Chapter 14

Photovoltaic Converters

Problem Solutions

Prob 14.1 What is the theoretical efficiency of cascaded photodiodes made of two semiconductors, one with a bandgap energy of 1 eV and the other with 2 eV when exposed to sunlight?

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We will take a 5800 K black body as representing the sun.

Cell #1 is transparent up to the frequency, $f_{g_1} = W_{g_1}/h = 1 \times 1.6 \times 10^{-19}/6.626 \times 10^{-34} = 242 \times 10^{12}$ Hz, while Cell #2 is transparent up to 484×10^{12} Hz.

To cascade the cells, it is necessary to place Cell #2 in front. Its efficiency will be

$$\eta_2 = 8.17 \times 10^{-43} \frac{f_{g_2}}{T^4} \int_{f_{g_2}}^{\infty} \frac{f^2}{e^{\frac{hf}{kT}} - 1} df$$

while that of Cell #1 (which sees only that part of the solar spectrum for which $f_{g_1} < f < f_{g_2}$), will be

$$\eta_1 = 8.17 \times 10^{-43} \frac{f_{g_1}}{T^4} \int_{f_{g_1}}^{f_{g_2}} \frac{f^2}{e^{\frac{hf}{kT}} - 1} df.$$

The efficiency of the two cells when cascaded is $\eta = \eta_1 + \eta_2$.

When evaluating the integrals, care must be exercised owing to the large numerical values involved. For instance, MATHEMATICA will not handle these integrals correctly. It is necessary to expressed the frequencies in THz (adjusting the constant in the integral accordingly) and to use an upper limit much lower than infinity. A little experimentation will show that an upper limit of $10f_{g_2}$ for the first integral will yield the correct result (increasing the limit further does not change the result).

One obtains

$$\eta_1 = 0.2883$$

and

$$\eta_2 = 0.2952$$

so that

$$\eta = 0.584.$$

The efficiency of the cascaded photocells would be 58.9%. It must be noted that the practical efficiency of such a pair of cells is much lower owing to the technical difficulties in building the device.

Solution of Problem 14.1

Prob 14.2 As any science fiction reader knows, there are many parallel universes each one with different physical laws. In the parallel universe we are discussing here, the black body radiation at a given temperature, T, follows a simple law:

$\frac{\partial P}{\partial f}$ is zero at f=0,

$\frac{\partial P}{\partial f}$ grows linearly with f to a value of 1 W m^{-2}THz^{-1} at 500 THz,

from 500 THz it decreases linearly to zero at 1000 THz.
P is the power density (W m^{-2}) and f is the frequency in Hz.

Translating the description of the blackbody radiation into mathematical language,

$$\frac{\partial P}{\partial f} = \frac{f}{500}, \qquad \text{for } 0 < f \leq 500 \quad \text{THz},$$

$$\frac{\partial P}{\partial f} = 2 - \frac{f}{500}, \qquad \text{for } 500 < f < 1000 \quad \text{THz},$$

$$\frac{\partial P}{\partial f} = 0, \qquad \text{for } f > 1000 \quad \text{THz}.$$

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a. – What is the total power density of the radiation?

$$P_T = \int_0^\infty \frac{\partial P}{\partial f} df = \int_0^{500} \frac{f}{500} df + \int_{500}^{1000} \left(2 - \frac{f}{500}\right) df$$

$$= \frac{1}{500} \left.\frac{f^2}{2}\right|_0^{500} + 2f\Big|_{500}^{1000} - \frac{1}{500} \left.\frac{f^2}{2}\right|_{500}^{1000}$$

$$= \frac{1}{500} \times \frac{1}{2}(250,000) + 2 \times (1000 - 500) - \frac{1}{500} \times \frac{1}{2}(10^6 - 25 \times 10)$$

$$= 250 + 1000 - 750 = 500 \quad \text{W m}^{-2}.$$

$$\boxed{\text{The total power density of the radiation is 500 W/m}^2.}$$

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b. What is the value of the bandgap of the photodiode material that results in the maximum theoretical efficiency of the photodiode exposed to the above radiation?

