1. **Gauss-Markov process.** Let \( X_0 = 0 \) and \( X_n = \frac{2}{3} X_{n-1} + Z_n \) for \( n \geq 1 \), where \( Z_1, Z_2, \ldots \) are i.i.d. \( \sim \mathcal{N}(0, 1) \). Find the mean and autocorrelation function of \( X_n \).

**Solution (10 points)**

Using the method of lecture notes 7, we can easily verify that \( \mathbb{E}(X_n) = 0 \) for every \( n \) and that

\[
R_X(n, m) = \mathbb{E}(X_n X_m) = \left( \frac{2}{3} \right)^{|n-m|} \frac{9}{5} \left[ 1 - \left( \frac{4}{9} \right)^{\min\{n,m\}} \right].
\]

2. **Sawtooth process.** Let \( X(t) = g(t-T) \), where \( g(t) \) is the periodic triangular waveform shown in Figure 1 and the delay \( T \) is a random variable with \( T \sim \mathcal{U}[0, 1) \).

\begin{figure}[h]
\centering
\begin{tikzpicture}
\draw[->] (0,0) -- (4,0) node[anchor=north] {t};
\draw[->] (0,0) -- (0,1) node[anchor=east] {g(t)};
\draw (0,0) -- (0.5,0) -- (1,0.5) -- (1.5,0) -- (2,1) -- (2.5,0) -- (3,0.5) -- (3.5,0);
\draw (0,0) -- (0,0.5) -- (1,0) -- (1,0.5) -- (2,0) -- (2,0.5) -- (3,0) -- (3,0.5);
\filldraw (0,0) circle (2pt);
\filldraw (0.5,0) circle (2pt);
\filldraw (1,0.5) circle (2pt);
\filldraw (1.5,0) circle (2pt);
\filldraw (2,1) circle (2pt);
\filldraw (2.5,0) circle (2pt);
\filldraw (3,0.5) circle (2pt);
\filldraw (3.5,0) circle (2pt);
\end{tikzpicture}
\caption{Periodic triangular waveform}
\end{figure}

Is \( X(t) \) a strict-sense stationary random process? Justify your answer.

**Solution (10 points)**

By definition, \( X(t) \) is stationary if the joint distribution of any set of samples does not depend on the placement of the time origin. This means that the joint cdf of \( X(t_1), \ldots, X(t_k) \) is the same as that of \( X(t_1 + \tau), \ldots, X(t_k + \tau) \), for all \( t_1, t_2, \ldots, t_k \) and all \( \tau \). First we show that the first-order cdf of \( X(t) \) is independent of time. If
0 \leq x \leq 1, then
\[ F_X(t)(x) = P\{X(t) \leq x\} = P\{g(t - \Delta) \leq x\} \]
\[ = \int_0^1 P\{g(t - \Delta) \leq x \mid \Delta = \tau\} f_\Delta(\tau) \, d\tau \]
\[ = \int_0^1 P\{g(t - \Delta) \leq x \mid \Delta = \tau\} \, d\tau \]
\[ = \int_0^1 P\{g(t - \tau) \leq x\} \, d\tau \quad \text{since } g(t) \text{ has period 1} \]
\[ = \int_0^1 P\{ \tau \geq 1 - x\} \, d\tau = \int_{1-x}^1 1 \, d\tau = 1 - (1 - x) = x. \]

In other words, \( X(t) \) is uniformly distributed between 0 and 1 for every \( t \).

To show that higher order distributions are time invariant, note that \( g(t) \) can be expressed in terms of a given \( g(t_1) \) as \( g(t) = (g(t_1) - (t - T)) \mod 1 \), where \( a \mod b = a - b\lfloor a/b \rfloor \). Thus, given \( X(t_1) = x_1 \), the sample function of the process is determined unambiguously, and its values at \( t_2, t_3, \ldots, t_k \) depend only on the relative positions of the \( t_i \)'s. Thus \( X(t) \) is strict-sense stationary.

3. QAM random process. Consider the random process
\[ X(t) = Z_1 \cos \omega t + Z_2 \sin \omega t, \quad -\infty < t < \infty, \]
where \( Z_1 \) and \( Z_2 \) are i.i.d. discrete random variables such that \( p_{Z_i}(+1) = p_{Z_i}(-1) = \frac{1}{2} \).

a. Is \( X(t) \) wide-sense stationary? Justify your answer.

b. Is \( X(t) \) strict-sense stationary? Justify your answer.

