Homework #7  
Due Wednesday, May 25

1. **Autocorrelation functions.** Find the autocorrelation functions of
   
a. random binary waveform $X(t)$:
   
   In a digital communication channel, the symbol “1” is represented by the fixed duration rectangular pulse
   
   $$g(t) = \begin{cases} 
   1 & \text{for } 0 \leq t < T \\
   0 & \text{otherwise} 
   \end{cases}$$

   and the symbol “0” is represented by $-g(t)$. The data transmitted over the channel is represented by the random process
   
   $$X(t) = \sum_{k=0}^{\infty} A_k g(t - kT), \quad t \geq 0,$$

   where the data $A_0, A_1, A_2, \ldots$ is an i.i.d. random sequence with
   
   $$A_i = \begin{cases} 
   +1 & \text{with probability } \frac{1}{2} \\
   -1 & \text{with probability } \frac{1}{2} 
   \end{cases}$$

   b. moving average process $Y_n$:
   
   Let $\{X_n : n \geq 1\}$ be a discrete-time white Gaussian noise process, that is, $X_1, X_2, X_3, \ldots$ are i.i.d. random variables with $X_n \sim \mathcal{N}(0, \sigma^2)$. Consider the moving-average process $\{Y_n : n \geq 2\}$ defined by
   
   $$Y_n = \frac{2}{3} X_{n-1} + \frac{1}{3} X_{n-2}, \quad n \geq 2.$$

   with $X_0$ defined to be 0.

2. **Sawtooth process.** Let $X(t) = g(t - T)$, where $g(t)$ is the periodic triangular waveform shown in Figure 1, and the delay $T$ is a random variable with $T \sim \text{U}[0, 1]$.

   ![Figure 1: Periodic triangular waveform](image)

   Is $X(t)$ a strict-sense stationary random process? Justify your answer.
3. **Crosscorrelation function inequalities.** Prove the following statements.
   If $X, Y$ are jointly WSS processes,
   
a. $|R_{XY}(\tau)| \leq \sqrt{R_X(0)R_Y(0)}$.
   
b. $|R_{XY}(\tau)| \leq \frac{1}{2}(R_X(0) + R_Y(0))$.

4. **LTI system with WSS process input.** Let $Y(t) = h(t) \ast X(t)$ and $Z(t) = X(t) - Y(t)$, as shown in Figure 2.

   ![Figure 2: Linear time-invariant system.](image)

   a. Find $S_Z(f)$.
   
b. Find $E(Z^2(t))$.
   
   Your answers should be in terms of $S_X(f)$ and the transfer function $H(f) = F(h(t))$.

5. **Linear system with feedback.** Given the system in Figure 3 where $X(t)$ and $Z(t)$ are independent zero-mean WSS random processes, and $h(t)$ is the impulse response of a linear time invariant system, find the power spectral density of $Y(t)$. Your answers should be in terms of $S_X(f), S_Z(f)$ and the transfer function $H(f) = F(h(t))$.

   ![Figure 3: Linear System with Feedback](image)

6. **Discrete-time LTI system with white noise input.** Let $\{X_n: -\infty < n < \infty\}$ be a discrete-time white noise process, i.e., $E(X_n) = 0$ and
   
   $$R_X(n) = \begin{cases} 
   1 & n = 0 \\
   0 & \text{otherwise} 
   \end{cases}$$

   The process is filtered using a linear time invariant system with impulse response
   
   $$h(n) = \begin{cases} 
   \alpha & n = 0 \\
   \beta & n = 1 \\
   0 & \text{otherwise} 
   \end{cases}$$
Find $\alpha$ and $\beta$ such that the output process $Y_n$ has

$$R_Y(n) = \begin{cases} 2 & n = 0 \\ 1 & |n| = 1 \\ 0 & \text{otherwise} \end{cases}$$