1. **Non-Gaussian Estimation**  Suppose that $X \sim \text{Exponential}(\lambda)$ and $Y \sim \text{Exponential}(x)$ conditional on $X = x$. We wish to estimate $X$ from the observation $Y$.

(a) Compute the joint distribution $f_{X,Y}(x,y)$ and the conditional distribution $f_{X|Y}(x|y)$.

(b) Find the MMSE estimate of $X$ given the observation $Y = y$.

(c) What is the MAP estimate of $X$ given $Y = y$, i.e. the value $x$ that maximizes $f_{X|Y}(x|y)$?

(d) What is the ML estimate of $X$ given $Y = y$, i.e. the value $x$ that maximizes $f_{Y|X}(y|x)$?