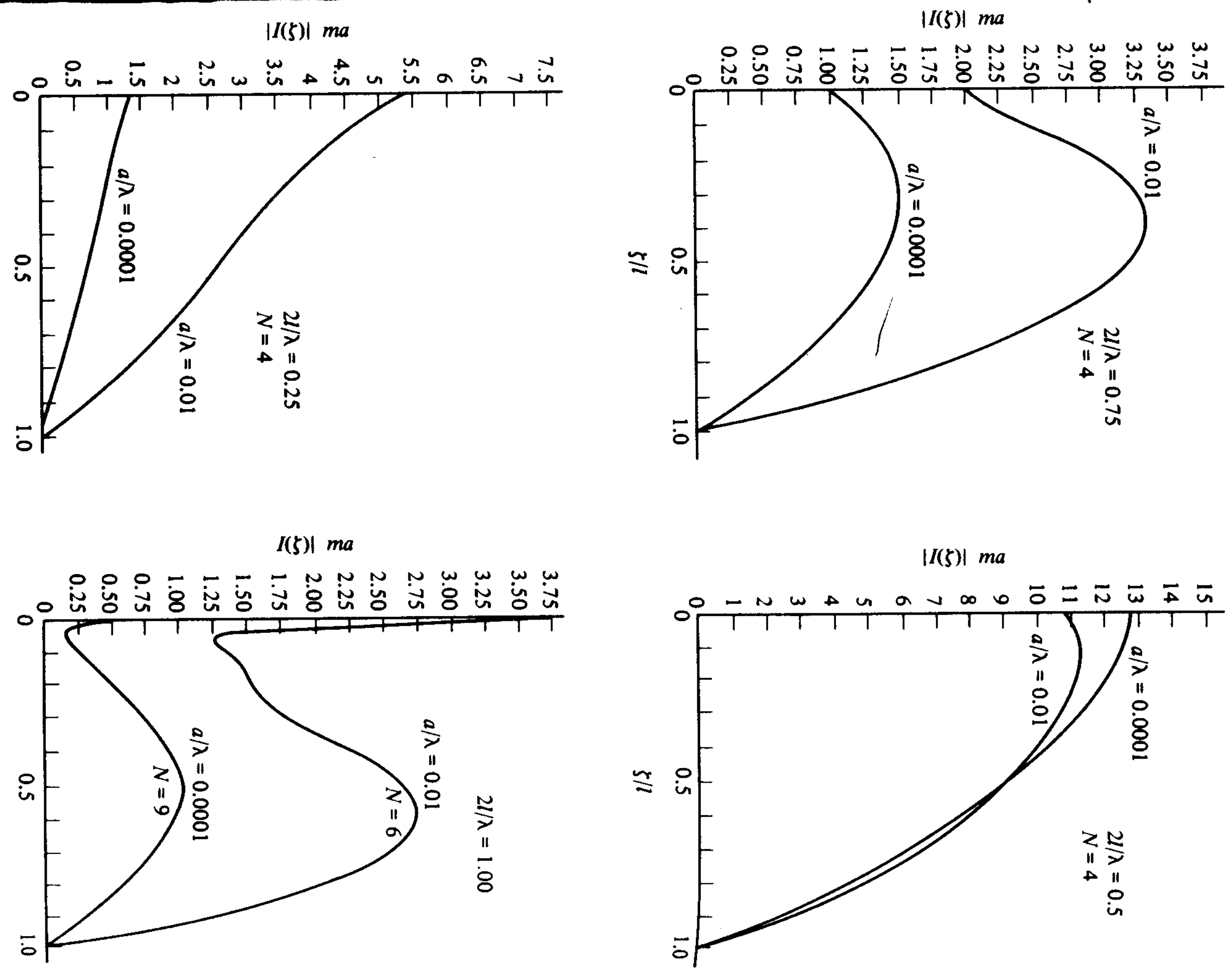

Fig. 7.10 The Magnitude of $I(z)$ for Center-Fed Cylindrical Dipoles of Various Lengths and Diameters; Solution Using Sinusoidal Basis Functions

Elliott

TABLE 7.5 Comparison of the input impedance of a center-fed cylindrical dipole using different computational methods

