EE262 **Handout # 35**

Homework # 7

Due next Wednesday

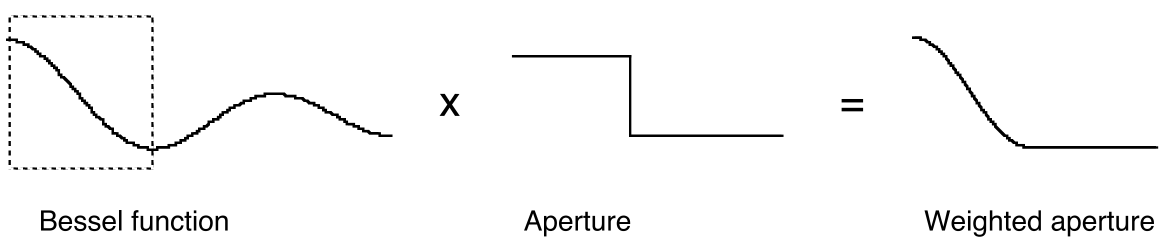
1. Show that the Hankel transform of r^{-1} is q^{-1} using Abel transforms.

2. You examine the sky with the Hubble space telescope, which has a round aperture 2.5 m in diameter. For visible light, with $\lambda = 0.5 \times 10^{-6}$ m, the aperture is uniformly illuminated.

a) What is the telescope **intensity** response to a point source ? For a cut through the center, what is the null to null width of the main lobe in seconds of arc ?

b) The pattern has a series of “rings” of light surrounding the central peak. What is the ratio of intensities of the main lobe to the first sidelobe ?

c) You know for rectangular apertures a cosine weighting reduces sidelobe levels, and you have calculated a cosine weighting on a circular aperture. Since cosines in rectangular coordinates correspond to $J_0(x)$ in circular coordinates, you now decide to weight the aperture according to the following:



Note that the Bessel function is scaled so that the edges of the aperture touch the minimum of the Bessel function.

What is the peak sidelobe ratio of the weighted aperture ? Why is this useful in detecting weak stellar companion stars ?

d) What are the half-power widths of the intensity pattern for the weighted and unweighted apertures ?

(You may evaluate the above numerically.)

3. We know that the Hankel transform of a circularly symmetric function is a one-dimensional representation of the 2-D Fourier transform. Compare the numerical results

of the Hankel transform of a function f(r) calculated using the integral definition of the Hankel transform vs. using a 2-D Fourier transform. Let

f(r)=(1/(pi*(a/2)^2))*rect(r/a)

and n be the first power of 2 greater than 4*a.

a) Set a=32. Sample f(r) at r = 0,1,2,...,n-1. Evaluate and plot the Hankel transform F(q) of your sampled version of f(r) using the integral definition of the Hankel transform for q = 0,1/2n,2/2n,...,(n-1)/2n. (You may find the Matlab function besselj() useful.)

b) Form a 2n x 2n 2-D array of values defined for x,y = -n,-n+1,...,n-1, with r=sqrt(x^2+y^2). Form the 2-D version of f(r) in part (a). Calculate the 2-D Fourier transform of your array, and plot the function along cuts through the origin at angles of 0 degrees and 45 degrees with respect to the u axis on the same plot as in (a). Don't forget to scale the axes so that the values are displayed as a function of q.

c) On the same plot, plot the analytical transform. How well do the plots agree? Explain.

d) Now repeat (a) through (c) using a =8. Plot your results again on a single graph. How well do plots agree now? Explain.

(Hint: You might want to plot the 2-D transforms to help explain the differences.)

4. Copy a file called "hw7p4data" from the class directory. This is an ascii file, type it out and examine it. It depicts a function in two columns: the first value is the argument of the function and the second value is the function itself. The first few entries look like this:

.0	.3534847
1.00000E-02	.3493845
2.00000E-02	.4093108

These data are a noisy projection of a circularly symmetric function as acquired by a tomographic scanner. In other words they represent the Abel transform of the original object.

a) Plot the data as they appear on the disk.

b) Invert the data to recover the original function using the relationships between Abel, Hankel, and Fourier transforms. Plot the recovered function.

(Hints: Since all functions are symmetric and real, we need to store only real values for positive arguments for all quantities and their transforms. It may be simpler to use a dft rather than an fft to obtain a properly sampled Fourier transform of the input sequence.)

c) Create an “image” of the resulting recovered function from part (b) by mapping the one dimensional function of r to a 2-d function of x and y, and submit your tiff file image of the original object.

5. Create a function $J_0(r)$ by integrating 2-D cosine functions over θ. That is, create many 2-D cosines at different angles to the x-axis, and weight and sum these to approximate the integral definition of the Bessel function. Calculate this integration over a 1024 x 1024 array of values using cosine frequencies of 4, 7, and 10 cycles over 1024 points. For each case, plot a radial cut through the result along with a theoretical Bessel function of the same frequency. Scale the numerical integration so that you get a close match to the analytic solution. Submit the plot and a tiff image of your numerical calculation.