Aperture Antennas

EE 252

Until now ... All antennas discussed have had reasonably well-known and simple distribution of current — $\bar{J}(\bar{r})$

... Radiation integral has served us well.

Now ... consider cases where $\displaystyle\int \frac{[\bar{J}]}{4\pi r}\,dv = \bar{A}$

would apply to $\bar{J}$ too complex to be useful.

/...

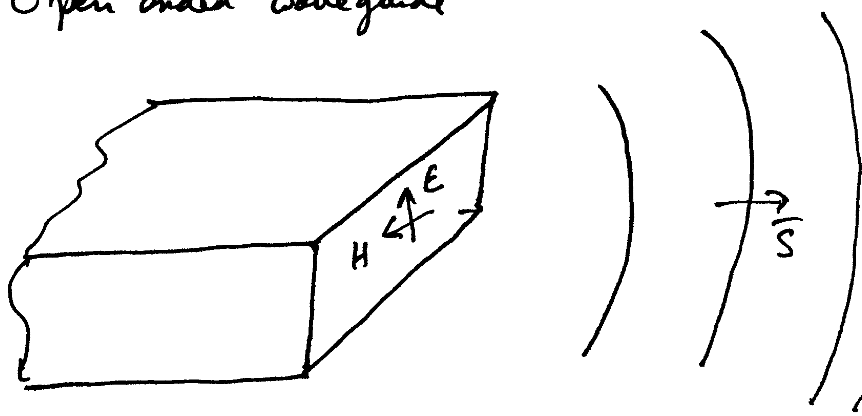
Often, ~~when~~ although $\bar{J}$ is complex the fields $\bar{E}(x,y,z)$, $\bar{H}(x,y,z)$ may be well-defined.

Usually either $\bar{J}$ is fairly simple or $\bar{E},\bar{H}$ are fairly simple.

The latter case makes up the ~~pr~~ topic of "aperture antennas", which we discuss next.

Examples of antennas with well-defined $\bar{E}, \bar{H}$ in some localized region, typically corresponding to a well-defined opening for $\bar{E} \times \bar{H}$ to "flow".

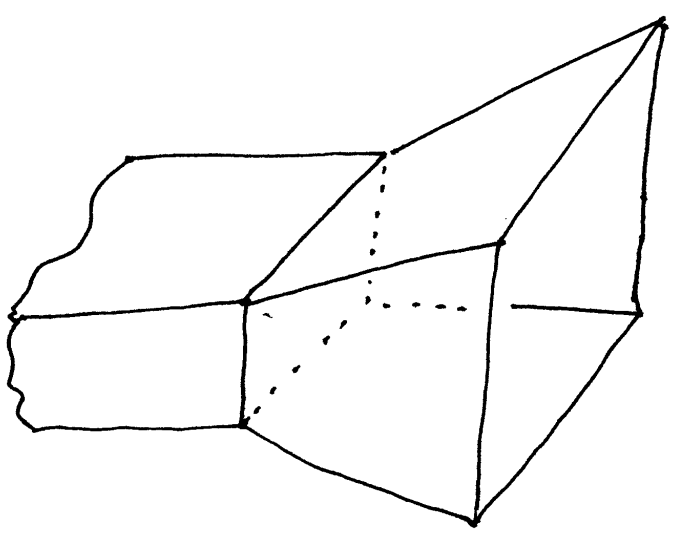
1. Open ended waveguide



/...

/... Examples ...

