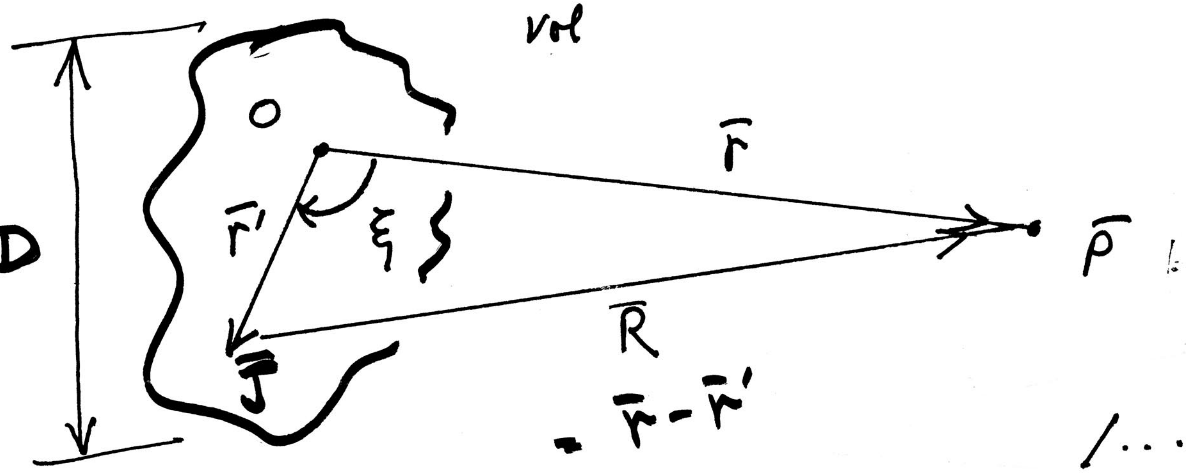
Definitions of Regions for Large Antennae

Consider:

$$\bar{A} = \frac{1}{4\pi} \int_{vol} \frac{\bar{J}(x', y', z')}{R} e^{-jkR} \, dv'$$



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$$R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

$$= |\bar{r} - \bar{r}'|^2 = r^2 + r'^2 - 2rr'\cos\xi$$

$$R = r\left(1 + \frac{r'^2}{r^2} - 2\frac{r'}{r}\cos\xi\right)^{1/2}$$

$$R = \underbrace{r - r'\cos\xi}_{\text{1st}} + \underbrace{\frac{r'^2}{r}\frac{\sin^2\xi}{2}}_{\text{2nd}} + \underbrace{\frac{1}{r^2}\left(\frac{r'^3}{2}\cos\xi\sin^2\xi\right)}_{\text{3rd}} + \cdots$$

Binomial Expansion: order of terms in r'

What is the Simplest Expansion for R ?

First Order $\qquad R = r - r' \cos \xi$

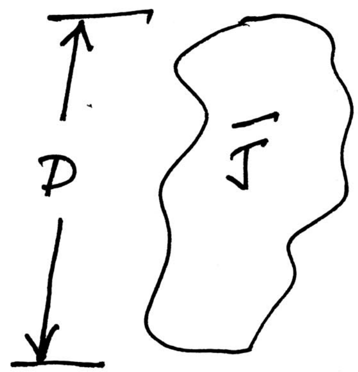
First neglected term is

$$\frac{1}{r} r'^2 \frac{\sin^2 \xi}{2} \xrightarrow[\xi = \frac{\pi}{2}]{\max} \frac{r'^2}{2r}$$

Which corresponds to a maximum phase error

$$\Delta \phi = k \Delta R = \frac{2\pi}{\lambda} \Delta R = \frac{2\pi}{\lambda} \frac{r'^2}{2r}$$

In the expression for $\bar{A}$ (the Radiation Int.)



Let D be the maximum dimension of the region for which $|J| \neq 0$

Then $r' \lesssim \frac{D}{2}$ (for some choice of origin)

With this picture in mind and some standard value for the maximum tolerable $\Delta\phi$ /...

