Homework #2
Due Sunday, July 16, 05:00 pm via Gradescope

1. Exercise 3.4 in Gallager.
   **Do not use jointly Gaussian random vectors, only use scalars for this question.**

2. Exercise 3.17 in Gallager.

3. Exercise 8.9 in Gallager.

4. Exercise 8.15 parts (a), (b) and (d) in Gallager.

5. In this problem we are going to implement the MAP rule for example 3 taught in class. Download the dataset from the course website [http://web.stanford.edu/class/ee278/homeworks/hw4-data.zip](http://web.stanford.edu/class/ee278/homeworks/hw4-data.zip) In example 3, we detect based on an observation \( Y \in \mathbb{R}^n \) following the statistical model
\[
Y = hX + gW + Z,
\]
where \( g \) and \( h \) are normalized to have norm 1, \( n = 10 \), \( W \sim \mathcal{N}(0, \sigma_w^2) \), \( Z \sim \mathcal{N}(0, \sigma_z^2I) \) and all vectors have the matching dimensions. We will further make the assumption that \( X \) is equally likely to be 0 or 1.

In practice, we may not know the parameters. So we need to first estimate the parameters and then estimate \( X \) from \( Y \).

(a) To estimate the noise level \( \sigma_z \), we first collect data in the form of \( k = 3000 \) observations which only contain noise \( Z \) but no signal \( hX \) nor interference \( gW \). Propose an estimator for \( \sigma_z^2 \). The data are stored as column vectors in \( Y_z \) in \( data.mat \). Implement your method and report the estimated value of \( \sigma_z^2 \). You can assume the noise is independent across the 3000 observations.

(b) We now want to further estimate \( g \) and \( \sigma_w \). We collect data in the form of another \( k = 3000 \) observations with both the interference \( gW \) and the noise \( Z \), but no signal \( hX \). Propose an estimator for \( \sigma_w^2 \) and \( g \). The data are stored as column vectors in \( Y_w \) in \( data.mat \). Implement your method and report the estimated value of \( \sigma_w^2 \) and \( g \). You can assume that while the interference and the noise are independent across the 3000 observations, the parameters to estimate remain the same.

(c) Finally, we need to estimate \( h \). Propose an estimator for \( h \) based on another \( k = 3000 \) observations which follows model (1). The observations are stored as column vectors in \( Y_x \) in \( data.mat \). Implement your method and report the estimated value of \( h \). You can assume the signals are independent across the observations.

(d) Based on the observations in part (c) and your estimated channel parameters, perform detection using the MAP rule on each of the 3000 observations. The true signals are stored in \( X \) as an \( 1 \times n \) row vector in \( X_label.mat \). Report the overall error rate. Your detector should not involve inverting a 10 by 10 matrix.

Please include your code. You can use any programming languages (e.g. Matlab, Python).