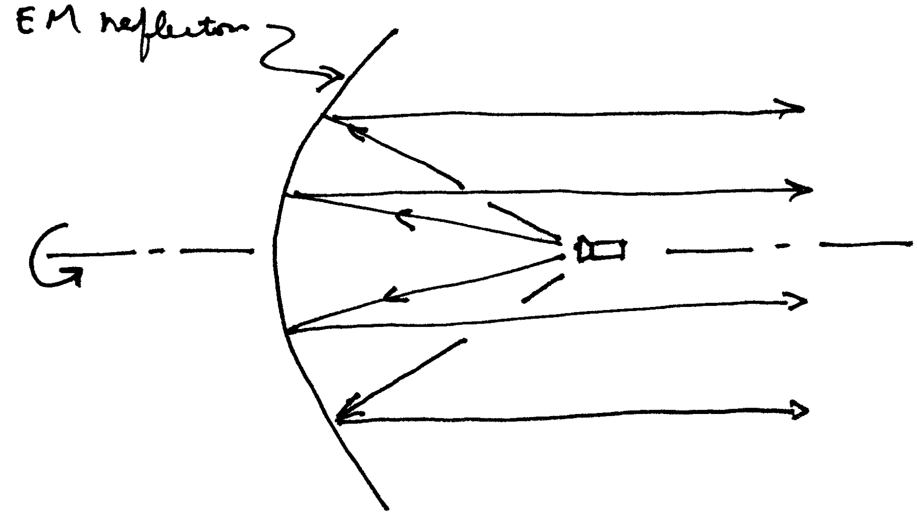
2. Horn antenna



/...

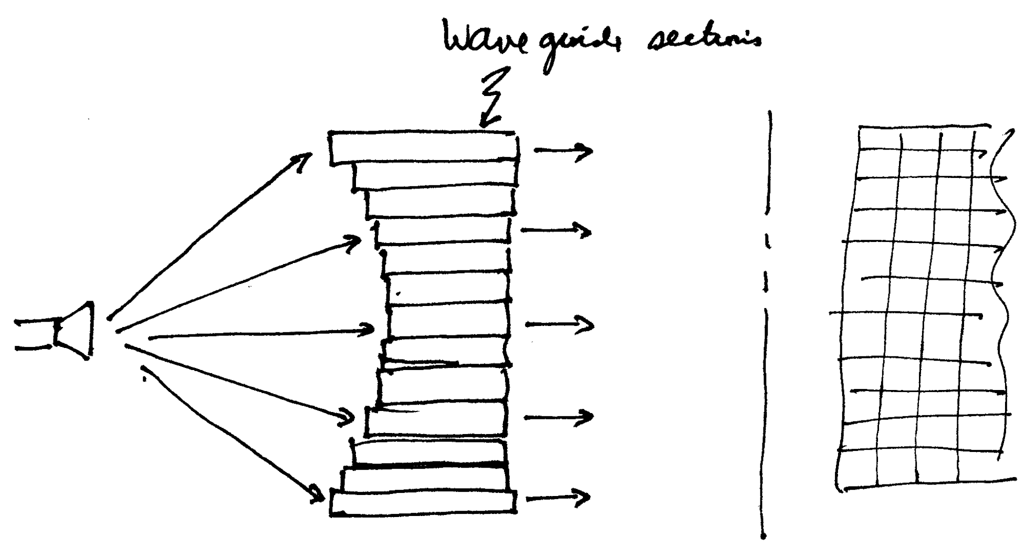
/...

3. Reflector types.



/...

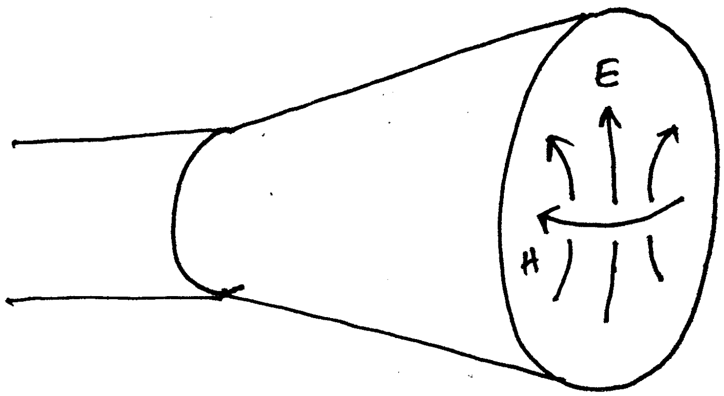
4. EM Lens



/...