We are led to

$$\Delta\phi \leq \Delta\phi_{standard} = \frac{2\pi}{\lambda}\frac{r'^2}{2r}$$

$$\Rightarrow \quad r > \frac{\pi}{\lambda}\frac{r'^2}{\Delta\phi_{STANDARD}}$$

or $$\boxed{r > \frac{2D^2}{\lambda}}$$ for $\Delta\phi = \frac{2\pi}{16} = \frac{\pi}{8}$ rad.

$= 22\frac{1}{2}^\circ$ phase

Which is the standard definition for the "Far-Field" of an antenna!

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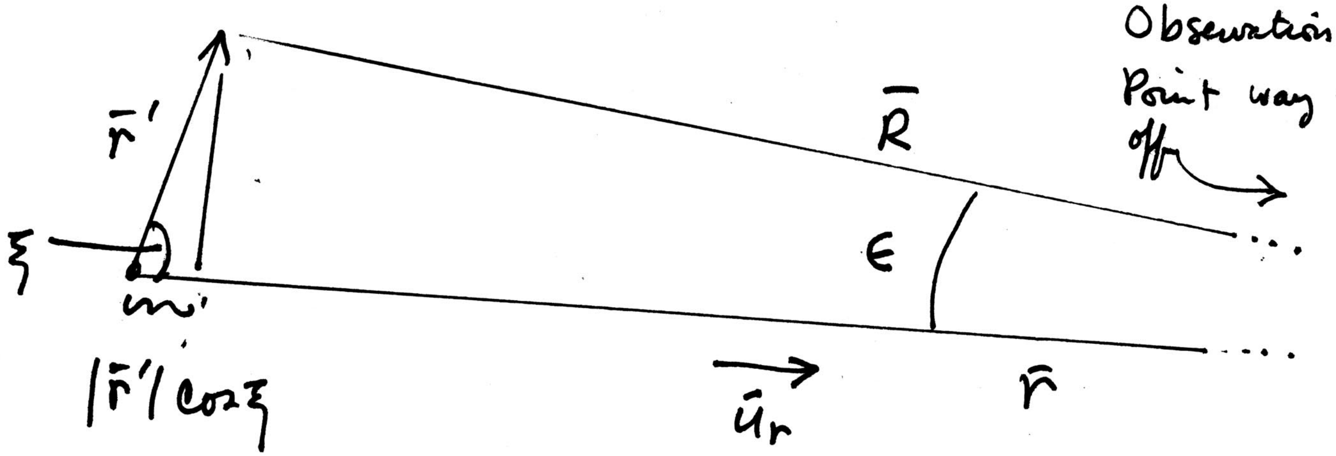
Far Field Relationship $\quad r > \frac{2D^2}{\lambda}$



$$R \simeq r - r'\cos\xi = \bar{r} - \bar{r}'\cdot\bar{u}_r$$

$$\left.\frac{2D^2}{\lambda}\right|_{\substack{D=46m\\ \lambda=1\\ f=3\times10^8}} = \frac{2(46)^2}{1} = 4.2\ km \longrightarrow 42\ km$$

$\lambda \to 0.1$

$f \to 3\times10^9$

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/... FAR Field (cont)

$$kR = \bar{k}\cdot\bar{R} = \bar{k}\cdot(\bar{r}-\bar{r}') \; ; \quad \bar{R} \overset{\text{approx.}}{/\!/} \bar{r}$$

$$\bar{A} = \underbrace{\frac{e^{-j\bar{k}\cdot\bar{r}}}{4\pi r}\,\mu}_{\text{Green's function - varies w/ } \bar{r}\cdot\bar{k} = kr} \underbrace{\int_{vol} \bar{J}(\bar{r}')\, e^{+j\bar{k}\cdot\bar{r}'}\, dv'}_{\text{function of } \bar{k}(\bar{r})\text{, varies with } \bar{u}_r\text{, but not with } |\bar{r}|\,!}$$

Green's function - varies w/ $\bar{r}\cdot\bar{k}$ $= kr$

function of $\bar{k}(\bar{r})$, varies with $\bar{u}_r$, but not with $|\bar{r}|$!

