1. Stationary Processes.

For each random process, sketch a few sample paths, and compute the mean function and covariance function. State whether each process is stationary or not. Each process is defined for \( n = 0, 1, 2, 3, \ldots \).

1. \( X_n \) are i.i.d. \( \mathcal{N}(0, 1) \) random variables.
2. \( X_n = A \) where \( A \sim \mathcal{N}(0, 1) \).
3. \( X_n = nA \) where \( A \sim \mathcal{N}(0, 1) \).
4. \( X_n = \cos(2\pi n) \).
5. \( X_n = \cos(\pi n) \).
6. \( X_n = \cos(\pi n + \Theta) \) where \( \Theta \) is a \( \text{U}[0, 2\pi] \) random variable.

2. AM modulation. Consider the discrete AM modulated random process

\[
Y_n = A_n \cos(\pi n + \Theta)
\]

for \( n \geq 1 \), where the amplitude \( A_n \) is a stationary zero-mean random process with covariance function \( K_A(n_1, n_2) = e^{-|n_1 - n_2|} \), the phase \( \Theta \) is a \( \text{U}[0, 2\pi] \) random variable, and \( A_n \) and \( \Theta \) are independent.

1. Find the mean function of \( Y_n \). Does it depend on \( n \)?
2. Find the covariance function \( K_Y(n_1, n_2) \). Does it depend only on \(|n_1 - n_2|\)?
3. Is \( Y_n \) stationary?

The following trigonometric identities may be useful:

\[
\cos(A) \cos(B) = \frac{1}{2} (\cos(A - B) + \cos(A + B))
\]
\[
\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)
\]