Normalized Length $2l/\lambda$	Normalized Radius a/λ	Input Impedance in Ohms: Storer's Two-Term Approximation	First-Order Approximation to Hallén's Equation	King-Middleton Second-Order Approximation
0.25	0.01	$11.63 - j185$	$15.99 - j240$	$13.98 - j166$
0.25	0.0001	$12.93 - j723$	$14.67 - j756$	$12.90 - j811$
0.50	0.01	$101 + j32.82$	$87.34 + j35.68$	$92.51 + j38.30$
0.50	0.0001	$80.15 + j42.61$	$79.08 + j43.52$	$79.89 + j43.47$
0.75	0.01	$566 + j3.10$	$437 + j318$	$543 + j32.2$
0.75	0.0001	$521 + j1019$	$433 + j1018$	$540 + j1016$
1.00	0.01	$290 - j363$	$559 - j594$	$177 - j339$
1.00	0.0001	$2370 - j2129$	$3052 - j2626$	$2233 - j2150$

TABLE 7.1 $\mathcal{A}_\theta(\theta)$ versus θ for idealized current distribution and for current distribution found using pulse functions

	$\mathcal{A}_\theta(\theta)$	
$\theta°$	$I(z) = I_m \sin k(l - z)$	$I(z)$ Found by Using Pulse Functions
0	0.000	$0.000\angle 1.163°$
6	0.082	$0.081\angle 1.141°$
12	0.165	$0.162\angle 1.098°$
18	0.249	$0.244\angle 1.029°$
24	0.333	$0.327\angle 0.940°$
30	0.418	$0.411\angle 0.833°$
36	0.503	$0.496\angle 0.717°$
42	0.587	$0.580\angle 0.596°$
48	0.668	$0.662\angle 0.475°$
54	0.746	$0.740\angle 0.361°$
60	0.816	$0.812\angle 0.258°$
66	0.879	$0.876\angle 0.169°$
72	0.930	$0.928\angle 0.096°$
78	0.968	$0.967\angle 0.043°$
84	0.992	$0.992\angle 0.011°$
90	1.000	$1.000\angle 0°$

TABLE 7.2 Approximations to input impedance for various cylindrical dipoles using pulse functions or sinusoidal functions as basis functions

Normalized Length $2l/\lambda$	Normalized Radius a/λ	Number of Pulse Functions N	Self-Impedance in Ohms $(c_1)^{-1}$ Pulse Functions	Self-Impedance in Ohms Sinusoidal Basis Functions
0.25	0.01	10	* 11.1 − j177 11.3 − j186	10.2 − j185
0.25	0.0001	10	12.9 − j737	12.5 − j739
0.50	0.01	20	97.3 + j27.8	90.2 + j22.2
0.50	0.0001	20	74.0 + j11.3	74.2 + j26.4
0.75	0.01	30	534 + j79.9	477 + j180
0.75	0.0001	30	424 + j827	403 + j882
1.00	0.01	40	178 − j344	40 − j255
1.00	0.0001	40	2724 − j1067	439 − j1445

