EE 252
Len Tyler **May 17, 2005**

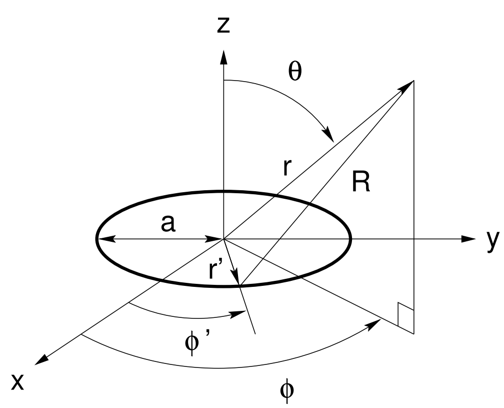
PROBLEM SET #6

Due Wednesday, May 25, 2005

Problems:

1. **Fields of a small, circular loop**
   Show that these fields are the dual of the ideal electric dipole fields by proceeding as follows:

   (a) Assume a loop of radius a ($ka \ll 1$) carrying a uniform current–there is no phase variation around the loop.



   (b) Express the source points $\mathbf{I}(\mathbf{r}') = I_\phi \hat{\boldsymbol{\phi}}$.

   (c) Express $\mathbf{I}(\mathbf{r}')$ in terms of $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$ at the observation point. To do this, write $\mathbf{I}$ in terms of its $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ (fixed) components. Then write $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ in terms of $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$ at (θ, ϕ). Substitution eliminating $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ completes this step.

   (d) Now you need $R = R(r, a, \theta, \phi, \phi')$. Start with the law of cosines in terms of ($\mathbf{r}$, $\mathbf{r}'$).

   (e) Plug into the radiation integral. You will have a radical in the denominator and in an exponent of the integrand.

   (f) Choose $\phi = 0$, without loss of generality, and expand the radical as a function of a. Retain only terms through first order in a.

   (g) Perform the integration.

   (h) What are the components of $\mathbf{A}$? *(Hint: only one is non-zero.)*

   (i) What are the fields?

(j) Compare with the results for ideal electric dipoles derived in class. Your results should be duals of the general dipole field expressions.

2. *Stutzman & Thiele*, Problem 7.1-6.

3. *Stutzman & Thiele*. Problem 7.3-4.