$\bar{k} = \frac{2\pi}{\lambda}\bar{u}_R \rightarrow \frac{2\pi}{\lambda}\bar{u}_r \, , \quad r \gg D$

Far Field General Case: Also called Fraunhofer Region -

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In the above note that the Green's function factor depends only on r, and $\bar{k}\cdot\bar{r}$. But $\bar{k}\cdot\bar{r} = \frac{2\pi}{\lambda} r$ depends only on r, also.

On the other hand, the integrand depends on $\bar{r}'$ and $\bar{k}\cdot\bar{r}' = \bar{u}_R\cdot\bar{r}'\cdot\frac{2\pi}{\lambda}$,

where $\bar{u}_R = \frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|}$. That is, the integral is a function of direction $\bar{u}_R$. Seen another way $\bar{k}\cdot\bar{r}' = \frac{2\pi}{\lambda} r' \cos\xi$.

Far Field Region or Fraunhofer Region

Characterized by retention of linear phase term in expansion of kR.

R must be precise on scale of $2\pi/\lambda$ in all phase calculations. (cf. Rad. Int.)

R must be precise only on scale of D for amplitude calculations. (cf. Rad. Int.)

What if $r < 2D^2/\lambda$, then what ?

$$R = r - r'\cos\xi + \frac{1}{r} r'^2 \frac{\sin^2\xi}{2} \Big| + \cdots$$

First neglected term is $\frac{1}{r^2}\left(\frac{r'^3}{2}\cos\xi \sin^2\xi\right)$

$$\frac{\partial}{\partial\xi} = 0 = -\sin^3\xi + 2\cos^2\xi\sin\xi = \sin\xi\left(-\sin^2\xi + 2\cos^2\xi\right)$$

$\xi = 0 \Rightarrow$ min error ; $\tan^2\xi = 2$, $\xi = 0.96$ rads ✓

$\Rightarrow$ max error.

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$$\frac{r'^3}{r^2}\left(\frac{\cos\xi\,\sin^2\xi}{2}\right)\Bigg|_{\xi = 0.96 \text{ radians}} = \frac{r'^3}{r^2}(0.19) = \Delta R \quad \left\{\begin{array}{l}\text{Error from}\\ \text{neglect of}\\ \text{cubic terms}\end{array}\right.$$

Applying the same criterion, that $\frac{2\pi}{\lambda}\cdot\Delta R < \pi/8$,

$$\frac{2\pi}{\lambda}\cdot\frac{r'^3}{r^2}\,0.19 < \frac{\pi}{8} \; ; \quad r^2 > \left[\frac{(16)(0.19)}{\lambda}\right] r'^3$$

$$r > 1.74\sqrt{\frac{r'^3}{\lambda}} \; ; \quad \boxed{r > 0.62\left[\frac{D^3}{\lambda}\right]^{1/2}}$$

Radiating Near-Field Region

$$\boxed{0.62\left[\frac{D^3}{\lambda}\right]^{1/2} < r < \frac{2D^2}{\lambda}}$$

1. Variation with $\bar{u}_r$ is now a function of distance.

$$\int_{vol} J(\ )\, e^{-j\bar{k}\cdot\bar{r}'}\, dr' \quad \text{now depends on } |\bar{r}|$$

(see below)

2. Fields have a significant curvature — hence <u>Fresnel Integrals</u> appear

Radiating Near-Field (cont.)