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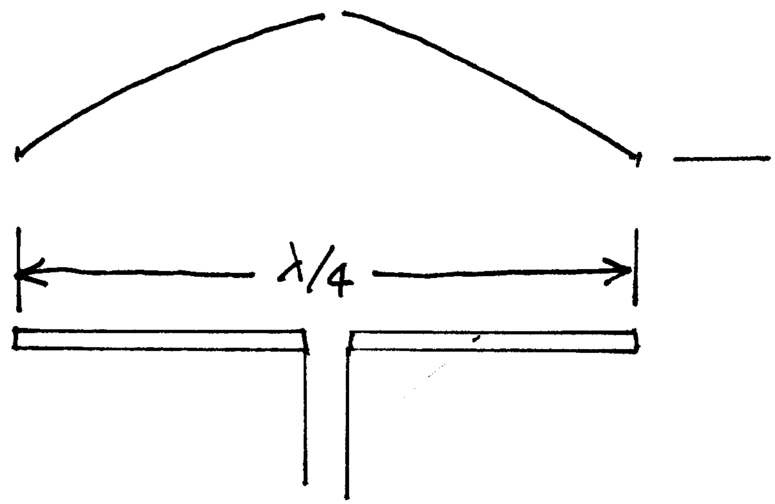
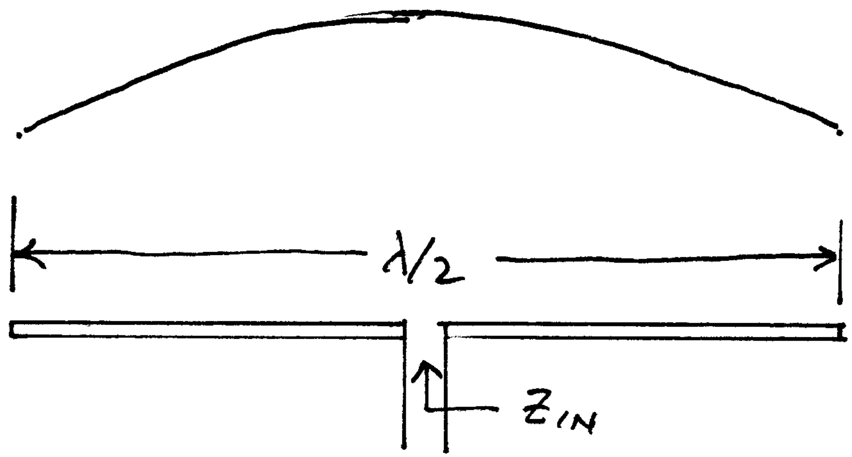
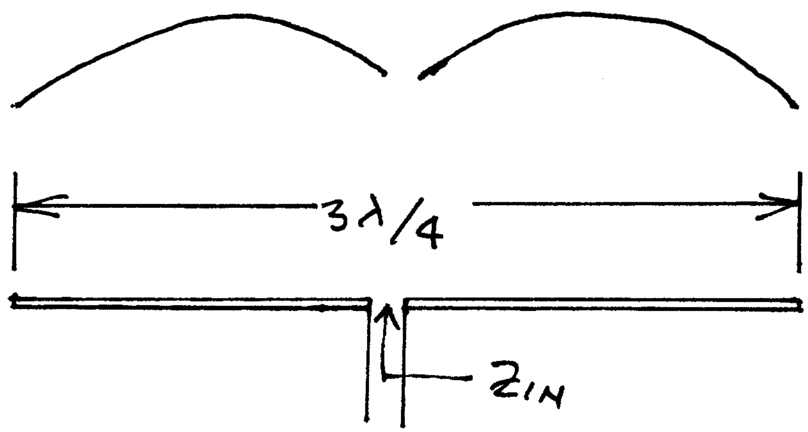


	a/λ
$Z_{IN} = 666.7\angle -89^\circ \approx 11.6 - j666.6$	0.0001
$Z_{IN} = 181.3\angle -87^\circ \approx 9.5 - j181$	0.01

From Fig 7.6, 7.7.



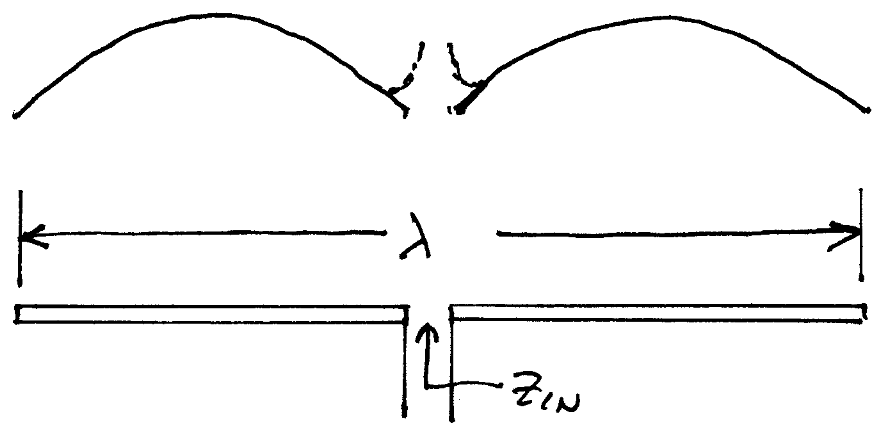
	a/λ
$Z_{IN} = 76.9 \angle +8^\circ = 76 + j10.7$	0.0001
$Z_{IN} = 105 \angle +23 = 96 + j104$	0.01



a/λ

$Z_{IN} = 1000\angle{+62} = 464 + j\,880$ $\quad$ 0.0001

$Z_{IN} = 571\angle{+32} = 484 + j\,302$ $\quad$ 0.01



		a/λ
$Z_{IN} = 4000\angle{-30} = 3464 - j2000$		0.0001
$Z_{IN} = 666\angle{-34} = 552 - j372$		0.01

