Section 5
EE278: Introduction to Statistical Signal Processing (Summer 2017)
Gates B01, 3:30-4:20pm, Friday, 28/07

1. midterm exam Q4
Suppose the random variable \( Y \) is a noisy measurement of the angular position \( X \) of an antenna, so \( Y = X + Z \), where \( Z \) denotes the additive noise. Assume the noise is independent of the angular position, i.e., \( X \) and \( Z \) are independent random variables, with \( X \) uniformly distributed in the interval \([-1, 1]\) and \( Z \) uniformly distributed in the interval \([-2, 2]\).

(a) Find the MMSE (Minimum Mean Squared Error) estimate \( \hat{X}(y) \).
(b) Calculate the MSE.

2. LLSE and linear regression.
Consider a set of \( n \) pairs of numbers \( \{(x_i, y_i), i = 1, \ldots, n\} \). Linear regression is the (non-probabilistic) problem of finding a line \( z_i = \alpha x_i + \beta \) through the points that minimizes the mean square error (MSE)

\[
\epsilon_{\text{MSE}} = \sum_{i=1}^{n} (z_i - y_i)^2
\]

(a) Find the values of \( \alpha \) and \( \beta \) that minimize MSE by differentiating \( \epsilon_{\text{MSE}} \).
(b) Show that if \( (x_i, y_i) \) are realizations of a pair of random variables \( (X, Y) \), \( z_i \) approaches the LLSE estimator for \( Y|X = x_i \) in the limit of large \( n \).

3. Random walk with random start.
Let \( X_0 \) be a random variable with pmf

\[
p_{X_0}(x) = \begin{cases} \frac{1}{5} & x \in \{-2, -1, 0, +1, +2\} \\ 0 & \text{otherwise} \end{cases}
\]

Suppose that \( X_0 \) is the starting position of a random walk \( \{X_n : n \geq 0\} \) defined by

\[
X_n = X_0 + \sum_{i=1}^{n} Z_i,
\]

where \( \{Z_i\} \) is an i.i.d. random process with \( P(Z_1 = -1) = P(Z_1 = +1) = \frac{1}{2} \) and every \( Z_i \) is independent of \( X_0 \).

(a) Does \( X_n \) have independent increments? Justify your answer.
(b) What is the conditional pmf of \( X_0 \) given that \( X_{11} = 2 \)?