/...

5. Circular Horns



etc!

General Procedure for Determining Performance
(Pattern, Radiated Power)

1. Replace fields in the aperture by an equivalent set of sources $\bar{J}, \bar{M}$.

2. Use $\bar{A}, \bar{F}$ calculated for the equivalent sources to find the far-field(s).

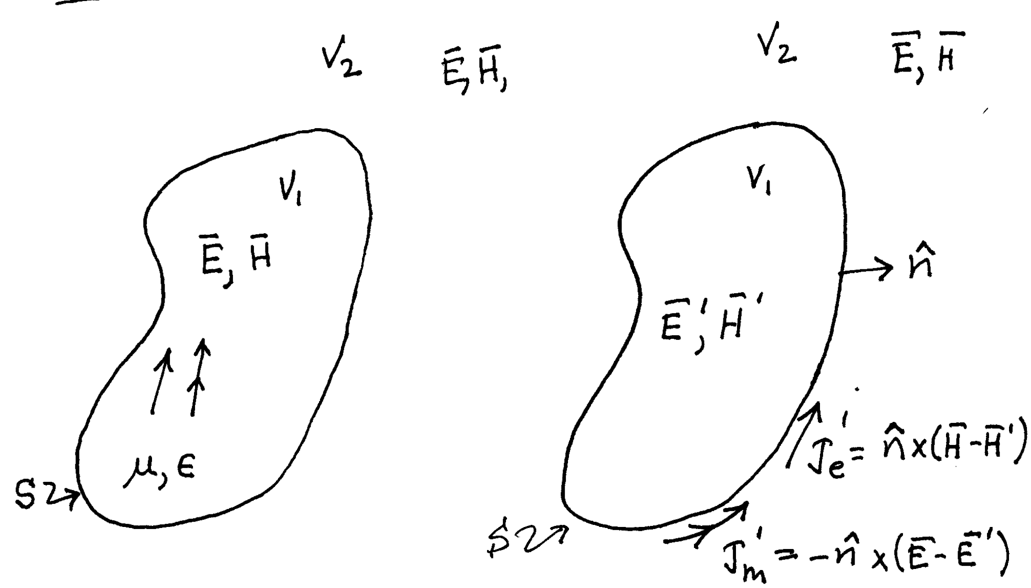
$$\boxed{\text{Ant. geom. Excitation}} \longrightarrow \underbrace{\boxed{\text{ESTIMATE APERTURE FIELDS}} \longrightarrow \boxed{\text{FIND EQUIVALENT SOURCES}}}_{\text{in the aperture}} \longrightarrow \boxed{\text{FIND RADIATION FIELDS}}$$

Field Equivalence Principle

Principle by which actual sources in a region (such as the vicinity of a transmitter) are replaced by "equivalent" sources. New sources are equivalent in the sense that they produce the same fields as the original sources, at the very least over a limited region.

Original problem is replaced w/ an equivalent problem which, it is hoped, is simpler.

Consider



Original Problem

Equivalent Problem

/...

/...

Above, the equivalent is found by replacing the original $\bar{J}_e, \bar{J}_m$ by $\bar{J}_e', \bar{J}_m'$, which yield the same fields on the boundary.

Current densities are equivalent w/i V_2 because the original fields are reproduced everywhere w/i V_2.

In general, fields w/i V_1 $\bar{E}', \bar{H}'$ are different from the originals and are in fact arbitrary. Typ. $\bar{E}' = \bar{H}' = 0$, then $\bar{J}_e' = \hat{n} \times \bar{H}$, $\bar{J}_m' = -\hat{n} \times \bar{E}$ on S !!!