Table 5-1 Simple Formulas for the Input Resistance of Dipoles

Length L	Input resistance R_{in} (ohms)
$0 < L < \frac{\lambda}{4}$	$20\pi^2\left(\frac{L}{\lambda}\right)^2$
$\frac{\lambda}{4} < L < \frac{\lambda}{2}$	$24.7\left(\pi\frac{L}{\lambda}\right)^{2.4}$
$\frac{\lambda}{2} < L < 0.637\lambda$	$11.14\left(\pi\frac{L}{\lambda}\right)^{4.17}$

Table 5-2 Wire Lengths Required To Produce a Resonant Half-Wave Dipole for a Wire Diameter of $2a$ and Length L

Length to diameter ratio, $L/2a$	Percent shortening required	Resonant length L	Dipole thickness class
5000	2	0.49λ	very thin
50	5	0.475λ	thin
10	9	0.455λ	thick

L = 2l

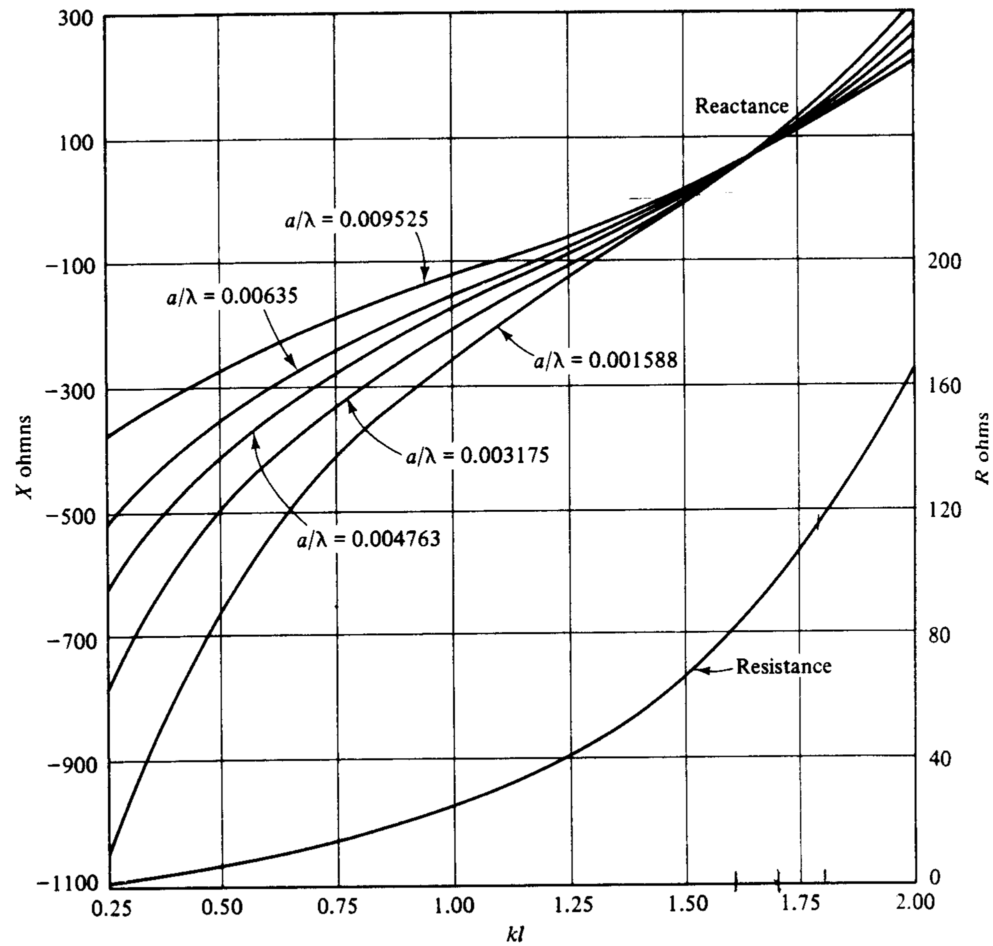


Fig. 7.11 The Resistance and Reactance of a Center-Fed Dipole versus kl and a/λ; Values Computed by the Induced EMF Method Using Equation 7.65

Definitions: Figs & Tables, immediately following.