The efficiency of the photodiode is given by

$$\eta = \frac{f_g \int_{f_g}^\infty \frac{1}{f} \frac{\partial P}{\partial f} df}{P_T}$$

Solution of Problem 14.2

Since P_T is independent of the bandgap, W_g, it is sufficient to find the maximum of $f_g \int_{f_g}^{\infty} \frac{1}{f} \frac{\partial P}{\partial f} df$ when f_g changes.

$$f_g \int_{f_g}^{\infty} \frac{1}{f} \frac{\partial P}{\partial f} df = f_g \left[\int_{f_g}^{500} \frac{1}{f} \frac{\partial P}{\partial f} df + \int_{500}^{1000} \frac{1}{f} \frac{\partial P}{\partial f} df \right]$$
$$= f_g \left[\int_{f_g}^{500} \frac{1}{f} \frac{f}{500} df + \int_{500}^{1000} \frac{1}{f} \left(2 - \frac{f}{500} \right) df \right]$$
$$= fg \left[\frac{1}{500} f \Big|_{f_g}^{500} + 2 \ln f \Big|_{500}^{1000} - \frac{1}{500} f \Big|_{500}^{1000} \right]$$
$$= f_g \left[1 - \frac{f_g}{500} + 2 \ln \frac{1000}{500} - \frac{1000 - 500}{500} \right]$$
$$= fg \left[1 + 2 \ln 2 - 1 - \frac{f_g}{500} \right] = f_g \left[1.386 - \frac{f_g}{500} \right].$$

η_{max} occurs when

$$\frac{d}{df_g} \left[1.386 f_g - \frac{f_g^2}{500} \right] = 0.$$

$$1.386 - \frac{2 f_g}{500} = 0$$

or

$$f_g = 346.6 \quad \text{THz}.$$

The corresponding bandgap energy is

$$W_g = 346.6 \times 10^{12} \times 6.62 \times 10^{-34} = 2.3 \times 10^{-19} \quad \text{J} \;\; \text{or} \;\; 1.43 \;\; \text{eV}.$$

Maximum efficiency is achieved with a semiconductor having a bandgap of 1.43 eV.

Solution of Problem 14.2

Prob 14.3 Under circumstances in which there is substantial recombination of carriers in the transition region of a diode, the v-i characteristic becomes

$$I = I_\nu - I_R\left[\exp(\tfrac{qV}{2kT}) - 1\right] \\ -I_0\left[\exp(\tfrac{qV}{kT}) - 1\right].$$

In solving this problem use the more complicated equation above rather than the equation give in the text which is

$$I = I_\nu - I_0\left[\exp\left(\frac{qV}{kT}\right) - 1\right].$$

A silicon diode has 1 cm^2 of effective area. Its reverse saturation current, I_0, is 400 pA, and the current, I_R, is 4 μA. These values are for $T = 300$ K. Assume that this is the temperature at which the diode operates. Assume also 100% quantum efficiency, i.e., that each photon with energy above 1.1 eV produces one electron-hole pair. Finally, assume no series resistance.

At one sun, the power density of light is 1000 W/m^2, and this corresponds to a flux of 2.25×10^{21} photons s^{-1} m^{-2} (counting only photons with energy above 1.1 eV).

a. What is the open-circuit voltage of this diode at 1 sun?

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The current through the diode is

$$I = I_\nu - I_R\left[\exp\left(\frac{qV}{2kT}\right) - 1\right] - I_0\left[\exp\left(\frac{qV}{kT}\right) - 1\right].$$

To find the open-circuit voltage, V_{oc}, we will set the load current, i, to zero:

$$I_\nu - I_R\left[\exp\left(\frac{qV_{oc}}{2kT}\right) - 1\right] - I_0\left[\exp\left(\frac{qV_{oc}}{kT}\right) - 1\right] = 0.$$

We expect that this open circuit voltage will be a few hundred millivolts. If so,

$$\left[\exp\left(\frac{qV_{oc}}{2kT}\right) - 1\right] \approx \exp\left(\frac{qV_{oc}}{2kT}\right)$$

and

$$\left[\exp\left(\frac{qV_{oc}}{kT}\right) - 1\right] \approx \exp\left(\frac{qV_{oc}}{kT}\right)$$

because kT/q is equal to 26 mV at 300 K. This simplifies our equation to

$$I_\nu - I_R\left[\exp\left(\frac{qV_{oc}}{2kT}\right)\right] - I_0\left[\exp\left(\frac{qV_{oc}}{kT}\right)\right] = 0.$$

Solution of Problem 14.3

Let $y \equiv \exp(\frac{qV_{oc}}{2kT})$, then

$$-I_0 y^2 - I_R y + I_\nu = 0,$$

and

$$y = \frac{I_R \pm \sqrt{I_R^2 + 4I_0 I_\nu}}{-2I_0}$$

The photon flux was given as 2.25×10^{21} s^{-1}m^{-2}, hence

$$I_\nu = \phi q A$$

or

$$I_\nu = 2.25 \times 10^{21} \times 1.6 \times 10^{-19} \times 10^{-4} = 0.036 \quad \text{A}$$