Solution (10 points)

a. We first check the mean.
\[ \mathbb{E}(X(t)) = \mathbb{E}(Z_1) \cos \omega t + \mathbb{E}(Z_2) \sin \omega t = 0 \cdot \cos \omega t + 0 \cdot \sin \omega t = 0. \]
The mean is independent of \( t \). Next we consider the autocorrelation function.
\[ \mathbb{E}(X(t + \tau)X(t)) = \mathbb{E}((Z_1 \cos \omega(t + \tau) + Z_2 \sin \omega(t + \tau))(Z_1 \cos \omega t + Z_2 \sin \omega t)) \]
\[ = \mathbb{E}(Z_1^2) \cos \omega(t + \tau) \cos \omega t + \mathbb{E}(Z_2^2) \sin \omega(t + \tau) \sin \omega t \]
\[ = \mathbb{E}(Z_1^2) \cos \omega(t + \tau - \omega t) = \cos \omega \tau \]
The autocorrelation function is also time invariant. Therefore \( X(t) \) is WSS.
b. Note that \( X(0) = Z_1 \cos 0 + Z_2 \sin 0 = Z_1 \), so \( X(0) \) has the same pmf as \( Z_1 \). On the other hand, 
\[
X(\pi/(4\omega)) = Z_1 \cos(\pi/4) + Z_2 (\sin \pi/4)
\]
\[
= \frac{1}{\sqrt{2}} (Z_1 + Z_2)
\]
\[
\sim \begin{cases} 
\frac{1}{4} & x = \pm \frac{2}{\sqrt{2}} = \sqrt{2} \\
\frac{1}{2} & x = 0 \\
0 & \text{otherwise}
\end{cases}
\]
This shows that \( X(\pi/(4\omega)) \) does not have the same pdf or even same range as \( X(0) \). Therefore \( X(t) \) is not first-order stationary and as a result is not SSS.

4. Stationary Gauss-Markov process. Consider the following variation on the Gauss-Markov process:
\[
X_0 \sim \mathcal{N}(0,a) \\
X_n = \frac{1}{2} X_{n-1} + Z_n, \quad n \geq 1,
\]
where \( Z_1, Z_2, Z_3, \ldots \) are i.i.d. \( \mathcal{N}(0,1) \), independent of \( X_0 \).
a. Find the mean and autocorrelation functions of \( X_n \).
b. Find \( a \) such that \( X_n \) is stationary.
c. Consider the sample mean \( S_n = \frac{1}{n} \sum_{i=1}^{n} X_i \), \( n \geq 1 \). Show that \( S_n \) converges to the process mean in probability even though the sequence \( X_n \) is neither i.i.d. nor uncorrelated. (Hence, \( X_n \) is a mean ergodic process.)

**SOLUTION (10 points)**

a. Using the method of lecture notes 6, we can easily verify that \( E(X_n) = 0 \) for every \( n \) and that
\[
R_X(n_1, n_2) = E(X_{n_1}X_{n_2}) = \frac{4}{3} 2^{-|n_1-n_2|}.
\]
b. We are asked to find \( a \) such that \( E(X_n) \) is independent of \( n \) and \( R_X(n_1, n_2) \) depends only on \( n_1 - n_2 \). For \( X_n \) to be stationary, \( E(X_n^2) \) must be independent of \( n \). Thus
\[
E(X_n^2) = \frac{1}{4} E(X_{n-1}^2) + E(Z_n^2) + E(X_{n-1}Z_n) = \frac{1}{4} E(X_n^2) + 1.
\]
Therefore, \( a = E(X_0^2) = E(X_n^2) = \frac{4}{3} \).
c. To prove convergence in probability, we first prove convergence in mean square and then use the fact that mean square convergence implies convergence in probability.
\[
E(S_n) = E \left( \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \sum_{i=1}^{n} 0 = 0.
\]
To show convergence in mean square we show that \( \text{Var}(S_n) \to 0 \) as \( n \to \infty \).

\[
\text{Var}(S_n) = \text{Var}\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = E \left( \left( \frac{1}{n} \sum_{i=1}^{n} X_i \right)^2 \right) \quad \text{(since } E(X_i) = 0) \\
= \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} R_X(i, j) = \frac{4}{3n^2} \left( n + 2 \sum_{i=1}^{n-1} (n-i)2^{-i} \right) \\
\leq \frac{4}{3n} \left( 1 + 2 \sum_{i=1}^{n-1} 2^{-i} \right) \leq \frac{4}{3n} \left( 1 + 2 \sum_{i=1}^{\infty} 2^{-i} \right) = \frac{4}{n}.
\]

Thus \( S_n \) converges to the process mean, even though the sequence is not i.i.d.