Consider two sets of fields, one driven by $\bar{J}_e$, the other driven by $\bar{J}_m$

a) $\nabla \times \bar{E}_1 = -j\omega\mu_0 \bar{H}_1$	e) $\nabla \times \bar{E}_2 = -\bar{J}_m - j\omega\mu_0 \bar{H}_2$
b) $\nabla \times H_1 = \bar{J}_e + j\omega\epsilon_0 \bar{E}_1$	f) $\nabla \times \bar{H}_2 = j\omega\epsilon_0 \bar{E}_2$
c) $\nabla \cdot \bar{E}_1 = \rho_e / \epsilon_0$	g) $\nabla \cdot \bar{E}_2 = 0$
d) $\nabla \cdot H_1 = 0$	h) $\nabla \cdot \bar{H}_2 = \rho_m$
$\nabla \cdot \bar{J}_e = -\dot{\rho}_e = -j\omega\rho_e$	$\nabla \cdot \bar{J}_m = -j\omega\rho_m$
$\nabla \cdot \bar{A} = -j\omega\epsilon\phi_e$	$\nabla \cdot \bar{F} = -j\omega\mu_0\phi_m$

/...

/... Potentials ?

$$\bar{H}_1 = \nabla \times \bar{A} \qquad\qquad \bar{E}_2 = -\nabla \times \bar{F}$$

from a) $\nabla \times (E_1 + j\omega \bar{A}) \equiv 0$ from e) $\nabla \times (H_2 + j\omega\epsilon \bar{F}) \equiv 0$

$$\bar{E}_1 = -j\omega \bar{A} - \nabla \Phi_e \qquad\qquad \bar{H}_2 = -j\omega\epsilon \bar{F} - \nabla \phi_m$$

Φ_e = electric scalar potential function

Φ_m = magnetic " " "

$\bar{A}$ = magnetic vector potential function

F = electric " " "

/...

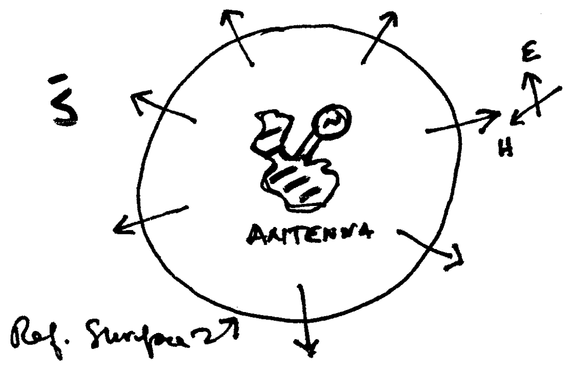
/... Potentials ...

$$\begin{Bmatrix} \bar{A} \\ F \end{Bmatrix} = \frac{1}{4\pi} \iiint \begin{Bmatrix} \bar{J}_e \\ \bar{J}_m \end{Bmatrix} \frac{e^{-jkR}}{R} \, dv'$$

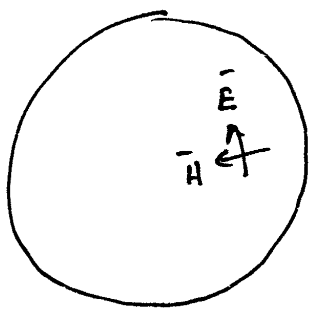
$$\begin{Bmatrix} \Phi_e \\ \Phi_m \end{Bmatrix} = \frac{1}{\underbrace{4\pi}_{\begin{Bmatrix} \epsilon_0 \\ 1 \end{Bmatrix}}} \iiint \begin{Bmatrix} \rho_e \\ \rho_m \end{Bmatrix} \frac{e^{-jkR}}{R} \, dv'$$

