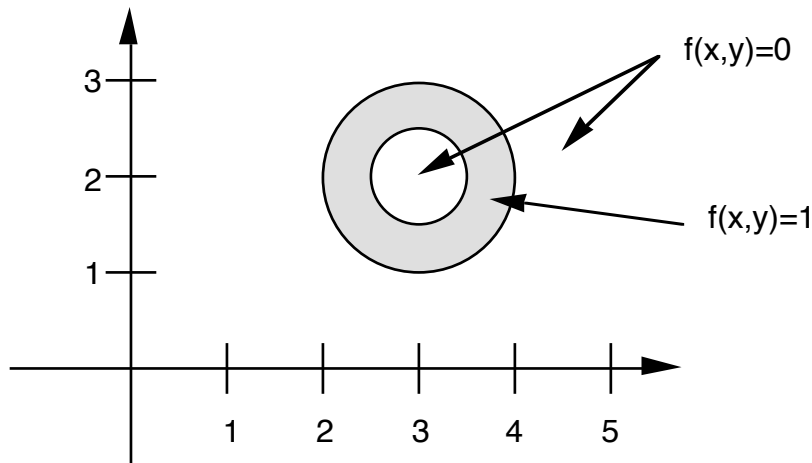


Homework # 4

Due next Wednesday

1. What is the Fourier transform of the function $f(x,y)$ defined by the following figure:



2. Prove the scaling theorem in 2-D, that is,

$$\text{if } f(x,y) \stackrel{2D}{\longleftrightarrow} F(u,v)$$

$$\text{then } f(ax,by) \stackrel{2D}{\longleftrightarrow} |ab|^{-1} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

3. Define functions as shown below. For each case, sketch the original function $f(x,y)$, and plot the 2-D Fourier power spectrum computed using fft techniques. You may plot the spectra with the DC term either in the upper left hand corner of the display or in the center, but label your axes accordingly. Submit tiff-format data files using the submiy script in the class computer submission directory. Assume each function is defined over a 256 x 256 point array.

a. $f(x,y) = \text{rect}(x/31, y/15)$, for $0 \leq x,y \leq 255$

b. $f(x,y) = \text{rect}(r/15)$, for $-128 \leq x,y \leq 127$

4. In the class homework data directory you will find an image entitled 'stanfordbw', a stretched image of the Stanford Quad similar to that used in HW 1. This file contains only the image data and consists of 173 lines each of length 540. The data are 8-bit

(byte) data. Using fft interpolation techniques, shift the image to the right by 27.3 pixels and down by 35.2 pixels, and place the shifted image in the class computer submission directory using the submit script. Your file should also be a byte file (8-bit data appropriately scaled.)

5. Download the file hw4prob5data from the class web area as listed under the “Homework” tab. This file is a coherent image of size 1024 x 1024, consisting of complex floating point data. The format is 1024 complex values representing line 1, followed by 1024 complex values representing line 2, and so forth. (Since the data are floating point values, you will have to be careful on the “endianness” of the data.)

a. Create an amplitude image file in byte format scaled so that the image displays well on your screen, and submit the byte file using the submit script.

b. Calculate the 2-D spectrum of the image. As before, you may plot the spectrum with the DC term either in the upper left hand corner of the display or in the center, but label and explain your axes accordingly.

c. Comment on the properties of the spectrum, including its width and position in the u,v plane. Can you speculate on how such an image may have been produced ?

6. Compare the time requirements for fft vs. dft algorithms. Copy the file hw4prob6data from the class web homework area. This is a 512 by 512 point complex file.

a. Calculate the spectrum using an fft algorithm, but this time be sure and plot the spectrum with the DC term in the upper left hand corner. Create a byte image display of the spectrum and submit. Time the calculation of the spectrum and list this value in your writeup.

b. Now, write your own implementation of a dft algorithm and use it to calculate the same spectrum as in part (a). Compare the two spectra and show that they are the same save for possible scaling and roundoff errors. Time this calculation and compare it to the fft technique. Report on the difference in execution times.

Hint: You will want to be fairly clever about how you write the dft, otherwise it will take until next week to calculate the result.