5. *Generating a random process with a prescribed psd.* The power spectral density \( S_X(f) \) of every WSS process is real, even, and nonnegative. In this problem you will show that, conversely, if \( S(f) \) is a real, even, nonnegative function such that \( \int_{-\infty}^{\infty} S(f) df < \infty \), then \( S(f) \) is the psd for some WSS random process. Let us consider the case that

\[
\int_{-\infty}^{\infty} S(f) df = 1.
\]

Define the random process

\[
X(t) = \cos(2\pi F t + \Theta),
\]

where \( F \sim S(f) \) and \( \Theta \sim U[0, 2\pi] \) are independent.

a. Show that \( X(t) \) is WSS.

b. Find the power spectral density of \( X(t) \). Interpret the result.

c. Consider the power spectral density

\[
S(f) = \frac{\alpha}{\alpha^2 + (\pi f)^2}, \quad -\infty < f < \infty.
\]

Use MATLAB to generate sample functions of \( X(t) \) for \( \alpha = 1, 5, 20 \).

**SOLUTION** (15 points)

a. First we find the mean of \( X(t) \).

\[
E(X(t)) = E\left( \cos(2\pi F t + \Theta) \right) \\
= E \left( 0 \mid F \right) = E_F(0 \mid F) = 0.
\]
Next the autocorrelation function.

\[ R_X(t_1,t_2) = E(X(t_1)X(t_2)) = E(\cos(2\pi F t_1 + \Theta) \cos(2\pi F t_2 + \Theta)) \]

\[ = E_F \left( E_{\Theta|F}(\cos(2\pi F t_1 + \Theta) \cos(2\pi F t_2 + \Theta)) \right) | F \]

\[ = E_F \left( \frac{1}{2} \cos(2\pi F (t_1 - t_2)) \right) \]

\[ = E_f \left( \frac{1}{2} \cos(2\pi F \tau) \right) \]

\[ = \frac{1}{2} \int_{-\infty}^{\infty} S(\alpha) \cos(2\pi \alpha \tau) \, d\alpha. \]

Since the mean is time invariant and the autocorrelation function depends only on the time difference, \( X(t) \) is WSS.

b. The power spectral density of \( X \) is

\[ S_X(f) = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\alpha) \cos(2\pi \alpha \tau) \, d\alpha \, e^{-j 2\pi f \tau} \, d\tau \]

\[ = \frac{1}{2} \int_{-\infty}^{\infty} S(\alpha) \int_{-\infty}^{\infty} \cos(2\pi \alpha \tau) e^{-j 2\pi f \tau} \, d\tau \, d\alpha \]

\[ = \frac{1}{2} \int_{-\infty}^{\infty} S(\alpha) \frac{1}{2} (\delta(f - \alpha) + \delta(f + \alpha)) \, d\alpha \]

\[ = \frac{1}{4}(S(f) + S(-f)) \]

\[ = \frac{1}{2} S(f) \quad \text{(since } S(f) \text{ is even)} \]

Notice that the psd of \( X(t) \) is a scalar multiple of the pdf of \( F \). This constructively proves that any function that is real, even, and nonnegative and has a finite integral can be a power spectral density for a WSS random process.

c. The pdf and cdf of the frequency random variable \( F \) are related as follows:

\[ f_F(f) = S(f) = \frac{\alpha}{\alpha^2 + (\pi f)^2} = \frac{d}{df} F_F(f). \]

Therefore

\[ F_F(f) = \frac{1}{2} + \frac{1}{\pi} \arctan \left( \frac{\pi f}{\alpha} \right). \]

Thus the frequency random variable \( F \) is a function of a uniform \( Z \sim U[0,1] \).

\[ F = F_F^{-1}(Z) = \frac{\alpha}{\pi} \tan(\pi(Z - \frac{1}{2})). \]

% The following MATLAB code generates 5 sample functions of X(t)
% for each of three values of alpha: 1, 5 and 20.