3. Region called "Radiating" because radiating power density is much greater than the reactive power density

4. Also called the <u>Fresnel Region</u>

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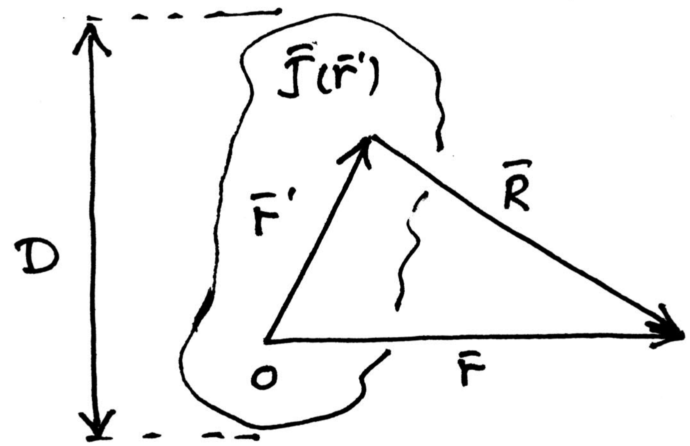
$$\bar{A} = \mu \underbrace{\int_{vol} \frac{\bar{J}(\bar{r}')\, e^{-jkR}}{4\pi R}\, dv'}_{\text{consider only the integral}}$$



"Far out" $\bar{r} \parallel \bar{R}$, so the integral depends only on the direction of $\bar{r}$, and $\bar{A} \propto 1/r$

Close in $\bar{r} \nparallel \bar{R}$, so R, and hence the integral depend on $\bar{u}_r$ and $|\bar{r}|$. In this case $\bar{A}$ does not vary as $1/r$, but as some higher power of $1/r$.

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(1)

Fresnel Region or Radiating Near-Field Region, keeps second-order term as well as r-dependence

Far Field or Fraunhofer Region - only need this

$$-jk\left(-r'\cos\xi + \frac{1}{r}r'^2\sin^2\xi\right) \cdots$$

$$\bar{A} = \frac{e^{-j\cancel{kr}}}{4\pi r}\mu\int_{vol}\bar{J}(\bar{r}')e\,dv'$$

... ...

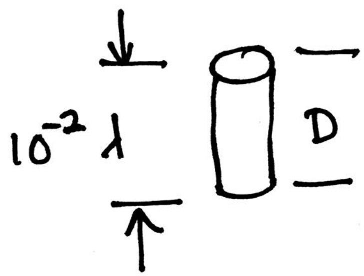
✓ Denominator easily approximated by $r \simeq R$

$\underline{r >> \lambda}$ for all of this!

Far-Field and Near-Field definitions are confusing when applied to small and large antennas alike.

$\frac{2D^2}{\lambda}$ makes no sense for an "ideal dipole".

e.g.



$$\frac{2D^2}{\lambda} = \frac{2\times10^{-4}\lambda^2}{\lambda} = 2\times10^{-4}\lambda \;(!)$$

$\lambda = 1\,m \Rightarrow 2\times10^{-4}\,m$ – ridiculous!

Fraunhofer and Fresnel Region concepts apply at distances beyond which the effect of individual current elements can be considered to vary as $1/r$, i.e., in the "local" far-field of the individual current elements. Think in terms of superposition of waves from individual radiators.

Reactive Region and Transition Region concepts are based on the behavior of the fields — they apply in the vicinity of the active current distribution

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Which concept you use will depend on the situation. Away from the immediate structure of large antennas the Fraunhofer/Fresnel approach is appropriate. In the vicinity of wire antennas or waveguides the field concepts are needed.

Some people use the term "near-field" to refer to reactive and transition regions.

