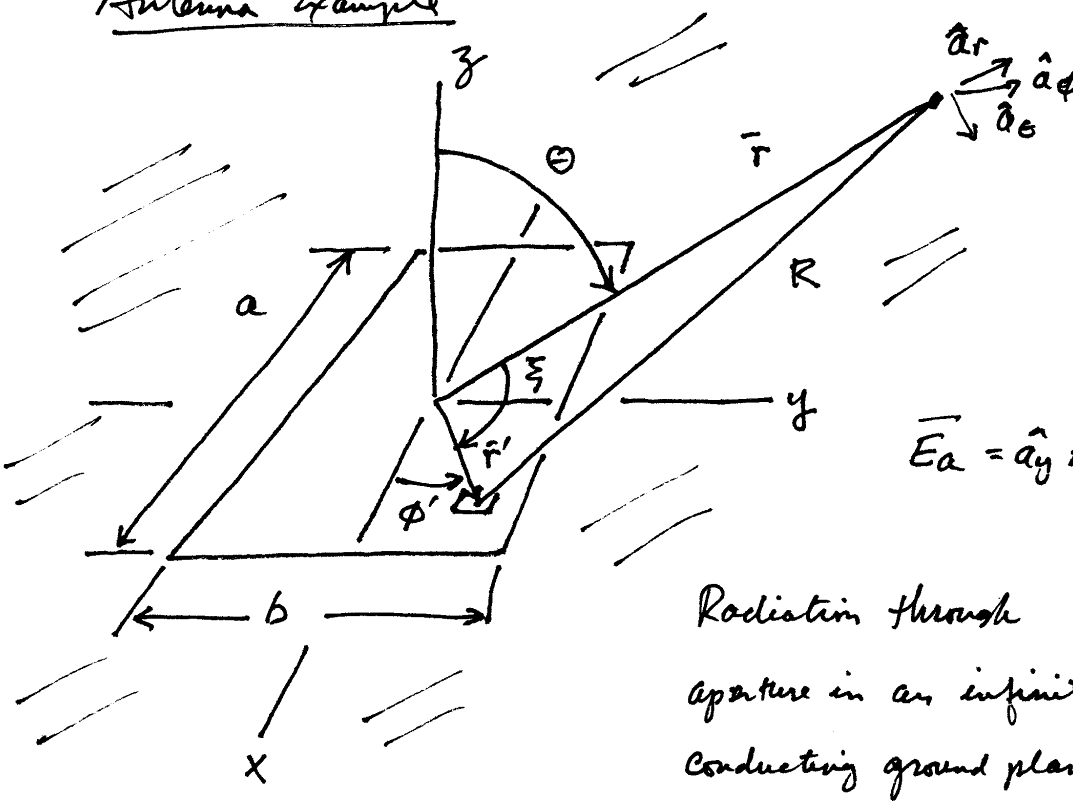
Antenna Example



$\overline{E_a} = \hat{a}_y E_0$

Radiation through aperture in an infinite, conducting ground plane

/...

$$|\bar{r}'|\cos\xi = \bar{r}' \cdot \hat{a}_r$$
$$= (x'\hat{a}_x + y'\hat{a}_y) \cdot (\hat{a}_x \sin\theta\cos\phi + \hat{a}_y \sin\theta\sin\phi + \hat{a}_z\cos\theta)$$

$$|\bar{r}'|\cos\xi = x'\sin\theta\cos\phi + y'\sin\theta\sin\phi$$

$$R \simeq r - r'\cos\xi \text{ , for phase calcs}$$
$$\simeq r \text{ , for amplitude}$$

$$-kR = = -kr + kr'\cos\xi$$

/...

/... <u>For far field</u>

$$\bar{A} \rightarrow \underbrace{\frac{e^{-jkr}}{4\pi r}}_{G} \underbrace{\iint \bar{J}_s \, e^{+j\bar{k}\cdot\bar{r}'} ds'}_{\bar{N}} = G\cdot\bar{N}$$

$$\bar{F} \rightarrow \underbrace{\frac{e^{-jkr}}{4\pi r}}_{G} \underbrace{\iint \bar{M}_s \, e^{+j\bar{k}\cdot\bar{r}'} ds'}_{\bar{L}} = G\cdot\bar{L}$$

But what are $\bar{J}_s$, $\bar{M}_s$ for this problem? We obtain these by a reduction procedure to obtain equivalent currents in an equivalent problem. ...