Figures and Tables beginning with "7"

$\frac{l}{2} \rightarrow l$, total length of Dipole is $2l$

$$\Omega = 2 \ln (2l/a) = 2 \ln \left[\frac{\text{length}}{\text{radius}}\right]$$

All cases center-fed.

$$kl = \frac{2\pi l}{\lambda}$$

$\frac{\lambda}{2}$ dipole , $kl = 1.57$

λ dipole , $kl = 3.14$

/...

Ω	length / radius	radius / length
100	$e^{50} = 10^{21}$	$\cong 0$
20	$e^{10} = 22{,}000$	0.000045
10	$e^{5} = 148$	0.0067
5	$e^{2.5} = 12.2$	0.08

Figs and Tables beginning with "S" are from SST

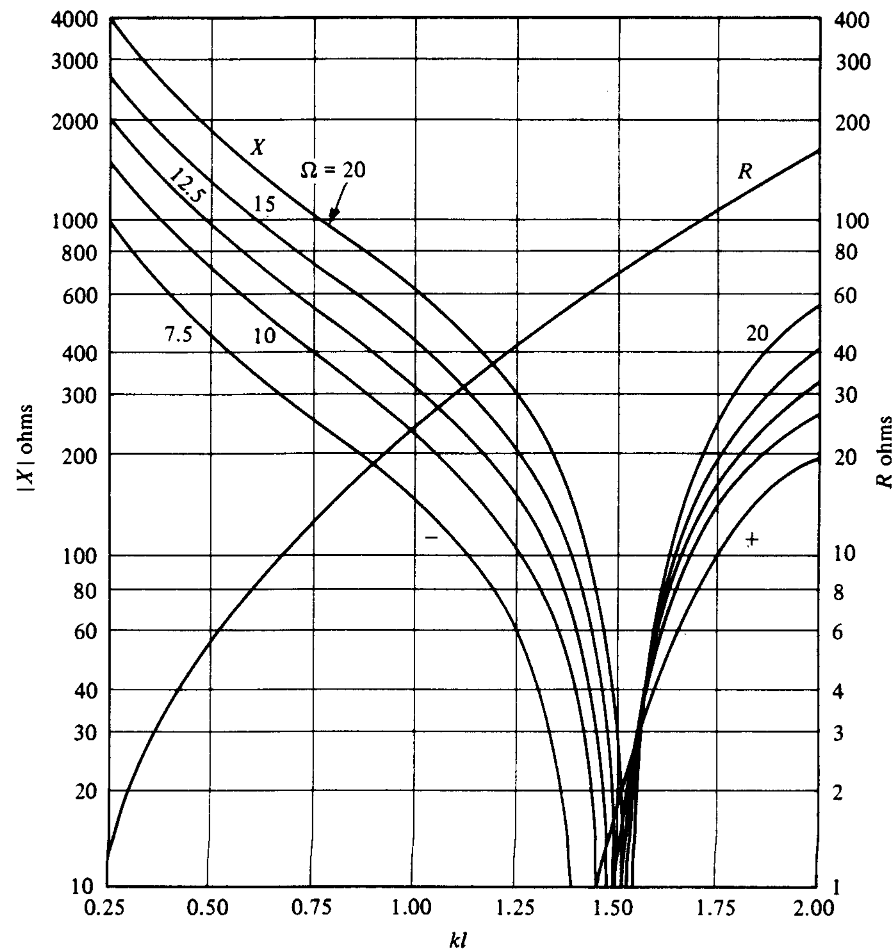


Fig. 7.12 The Resistance and Reactance of a Center-Fed Dipole versus kl and Ω; Values Computed by the Induced EMF Method Using Equation 7.65