alpha_list = [ 1 5 20 ];
```matlab
% MATLAB code for generating sample paths

% Initialize t
t = 0:.001:1;

% Loop over alpha values
for alpha_index = 1 : 3
    alpha = alpha_list( alpha_index );
    Z = rand( 5, 1 );
    F = (alpha/pi)*tan( pi*( Z - 1/2 ) );
    Theta = pi*( 2*rand( 5, 1 ) - 1 );
    X = cos( 2*pi*F*t + repmat( Theta, 1, 1001 ) );
    subplot( 3, 1, alpha_index );
    plot( t, X );
end

% Sample output is shown in the figure below.
```

Sample output is shown in the figure below.
6. **Windowed Poisson process.** Let $N(t)$, for $t \geq 0$, be a Poisson process with rate $\lambda > 0$, and let $X(t)$, for $t \geq 0$, be defined by $X(t) = N(t + 1) - N(t)$. Thus, $X(t)$ is the number of events of $N$ during the time window $(t, t + 1]$.

a. Sketch a typical sample path of $N$, and the corresponding sample path of $X$.

b. Find the mean function $\mu_X(t)$, for $t \geq 0$ and the autocorrelation function $R_X(t_1, t_2)$ for $t_1, t_2 \geq 0$. Express your answer in a simple form.

c. Is $X$ a Markov process? Why or why not?

d. Consider the process $Y(t) = (1/t) \int_0^t X(s) ds$. Determine whether $Y(t)$ converges in the mean square sense as $t \to \infty$.

**Solution** (15 points)

a. An example with $\lambda = 1$ is shown in the figure below.
b. We can compute the mean function as follows

\[
\mu_X(t) = E[X(t)] \\
= E[N(t + 1)]E[N(t)] \\
= \lambda(t + 1) - \lambda t \\
= \lambda.
\]

The autocorrelation function is

\[
R_X(t_1, t_2) = E[X(t_1)X(t_2)] \\
= E[(N(t_1 + 1)N(t_1))(N(t_2 + 1)N(t_2))] \\
= R_N(t_1 + 1, t_2 + 1) - R_N(t_1 + 1, t_2) - R_N(t_1, t_2 + 1) + R_N(t_1, t_2) \\
= \lambda(\min\{t_1 + 1, t_2 + 1\} - \min t_1 + 1, t_2 - \min\{t_1, t_2 + 1\} + \min\{t_1, t_2\}) \\
+ \lambda^2((t_1 + 1)(t_2 + 1) - (t_1 + 1)t_2 - t_1(t_2 + 1) + t_1t_2) \\
= \begin{cases} 
\lambda(1 - |t_1 - t_2|) + \lambda^2 & \text{if } |t_1 - t_2| \leq 1, \\
\lambda^2 & \text{otherwise.}
\end{cases}
\]

where the last step can be seen by considering the four cases

\[
t_1 \in [0, t_2 - 1), \\
|t_1 - t_2| \leq 1, \\
|t_1 - t_2| > 1.
\]

one by one and computing the min terms for each of them. \textbf{Remark:} Since \(\mu_X(t)\) does not depend on \(t\) and \(R_X(t_1, t_2)\) depends only on \(|t_1 - t_2|\), it follows that \(X(t)\) is WSS.

c. \(X(t)\) is not a Markov process. To prove this, we need to find times \(t_1 < t_2 < t_3\) and values \(x_1, x_2, x_3\) such that

\[
P\{X(t_3) = x_3 \mid X(t_2) = x_2, X(t_1) = x_1\} \neq P\{X(t_3) = x_3 \mid X(t_2) = x_2\}.
\]

There are several choices that work. We will use \(t_1 = 0, t_2 = 0.5, t_3 = 1.0\), and \(x_1 = 0, x_2 = 1, x_3 = 0\). Define the random variables

\[
A = N(0.5) - N(0), \\
B = N(1.0) - N(0.5), \\
A = N(1.5) - N(1.0), \\
A = N(2.0) - N(1.5),
\]

each of which is Poisson-distributed with mean \(\lambda/2\). The four random variables are independent from each other since the time intervals do not overlap. The
right hand side of (1) is then
\[ P\{C + D = 0 \mid B + C = 1\} = \frac{P\{C = 0, D = 0, B + C = 1\}}{P\{B + C = 1\}} \]
\[ = \frac{P\{C = 0, D = 0, B = 1\}}{P\{B + C = 1\}} \]
\[ = \frac{P\{C = 0\} P\{D = 0\} P\{B = 1\}}{P\{B + C = 1\}} \]
\[ = \frac{\lambda/2e^{-(3/2)\lambda}}{\lambda e^{-\lambda}} \]
\[ = \frac{1}{2}e^{-\lambda/2}. \]