/

Given an antenna - look for a surface where the fields are reasonably well-known

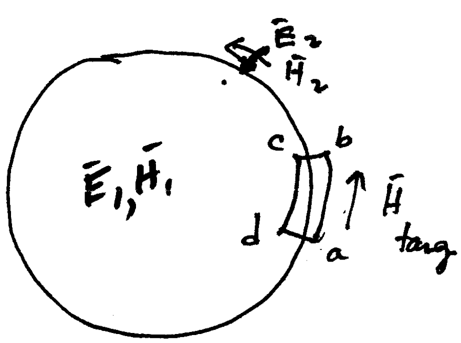


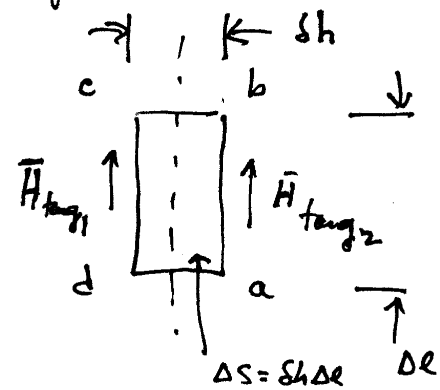
Then use that surface as a bed for new sources such that the radiation field outside can be completely described by sources on the surface.



How to get sources?

We are familiar with B. C. for electric current.

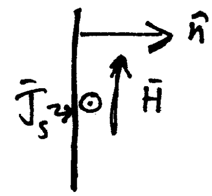




$$\int_S \nabla \times \bar H \cdot d\bar s = \oint \bar H \cdot d\bar \ell = \int_S \bar J_e \cdot d\bar s + \int_S \cancelto{0}{j\omega\epsilon \bar E \cdot d\bar s} \quad \delta h \to 0$$

E finite

So $\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_{\substack{\text{surface} \\ \text{electric}}} \left(= \bar{K}_e\right)$



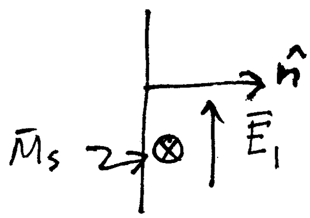
$\rightarrow \hat{n} \times \bar{H} = \bar{J}_s$ when $\bar{H}_1 = 0$

What about B.C. for magnetic current?

Use:

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} - \bar{J}_m \xrightarrow[\text{thm}]{\text{stokes}} -\hat{n} \times (\bar{E}_1 - \bar{E}_2) = \bar{M}_s$$

$$= \bar{K}_m$$



$-\hat{n} \times \bar{E}_1 = \bar{M}_s$ when $\bar{E}_2 = 0$.

How do $\bar{E}_\perp$ and $\bar{H}_\perp$ depend on

ρ_e and ρ_m?

$$\hat{n} \cdot \bar{E} = \frac{\rho_{surf.}}{\epsilon_0}$$

$$\hat{n} \cdot \bar{H} = \rho_m$$

<u>Bottom line</u> $\bar{J}_s$, $\bar{M}_s$, ρ_m, ρ_e in combination can reproduce fields everywhere over a closed surface with arbitrary fields inside.

Since M. Eqs. have unique sol'ns, subject to B.C., and maintaining same fields on on the surface* leaves the B.C. unchanged, the sol'n outside the surface is unchanged!

* choose the surface for B.C.'s just above the current sheets, if you like.

To calculate radiation fields use the standbys

$$\bar{A} = \frac{e^{-jkr}}{4\pi r} \iint_S \bar{J}_s(\bar{r}')\, e^{j\bar{k}\cdot\bar{r}'}\, ds'$$

far field

$$\boxed{\bar{E}_A = -j\omega\mu\, \bar{A}_T \;;\; \bar{H}_A = \hat{u}_r \times \frac{\bar{E}_A}{\eta_0}}$$

/...