Introducing all the pertinent data into the equation for y,

$$y = \frac{4 \times 10^{-6} \pm \sqrt{(4 \times 10^{-6})^2 + 4 \times 400 \times 10^{-12} \times (0.036)}}{-2 \times 400 \times 10^{-12}} = 5724.$$

$$V_{OC} = 2KT/q \ln\ y = 2 \times 0.026 \times \ln 5607 = 0.447 \quad \text{V}.$$

$\boxed{\text{The open-circuit voltage at 1 sun is 0.447 V.}}$

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b. At what voltage does the diode deliver maximum power to the load?

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As long as the voltage is much larger than kT/q, the power output $P = VI$ is

$$P = V\left[0.036 - 4 \times 10^{-6} \exp\frac{V}{0.0518} + 400 \times 10^{-12} \exp\frac{V}{0.0259}\right]. \qquad (1)$$

The easiest way to determine what value of V maximizes this power is to use EXCEL to tabulate P as a function of V and observe which value of the voltage leads to the largest P. The result is

$\boxed{\text{The voltage at maximum power is 0.355 V.}}$

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c. What is the maximum power the diode delivers?

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Introducing the voltage associated with maximum power into Equation 1, we find that the maximum power delivered to the load is 11.3×10^{-3} W.

$\boxed{\text{The maximum power delivered to the load is 11.3 mW.}}$

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d. What is the load resistance that draws maximum power from the diode?

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Solution of Problem 14.3

This means that the load current must be $11.35 \times 10^{-3}/0.356 = 0.032$ A, corresponding to a load resistance of $R_L = 0.356/0.032 = 11.3\ \ \Omega$.

$\boxed{\text{The load resistance that maximizes the load power is 11.3 ohms.}}$

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e. What is the efficiency of the diode?

The input power density is 1000 W/m^2 and the cell area is 10^{-4} m^2. Hence, the input power is 0.1 W. The cell efficiency is then

$$\eta = 0.0113/0.1 = 0.113 \quad \text{or} \quad 11.3\%.$$

$\boxed{\text{The efficiency of the diode is 11.3\%.}}$

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f. Now use a concentrator so that the diode will receive 1000 suns. This would cause the operating temperature to rise and would impair the efficiency. Assume, however, that an adequate cooling system is used so that the temperature remains at 300 K. Use 100% concentrator efficiency.

If we repeat the above calculations for 1000 suns, we can compare the performance of the cell at the two light levels:

#of suns	I_ν (A)	V_{OC} (V)	$V_{max\ pwr}$ (V)	$I_{max\ pwr}$ (A)	$P_{max\ pwr}$ (W)	R_L (Ω)	η
1	0.036	0.447	0.355	0.032	0.0113	11.3	11.3%
1000	36	0.655	0.573	34.3	19.6	0.017	19.6 %

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g. In fact, the concentrator is only 50% efficient. Is there still some advantage in using this diode with the concentrator?

If the collector is only 50% efficient, the photocell will be operating at 500 suns and, to find the power, we could repeat the procedure above. We know that the power will be substantially less than half that at 1000 suns because of the decreasing efficiency with lower light power densities. We can *estimate* the power by interpolating the efficiencies at 1000 suns and at 1 sun. We will get $\eta \approx 15\%$. The power would then be

$$P = \frac{19.6}{2}\,\frac{15}{19.6} = 7.5 \quad \text{W}.$$

We have to compare the cost of 1 cell with concentrator delivering 7.5 W with $7.5/0.011 = 682$ cells without concentrator. It all depends on the cost of the concentrator, the cooling system, and the more accurate tracking system necessary for the high power density solution.

Solution of Problem 14.3

Prob 14.4 Assume that you are dealing with perfect black-body radiation ($T = 6000$ K). We want to examine the theoretical limits of a photodiode. Assume no light losses by surface reflection and by parasitic absorption in the diode material. Consider silicon (bandgap energy, W_g, of 1.1 eV).

Clearly, photons with energy less than W_g will not interact with the diode because it is transparent to such radiation.

a. What percentage of the power of the black-body radiation is associated with photons of less than 1.1. eV?