The left hand side of (1) is
\[ P\{C + D = 0 \mid B + C = 1, A + B = 0\} = P\{C = 0, D = 0 \mid A = 0, B = 0, C = 1\} \]
\[ = 0 \]
which is different from the right hand side for every \( \lambda \). Therefore, the process \( X(t) \) is not a Markov process for any \( \lambda \).

d. We are interested in \( Y(t) = (1/t) \int_0^t X(s)ds \). We are going to show that \( Y(t) \) converges to \( \lambda \) in mean square as \( t \to \infty \). To this end, we first compute the second moment. Following the integrator example in the lecture notes (page 8-3), it is easy to see that
\[ E[Y(t)^2] = R_Y(t, t) \]
\[ = \frac{1}{t^2} \int_0^t \int_0^t R_X(\tau_1, \tau_2) \ d\tau_2 \ d\tau_1 \]
\[ = \lambda^2 + \frac{\lambda}{t^2} \int_0^t \int_0^t (1 - |\tau_1 - \tau_2|) \ d\tau_2 \ d\tau_1, \]
where we have substituted the equation for \( R_X(\tau_1, \tau_2) \). The region over which the double integral is taken is a diagonal stripe as defined by the three inequality conditions. A moments thought reveals that its area is exactly \( 2t - 1 \). For every \( (\tau_1, \tau_2) \) in the integration region, the integrand is less than or equal to 1. Thus, the double integral is less than or equal to \( 2t - 1 \), and we conclude
\[ E[Y(t)^2] \leq \lambda^2 + \frac{\lambda(2t - 1)}{t^2}. \]
It follows that
\[ E[(Y(t) - \lambda)^2] = E[Y(t)^2] - 2\lambda E[Y(t)] + \lambda^2 \]
\[ \leq \frac{\lambda(2t - 1)}{t^2}, \]
where we have used $E[Y(t)] = \lambda$. The last expression converges to zero as $t \to \infty$.

7. Modified telegraph process. Let $X(t), Y(t),$ and $W(t)$ be independent random processes; $X(t)$ and $Y(t)$ are zero-mean stationary Gaussian processes with $R_X(\tau) = R_Y(\tau) = e^{-|\tau|}$. $W(t)$ is the random telegraph process,

$$W(t) = A(-1)^N(t),$$

where $N(t)$ is a Poisson process with parameter $\lambda$, and the random variable

$$A = \begin{cases} 
1 & \text{with probability 0.5} \\
-1 & \text{with probability 0.5.}
\end{cases}$$

$A$ and $N(t)$ are independent. Now define the new process $Z(t)$ as

$$Z(t) = \begin{cases} 
X(t) & \text{if } W(t) = 1 \\
Y(t) & \text{if } W(t) = -1.
\end{cases}$$

a. Find the first order distribution of $Z(t)$.

b. Is $Z(t)$ a Gaussian random process? Justify your answer.

c. Is $Z(t)$ WSS? Justify your answer.

SOLUTION (15 points)

a. We know that

$$Z(t) = \begin{cases} 
X(t) & \text{if } W(t) = +1 \\
Y(t) & \text{if } W(t) = -1.
\end{cases}$$

We can write

$$P\{Z(t) \leq z\} = P\{W(t) = +1\}P\{Z(t) \leq z | W(t) = +1\} + P\{W(t) = -1\}P\{Z(t) \leq z | W(t) = -1\}$$

$$= \frac{1}{2}(P\{X(t) \leq z | W(t) = +1\} + P\{Y(t) \leq z | W(t) = -1\})$$

$$= \frac{1}{2}(\phi(z) + \phi(z))$$

$$= \phi(z),$$

where we have used the fact that $X(t)$ and $Y(t)$ are independent of $W(t)$, and where $\phi$ is the cdf of the normal distribution. Thus, $Z(t) \sim \mathcal{N}(0, 1)$, independent of $t$.

