1. **Deeper Kalman filter recursions** Reformulate the scalar system:

\[
\begin{align*}
X_0 & \sim \mathcal{N}(0, 1) \\
X_1 &= X_0 + W_1 \\
X_n &= X_{n-1} + 0.5X_{n-2} + W_n, \quad n = 2, 3, \ldots \\
Y_n &= X_n + 0.4X_{n-1} + Z_n, \quad n = 1, 2, \ldots
\end{align*}
\]

as a vector dynamical system. Here, \(W_n\)’s and \(Z_n\)’s are independent and \(\mathcal{N}(0, 1)\) distributed, and independent of \(X_0\) and \(X_1\).

2. **Signals and Systems** Consider the system with input \(x_n\) and output \(y_n\) given by

\[
y_n = \frac{1}{2}y_{n-1} + x_n.
\]

1. The initial condition is: if \(x_n = 0 \ \forall n < N\), then \(y_n = 0 \ \forall n < N\). Is this an LTI system?

2. The initial condition is \(y_n = 0\) for \(n < 0\). Is this an LTI system?

3. What is the impulse response of the LTI version of this system?

4. If we give the input

\[
x_n = u_n = \begin{cases} 
0 & n < 0 \\
1 & n \geq 0
\end{cases}
\]

calculate the output \(y_n\) of the LTI system.

5. Verify that you get the same result as part (c) if you convolve the input and the impulse response.