/... While for magnetic surface current

$$\bar{F} = \frac{e^{-jkr}}{4\pi r} \iint \bar{M}_s(\bar{r}') e^{j\bar{k}\cdot\bar{r}'}\, dS'$$

far field

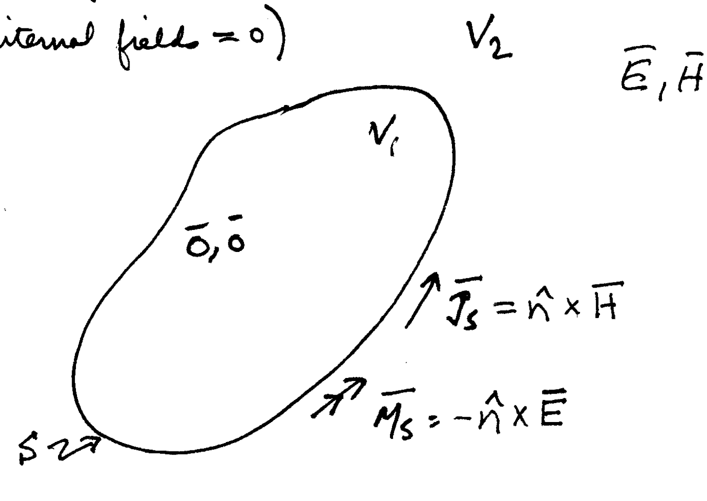
$$\boxed{\bar{H}_F = -j\omega\epsilon\, \bar{F}_T \; ; \quad \bar{E}_F = \eta\, \bar{H}_F \times \hat{u}_r}$$

$$\boxed{\bar{E} = \bar{E}_A + \bar{E}_F \; , \; \bar{H} = \bar{H}_F + \bar{H}_A \quad \text{in far field}}$$

total fields are the sum of those from electric and magnetic sources.

Three Equivalent Problems

1. Love's Equivalent (1901)
(choose internal field = 0)



/...

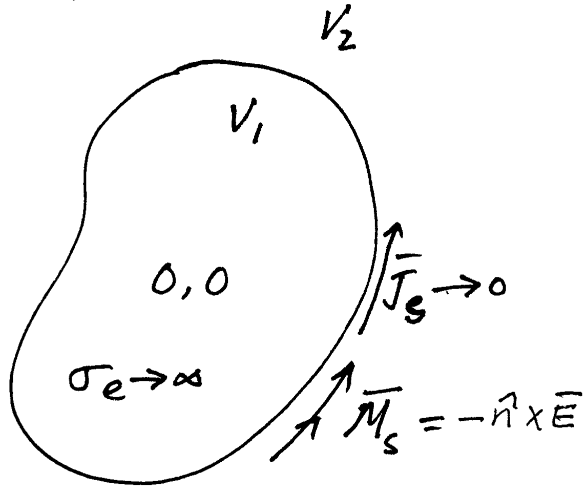
/...

2. <u>Electric Conductor Equivalent</u>

$\bar{A} \neq \int$

$\underbrace{\bar{F} \neq \int}$

Radiation integrals based on source currents breakdown.



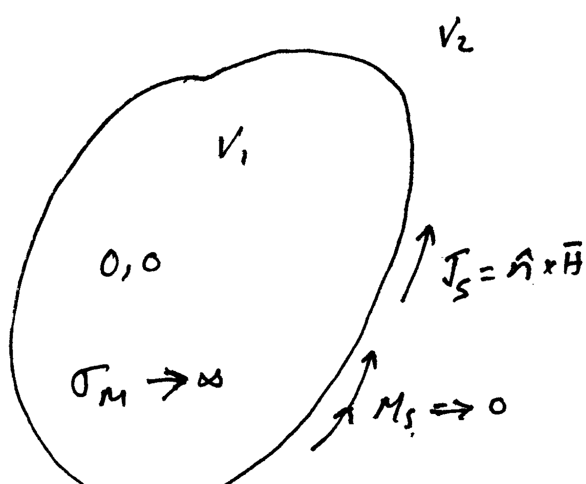
Note $\bar{J}_S = 0 \not\Rightarrow \bar{J} = 0$ (!)

$\boxed{\bar{J}_{total} \neq 0}$

$J_S \to 0$ because conductor shorts out equivalent surface current sources for electric current.

/...

3. <u>Magnetic Conductor Equivalent</u>



E, H