b. We consider the joint distribution of two samples $Z(t_1)$ and $Z(t_2)$, where $t_1 < t_2$. 
Their joint cdf is
\[
P \{ Z(t_1) \leq z_1, Z(t_2) \leq z_2 \}
= \sum_{w_1, w_2 \in \{+1, -1\}} P \{ W(t_1) = w_1, W(t_2) = w_2 \}
\cdot P \{ Z(t_1) \leq z_1, Z(t_2) \leq z_2 \mid W(t_1) = w_1, W(t_2) = w_2 \}. \tag{2}
\]
If \( Z(t_1) \) and \( Z(t_2) \) are from the same process (either both from \( X(t) \) or both from \( Y(t) \)), then their joint distribution is
\[
\begin{bmatrix} Z(t_1) \\ Z(t_2) \end{bmatrix} \mid \{ W(t_1) = W(t_2) \} \sim \mathcal{N} \left( 0, \begin{bmatrix} 1 & e^{-(t_2-t_1)} \\ e^{-(t_2-t_1)} & 1 \end{bmatrix} \right), \tag{3}
\]
where we have used the autocorrelation function of \( X(t) \) and \( Y(t) \). If they are from different processes (one from \( X(t) \) and one from \( Y(t) \)), then their joint distribution is
\[
\begin{bmatrix} Z(t_1) \\ Z(t_2) \end{bmatrix} \mid \{ W(t_1) = W(t_2) \} \sim \mathcal{N} \left( 0, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), \tag{4}
\]
since \( X(t) \) and \( Y(t) \) are independent. Substituting into (2), the joint cdf of \( Z(t_1) \) and \( Z(t_2) \) is
\[
P \{ Z(t_1) \leq z_1, Z(t_2) \leq z_2 \}
= P \{ W(t_1) = W(t_2) \} \Phi_1(z_1, z_2) + P \{ W(t_1) \neq W(t_2) \} \Phi_2(z_1, z_2)
= \frac{1 + e^{-2\lambda(t_2-t_1)}}{2} \Phi_1(z_1, z_2) + \frac{1 - e^{-2\lambda(t_2-t_1)}}{2} \Phi_2(z_1, z_2)
\]
where we have used the probability for an even/odd number of events in the interval \((t_1, t_2]\) as computed in problem 1(c). Taking a linear combination of two different Gaussian cdfs does not result in a Gaussian cdf. (This is different from taking a linear combination of two Gaussian random variables.) Hence, the joint distribution of \( Z(t_1) \) and \( Z(t_2) \) is not Gaussian, and \( Z(t) \) is not a GRP.

c. It follows from part (a) that the mean and variance of \( Z(t) \) are independent of \( t \). To decide whether \( Z(t) \) is WSS, we only need to compute the correlation function \( R_Z(t_1, t_2) \). In part (b), we already computed the joint distribution of \( Z(t_1) \) and \( Z(t_2) \) for \( t_1 < t_2 \). The joint pdf is
\[
f_{\{Z(t_1), Z(t_2)\}}(z_1, z_2) = c_1 \phi_1(z_1, z_2) + c_2 \phi_2(z_1, z_2),
\]
where
\[ c_1 = \frac{1 + e^{-2\lambda(t_2-t_1)}}{2} \]
\[ c_2 = \frac{1 - e^{-2\lambda(t_2-t_1)}}{2} \]
\[ \phi_1(z_1, z_2) = \mathcal{N} \left( 0, \begin{bmatrix} 1 & e^{-(t_2-t_1)} \\ e^{-(t_2-t_1)} & 1 \end{bmatrix} \right) \]
\[ \phi_2(z_1, z_2) = \mathcal{N} \left( 0, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right). \]

The correlation function is
\[ R_Z(t_1, t_2) = \mathbb{E}[Z(t_1)Z(t_2)] \]
\[ = \int \int z_1 z_2 f_{Z(t_1), Z(t_2)}(z_1, z_2) \, dz_1 \, dz_2 \]
\[ = c_1 \int \int z_1 z_2 \phi_1(z_1, z_2) \, dz_1 \, dz_2 + c_2 \int \int z_1 z_2 \phi_2(z_1, z_2) \, dz_1 \, dz_2 \]
\[ = c_1 e^{-(t_2-t_1)} + c_2 \cdot 0 \]
\[ = \frac{1 + e^{-2\lambda(t_2-t_1)}}{2} e^{-(t_2-t_1)} \]

This is a function only of \( t_2 - t_1 \), not of \( t_1 \) and \( t_2 \) individually. Thus \( Z(t) \) is WSS.