/....

Reduction of Aperture Problem to Equivalent — How is it Done?

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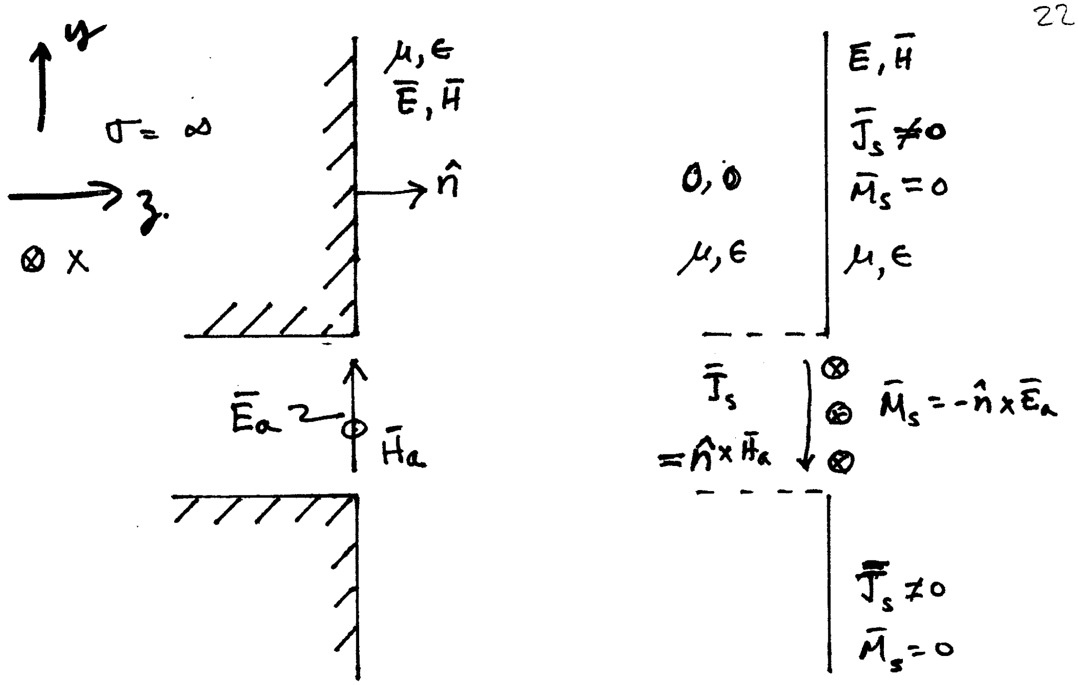

a) Original Problem

b) Love's Equivalent

/...

/... Reduction to Equivalent (2)

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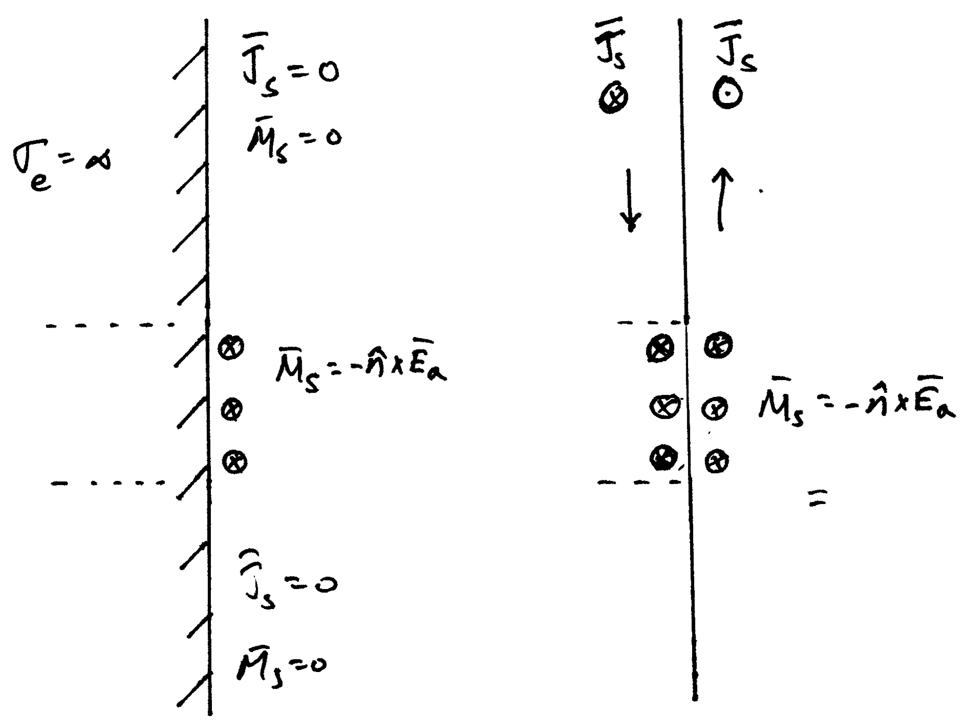


c) Electric Conductor

d) Image

/..