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Equation 19 in Chapter 14 yields the fraction of the radiation that is transmitted through the semiconductor:

$$G_L = \frac{1}{P}\int_0^{f_g} \frac{\partial P}{\partial f} df$$

The spectral power distribution of a black body is (Equation 3, text):

$$\frac{\partial P}{\partial f} = A\frac{f^3}{e^{\frac{hf}{kT}} - 1}$$

where A is a constant.

The total power in the spectrum is (Equation 7)

$$P = A\left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15}.$$

From all this,

$$G_L = \left(\frac{h}{kT}\right)^4 \frac{15}{\pi^4}\int_0^{f_g} \frac{f^3}{e^{\frac{hf}{kT}} - 1} df = 630 \times 10^{-60}\int_0^{f_g} \frac{f^3}{e^{\frac{hf}{kT}} - 1} df$$

Let $f^* \equiv 10^{-12} f$; $f^3 = 10^{36} f^*$; $df = 10^{12} df^*$, and

$$G_L = 630 \times 10^{-12}\int_0^{f_g^*} \frac{f^*3}{e^{0.008 f^*} - 1} df^*$$

$$f_g = qW_g/h = 1.6 \times 10^{-19} \times 1.1/663 \times 10^{-36} = 266 \times 10^{12}$$

and

$$f_g^* = 266$$

Using numerical integration, $G_L = 0.206$

The percentage of the light power associated with photons with less energy than 1.1 eV is 20.6%

b. What would the photodiode efficiency be if all the energy of the remaining photons were converted to electric energy ?

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Solution of Problem 14.4

The absorbed photons represent 0.794 of the total energy. If this were the output of the cell, the efficiency would be 0.794. or

> If all the absorbed energy appeared in the load, the efficiency would be 79.4%.

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c. Would germanium ($W_g = 0.67$ eV) be more or less efficient?

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> Germanium, would absorb a larger fraction of the spectrum and would be correspondingly more efficient.

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d. Using Table 12.1 in the text, determine the percentage of the solar energy absorbed by silicon.

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From Table 12.1, one finds that solar photons with more than 250 THz contain 0.796 of the total energy, while those with more than 273 THz contain 0.757 of this energy. By interpolation, photons above 266 contain 0.769 or 76.9% of the solar energy. This differs somewhat from the black-body results.

> If real sun-light were used (instead of black-body radiation), silicon would absorb 76.8% rather than 79.4% of the radiation.

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e. A photon with 1.1 eV will just have enough energy to produce one electron-hole pair, and under ideal conditions, the resulting electron would be delivered to the load under 1.1 V of potential difference. On the other hand, a photon of, say, 2 eV, will create pairs with 0.9 eV excess energy. This excess will be in the form of kinetic energy and will rapidly be thermalized, and, again, only 1.1 eV will be available to the load. Thus, all photons with more than W_g will, at best, contribute only W_g units of energy to the load.

Calculate what fraction of the black-body radiation is available to a load connected to an ideal silicon photodiode.

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The fraction of the black-body radiation available to a load is given by the efficiency formula from the text :

$$\eta = 8.17 \times 10^{-43} \frac{f_g}{T^4} \int_{f_g}^{\infty} \frac{f^2}{e^{\frac{hf}{kT}} - 1} df.$$

Solution of Problem 14.4

If all frequencies are expressed in THz,

$$\eta = 8.17 \times 10^5 \frac{f_g}{T^4} \int_{f_g}^{\infty} \frac{f^2}{e^{\frac{hf}{kT}} - 1} df = 0.438.$$

$$\boxed{\text{The load sees 43.8\% of the total radiation.}}$$

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f. The short circuit current of a diode under a certain illumination level is 10^7 times the diode reverse saturation current. What is the relative efficiency of this diode compared with that of the ideal diode considered above?

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The power output of a photodiode is maximum when its output voltage, V_m, satisfies

$$\left(1 + \frac{qV_m}{kT}\right) \exp\left(\frac{qV_m}{kT}\right) = \frac{J_\nu}{J_0} + 1$$

For $I_\nu / I_0 = 10^7$, the numerical solution of this equation is $V_m = 0.35$ V. The load current is then very nearly the short-circuit current, $I_\nu = q\phi_g S$, so that the power output of the diode is $P_U = V_m q \phi_g S$ where S is the area of the cell. Using Equation 19,

$$P_U = V_m q S \frac{1}{h} \int_{f_g}^{\infty} A \frac{f^2}{e^{\frac{hf}{kT}} - 1} df.$$

The total power in the spectrum is

$$P = SA \left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15} = 1.58 \times 10^{57} SA$$

The efficiency of the cell is

$$\eta = \frac{P_u}{P} = \frac{V_m q \frac{1}{h} \int_{f_g}^{\infty} \frac{f^2}{e^{\frac{hf}{kT}} - 1} df}{1.58 \times 10^{57}} = 53.6 \times 10^{-9} \int_{f_g}^{\infty} \frac{f^2}{e^{\frac{hf}{kT}} - 1} df = 0.14.$$

Compared with the ideal cell, the real one has only 0.14/0.438 or 32% of the ideal efficiency.