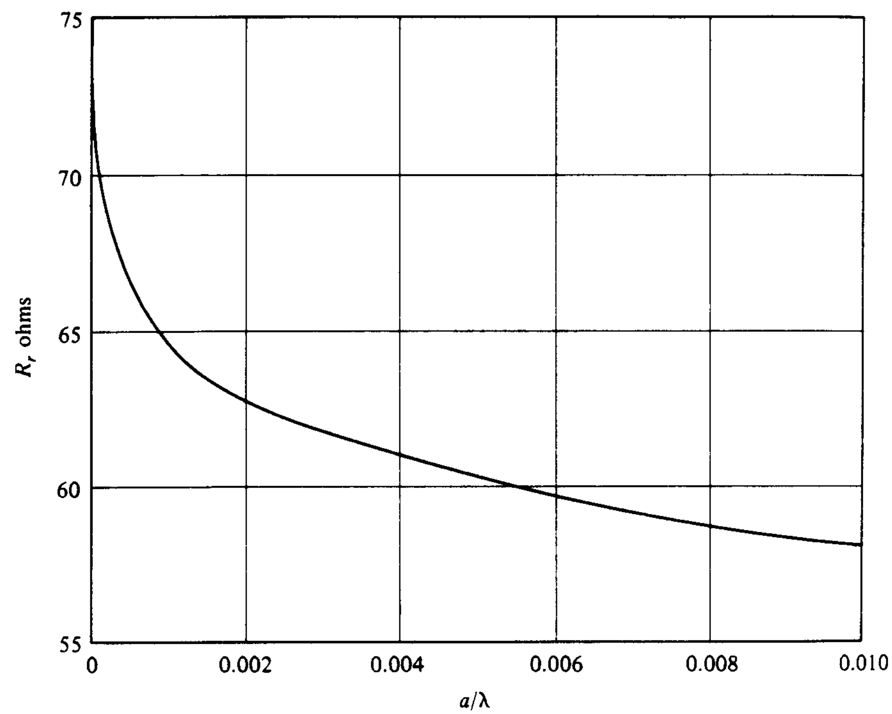


Fig. 7.14 Resonant Resistance versus Radius for Center-Fed Cylindrical Dipoles

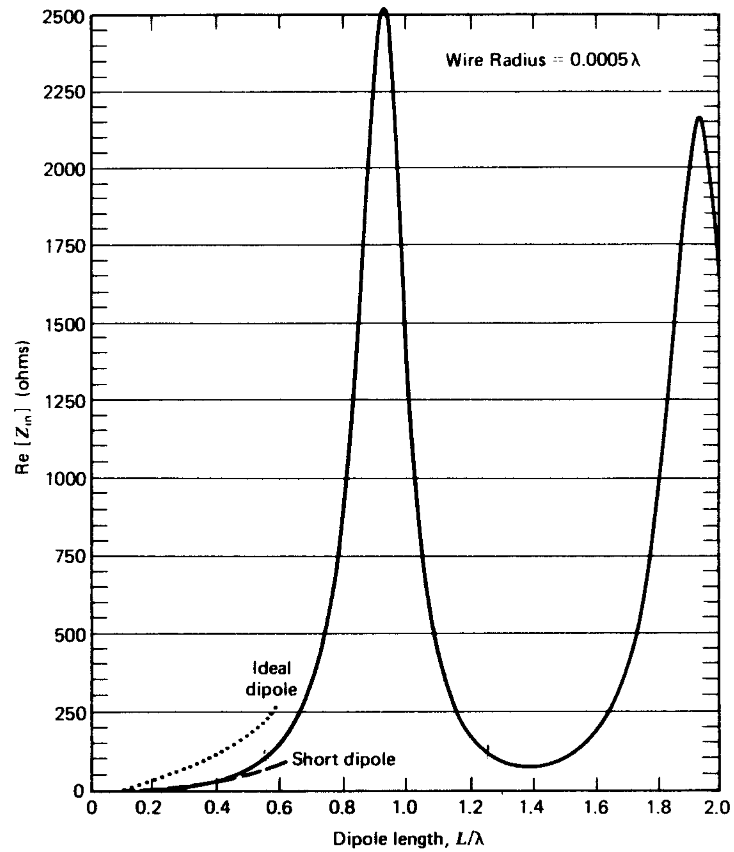


Figure 5-5 Calculated input resistance of a center-fed wire dipole of 0.0005λ radius as a function of length L (solid curve). Also shown is the input resistance $R_{in} = 80\pi^2(L/\lambda)^2$ of an ideal dipole with a uniform current distribution (dotted curve) and the input resistance $R_{in} = 20\pi^2(L/\lambda)^2$ of a short dipole with a triangular current distribution approximation (dashed curve).