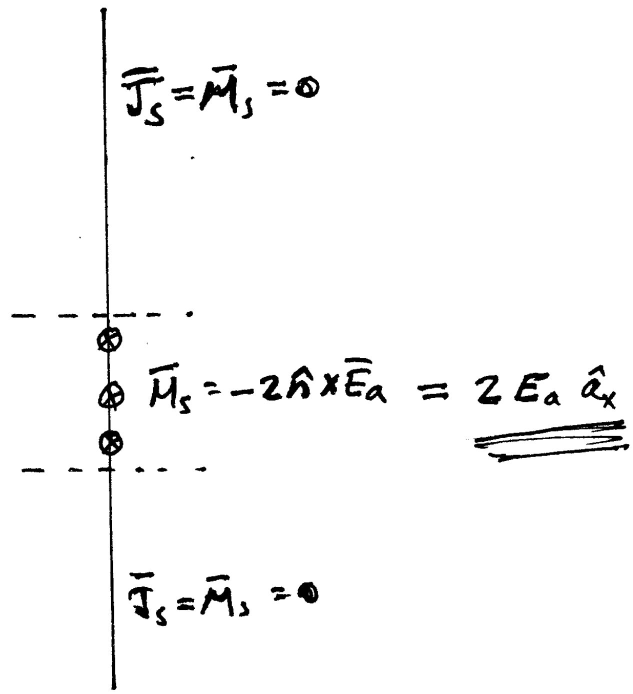
/ ... Reduction to Equivalent (3)

$\bar{M}_s = -2\hat{n} \times \bar{E}_a$

$= +2E_o \bar{a}_x$

on the aperture

$\bar{J}_s = 0$



So, in above, we essentially have found the radiation fields (far-field) from the fields in the aperture. Note that this relationship is a Fourier transform.

Also note, for this particular example, the critical role played by the infinite conducting sheet — it is the presence of this sheet that allows us to set all fields outside the active aperture to zero.

Evaluation of Fields

$$\bar{N} = \iint \bar{J}_s \, e^{+jkr'\cos\xi} \, ds'$$

$$\bar{L} = \iint \bar{M}_s \, e^{+jkr'\cos\xi} \, ds'$$

w/ integrals over the aperture

$$\bar{A} = \frac{e^{-jkr}}{4\pi r} \bar{N} = G\bar{N}$$

$$\bar{F} = \frac{e^{-jkr}}{4\pi r} \bar{L} = G\bar{L}$$

expression valid in far-field only

/...

/... Evaluation (2)

But $\bar{J}_s = 0$ everywhere on aperture, so $\bar{N} = 0$

while $\bar{M}_s \neq 0$, so $\bar{L}_T = L_\theta \hat{a}_\theta + L_\phi \hat{a}_\phi$ in far-field

Fields? In general $\bar{E}_A \to -j\omega\mu \bar{A}_T$; $\bar{H}_F = -j\omega\epsilon \bar{F}_T$

$$(E_A)_\theta = -j\omega\mu A_\theta \quad ; \quad (H_A)_\theta = -(E_A)_\phi/\eta = +j\frac{\omega\mu}{\eta} A_\phi$$

$$(E_A)_\phi = -j\omega\mu A_\phi \quad ; \quad (H_A)_\phi = +(E_A)_\theta/\eta = -jkA_\theta$$

$$(H_F)_\theta = -j\omega\epsilon F_\theta \quad ; \quad (E_F)_\theta = \eta (H_F)_\phi = -jkF_\phi$$

$$(H_F)_\phi = -j\omega\epsilon F_\phi \quad ; \quad (E_F)_\phi = -\eta (H_F)_\theta = +jkF_\theta$$

/...

$$E_\theta = (E_A)_\theta + (E_F)_\theta = -j\omega\mu A_\theta - jkF_\phi$$

$$E_\phi = \quad = -j\omega\mu A_\phi + jkF_\theta$$

$$H_\theta = \quad = jkA_\phi - j\omega\epsilon F_\theta$$

$$H_\phi = \quad = -jkA_\theta - j\omega\epsilon F_\phi$$

while $\bar{A}_\theta = G\bar{N}_\theta \qquad \bar{F}_\theta = G\bar{L}_\theta$

$\bar{A}_\phi = G\bar{N}_\phi \qquad \bar{F}_\phi = G\bar{L}_\phi$ } transverse components only

/...

