1. Exercise 10.2 in Gallager.

2. Consider the estimation problem:

\[ Y_1 = X + Z_1 \]
\[ Y_2 = X + Z_2 \]

where \( X, Z_1, Z_2 \) are mutually independent Gaussian random variables. \( X \) has mean \( \bar{X} \) and variance \( \sigma_0^2 \), while \( Z_1, Z_2 \) have mean 0 and variance 1.

(a) Compute the conditional distribution of \( X \) given \( Y_1 = y_1 \).

(b) Compute the conditional distribution of \( (X, Z_1) \) given \( Y_2 = y_2 \).

(c) Using (a) and (b) compute the conditional distribution of \( X \) given \( Y_1 = y_1 \) and \( Y_2 = y_2 \) in a successive fashion by first conditioning on \( Y_1 = y_1 \) and then on \( Y_2 = y_2 \).

(d) Check that your answer in part (c) is the same as what you would have got by computing the conditional distribution directly.

3. Consider the MMSE estimation problem with a \( n \)-dimensional observation:

\[ Y = hX + Z, \]

where \( X \sim \mathcal{N}(0, 1) \) and \( Z \sim \mathcal{N}(0, \sigma^2 I) \) and independent of \( X \).

(a) Show that \( V = h^T Y \) is a sufficient statistic for the estimation problem. Recall that a sufficient statistic \( V = v(Y) \) is such that \( X, V, Y \) forms a Markov chain.

(b) Does \( V \) remain a sufficient statistic if \( X \) is non-Gaussian?

4. Exercise 10.3 in Gallager.

5. Consider the vector dynamical system:

\[ X_1 \sim \mathcal{N}(0, K_1) \]
\[ X_{n+1} = AX_n + BW_{n+1} \]
\[ Y_n = CX_n + DZ_n \]

where \( W_1, \ldots, Z_1, \ldots \), are independent and \( W_n \sim \mathcal{N}(0, K_w) \) and \( Z_n \sim \mathcal{N}(0, K_z) \).

(a) Reformulate the scalar system:

\[ X \sim \mathcal{N}(0, 1) \]
\[ X_{n+1} = X_n + 0.5X_{n-1} + 0.2X_{n-2} + W_{n+1} \]
\[ Y_n = X_n + 0.4X_{n-1} + Z_n \]

as a vector dynamical system. Here, \( W_n \)'s and \( Z_n \)'s are independent and \( \mathcal{N}(0, 1) \) distributed.
(b) For the general vector dynamical system, derive the recursion for the Kalman filter estimates and for the covariance matrices of the errors. Assuming that the error covariance matrices approach a limit as $n \to \infty$, characterize the limit in terms of the solution of an equation.