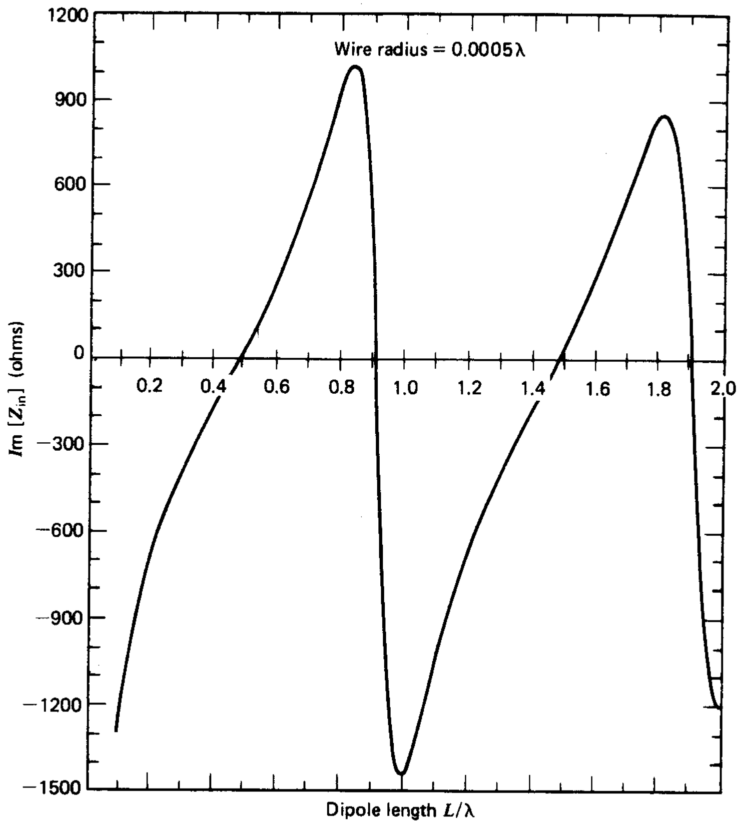


Figure 5-6 Calculated input reactance of center-fed wire dipole of radius 0.0005λ as a function of length L.

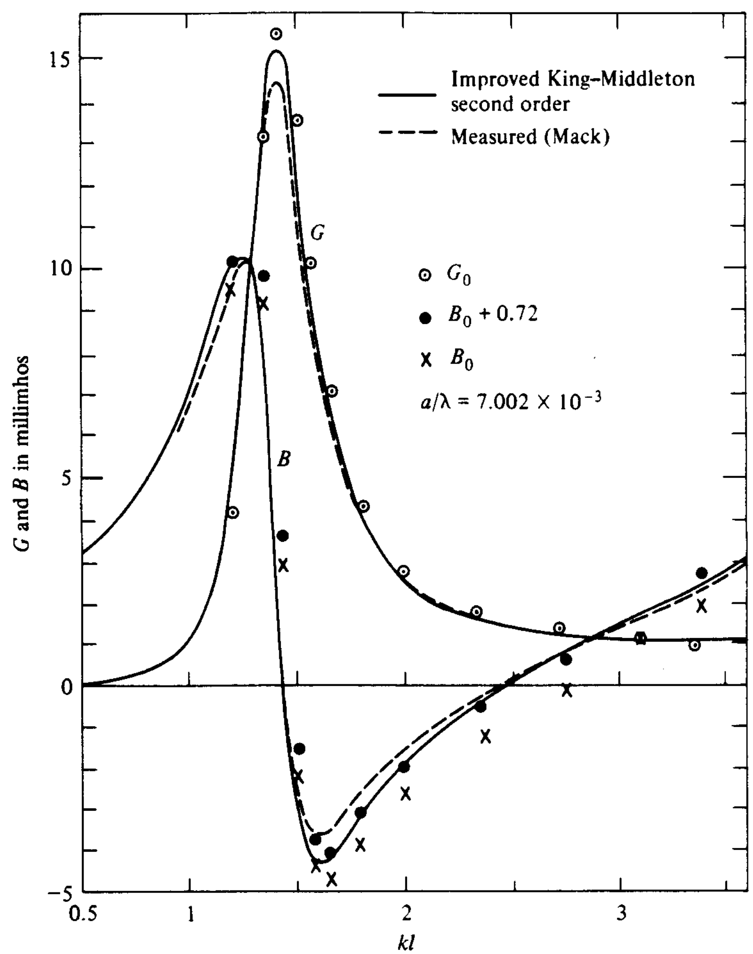


Fig. 7.17 A Comparison of the Improved King-Middleton Second-Order Admittance and the Measured Admittance of a Center-Fed Cylindrical Dipole (Measurements by Mack[28]) (© 1967 IEEE. Reprinted from R. W. P. King, *IEEE Proceedings*, pp. 2–16, 1967.)