/...

So the field expressions become, in terms of the equivalent currents ...

$$E_\theta = -jk\frac{e^{-jkr}}{4\pi r}\left(L_\phi + \eta N_\theta\right)$$

$$E_\phi = +jk\frac{e^{-jkr}}{4\pi r}\left(L_\theta - \eta N_\phi\right) \qquad \boxed{H_r = E_r = 0}$$

$$H_\theta = +jk\frac{e^{-jkr}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right)$$

$$H_\phi = -jk\frac{e^{-jkr}}{4\pi r}\left(N_\theta + \frac{L_\phi}{\eta}\right)$$

/...

/...

$$\bar{N} = \iint \bar{J}_s \, e^{+jkr'\cos\xi} ds'$$

$$= \iint (\hat{a}_x J_x + \hat{a}_y J_y + \hat{a}_z J_z) \, e^{+jkr'\cos\xi} ds'$$

$$\bar{L} = \iint \bar{M}_s \, e^{+jkr'\cos\xi} ds'$$

$$= \iint (\hat{a}_x M_x + \hat{a}_y M_y + \hat{a}_z M_z) \, e^{+jkr'\cos\xi} ds'$$

Then, following the transformation used for the circular loop...

/...

/...

$$\hat{a}_x = \hat{a}_r \sin\theta\cos\phi + \hat{a}_\theta \cos\theta\cos\phi - \hat{a}_\phi \sin\phi$$

$$\hat{a}_y = \hat{a}_r \sin\theta\sin\phi + \hat{a}_\theta \cos\theta\sin\phi + \hat{a}_\phi \cos\phi$$

$$\hat{a}_z = \hat{a}_r \cos\theta \quad - \hat{a}_\theta \sin\theta \quad + \hat{a}_\phi \cdot 0$$

$$N_\theta = \iint (J_x \cos\theta\cos\phi + J_y \cos\theta\sin\phi - J_z \sin\phi) e^{+jkr'\cos\xi} ds'$$

$$N_\phi = \iint (-J_x \sin\phi + J_y \cos\phi) \, e^{+jkr'\cos\xi} ds'$$

$$L_\theta = - - - -$$

$$L_\phi = - - - -$$

Back to the retangular aperture . . .

$$J_s = 0, \quad \bar{M}_s = 2E_0 \hat{a}_x + \hat{a}_y \cdot 0$$

$$L_\theta = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} (M_x \cos\theta \cos\phi)\, e^{jk(x' \sin\theta\cos\phi + y' \sin\theta \sin\phi)}\, dx'dy'$$

$$L_\theta = 2E_0 ab \cos\theta \cos\phi \left[\frac{\sin X}{X} \cdot \frac{\sin Y}{Y} \right]$$

$$X = \frac{ak \sin\theta \cos\phi}{2}, \quad Y = \frac{bk \sin\theta \sin\phi}{2}$$

$$L_\phi = -2E_0 ab \sin\phi \left[\frac{\sin X}{X} \cdot \frac{\sin Y}{Y} \right]$$

/...

So, finally (!)

$$E_\theta = -jk \frac{e^{-jkr}}{4\pi r} (L_\phi)$$

$$= -jk \frac{ab E_0}{2\pi r} e^{-jkr} \sin\phi \left[\frac{\sin X}{X} \cdot \frac{\sin Y}{Y} \right]$$

$$E_\phi = +jk \frac{e^{-jkr}}{4\pi r} (L_\theta)$$

$$= +jk \frac{ab E_0}{2\pi r} e^{-jkr} \cos\theta \cos\phi \left[\frac{\sin X}{X} \cdot \frac{\sin Y}{Y} \right]$$

$$H_\theta = -\frac{E_\phi}{\eta}; \quad H_\phi = \frac{E_\theta}{\eta}; \quad H_r = E_r = 0$$

This is an exact soln to stated problem!