Summary of Antenna Field Regions

Four Regions:

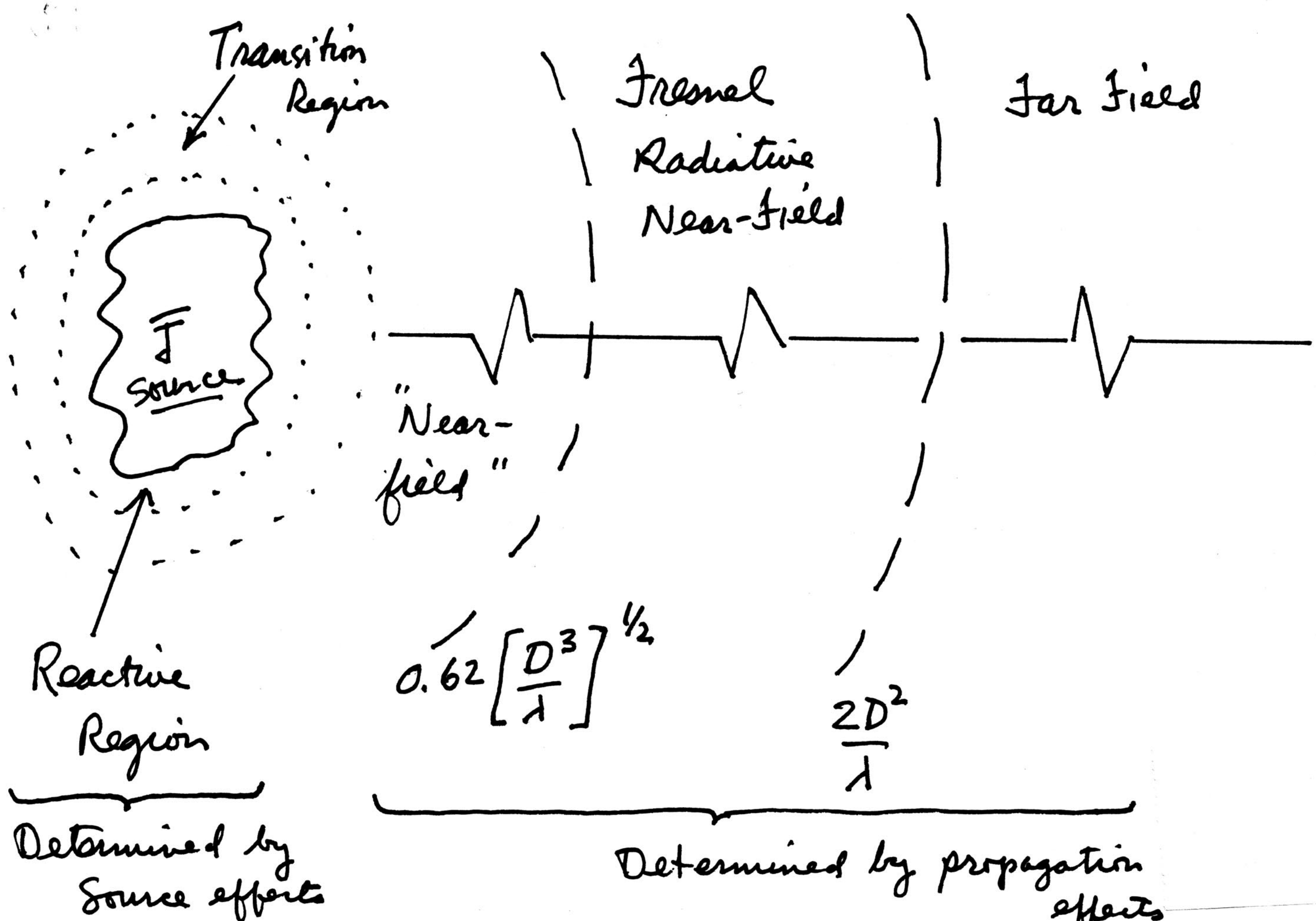
Reactive (sometimes "near-field" or "fringing-field")

Transition (sometimes "intermediate")

Radiating Near-Field (sometimes just "near-field")

Far-Field (always far-field)

Summary



Transition Region
Fresnel Radiative Near-Field
Far Field
$\bar{J}$ Source
"Near-field"
Reactive Region
$0.62\left[\frac{D^3}{\lambda}\right]^{1/2}$
$\frac{2D^2}{\lambda}$
Determined by Source effects
Determined by propagation effects