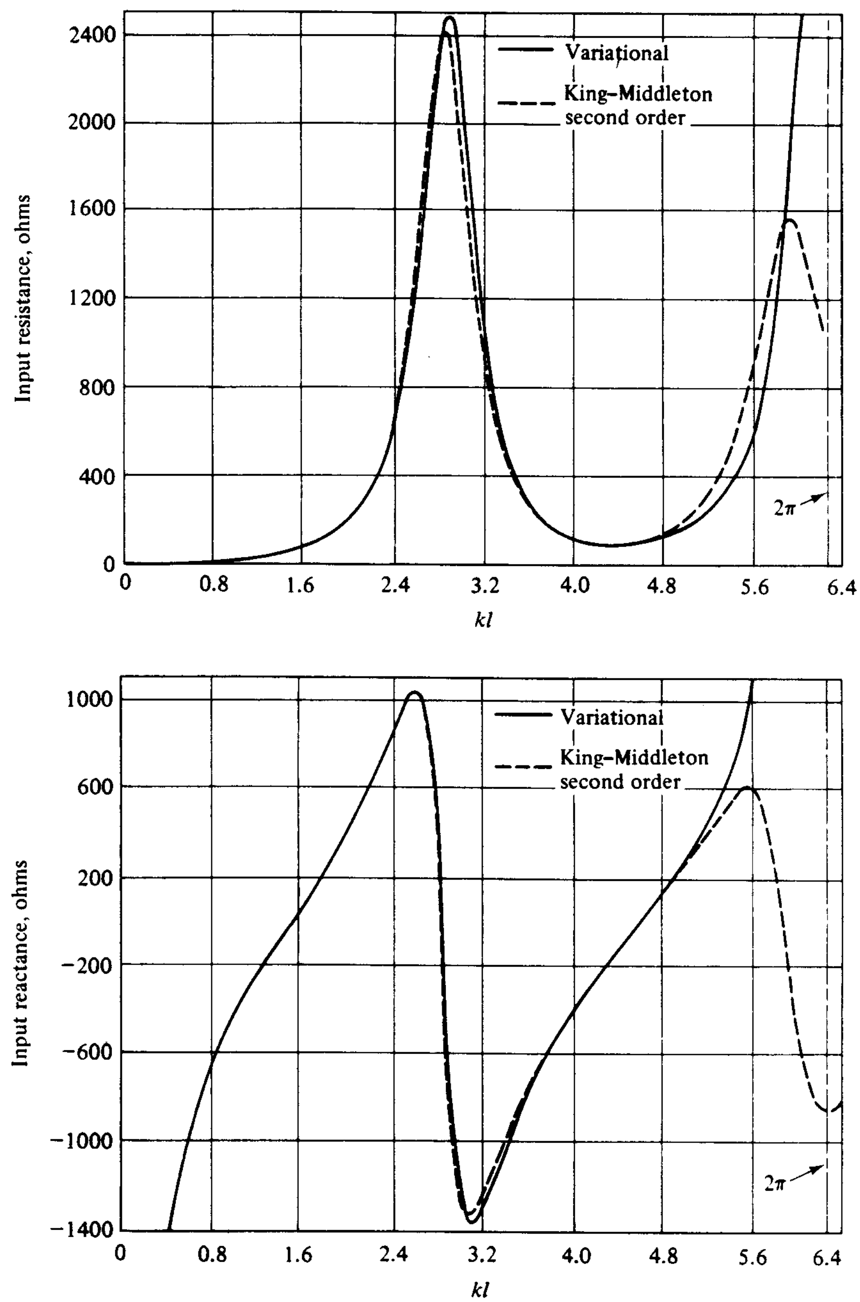


Fig. 7.16 A Comparison of Storer's Variational Solution and King-Middleton Second Order Values for the Input Impedance of a Center-Fed Cylindrical Dipole; $\Omega = 2 ln(2l/a) = 15$ (Reprinted from J. E. Storer, Cruft Laboratory Report No. 101, 1950, Courtesy of Harvard University.)