Perhaps a simpler way of reaching this same result is to realize that the ideal cell delivers 1.1 V (the bandgap voltage) to the load, whereas the real cell delivers only 0.35 V. Hence, their relative efficiency is 0.35/1.1=32%.

Solution of Problem 14.4

Prob 14.5 Consider radiation with the normalized spectral power density distribution given by:

$\frac{\partial P}{\partial f} = 0$ **for** $f < f_1$ **and** $f > f_2$**,**

$\frac{\partial P}{\partial f} = 1$ **for** $f_1 < f < f_2$

where $f_1 = 100$ **THz and** $f_2 = 1000$ **THz.**

a. What is the theoretical efficiency of a photodiode having a bandgap energy of $W_g = hf_1$**?**

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The power density of the radiation is

$$P = \alpha \int_{f_1}^{f_2} \frac{\partial P}{\partial f} = \alpha \int_{f_1}^{f_2} df = \alpha(f_2 - f_1)$$

where α is a constant.

The efficiency of the cell is

$$\eta = \frac{f_g}{P}\alpha \int_{f_1}^{f_2} \frac{1}{f} \frac{\partial P}{\partial f} df = \frac{f_g}{f_2 - f_1} \int_{f_1}^{f_2} \frac{df}{f} = \frac{f_g}{f_2 - f_1} \ln \frac{f_2}{f_g}.$$

For $f_g = f_1$,

$$\eta = \frac{f_1}{f_2 - f_1} \ln \frac{f_2}{f_1} = \frac{1}{10 - 1} \ln \frac{10}{1} = 0.256.$$

$$\boxed{\text{The efficiency of the photocell is 25.6\%.}}$$

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b. What bandgap energy maximizes the efficiency of the diode?

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$$\frac{d\eta}{df_g} = \frac{1}{f_2 - f_1} \ln \frac{f_g}{f_g} - \frac{f_g}{f_2 - f_1} \frac{1}{f_g} = \frac{1}{f_2 - f_1} \left(\ln \frac{f_2}{f_g} - 1 \right) = 0,$$

$$\ln \frac{f_2}{f_g} = 1,$$

$$f_2 = e\, f_g \quad \text{or} \quad f_g = e^{-1} f_2.$$

$$f_g = \frac{1000}{e} = 368 \quad \text{THz}.$$

This corresponds to 2.44×10^{-19} J or 1.52 eV.

$$\boxed{\text{The bandgap energy that maximizes the efficiency is 1.52 eV.}}$$

Solution of Problem 14.5

c. If the bandgap energy is $h \times 500$ THz, and, if the material is totally transparent to radiation with photons of less energy than W_g, what fraction of the total radiation power goes through the diode and is available on its back side?

$$G_L = \frac{1}{P}\alpha \int_0^{f_g} \frac{\partial P}{\partial f} df = \frac{1}{f_2 - f_1} \int_{f_1}^{f_g} df = \frac{f_g - f_1}{f_2 - f_1},$$

$$G_L = \frac{500 - 100}{1000 - 100} = \frac{4}{9} = 0.444$$

$$\boxed{\text{44.4\% of all the radiation goes through the diode material.}}$$

d. If behind the first diode, one mounts a second one with $W_g = h \times 100$ THz, what is the efficiency of the two cascaded diodes taken together?

The power delivered by the first cell is

$$P_1 = \eta P = \frac{f_{g_1}}{f_2 - f_1} \ln\left(\frac{f_2}{f_{g_1}}\right) P = \frac{500}{1000 - 100} \ln\left(\frac{1000}{500}\right) P = 0.385P.$$

The power received by the second cell is

$$P_{in,\ 2} = G_L P = 0.444P.$$

The power delivered by the second cell is

$$\begin{aligned} P_2 = \eta P_{in,\ 2} &= \frac{f_{g_2}}{f_{g_2} - f_{g_1}} \ln\left(\frac{f_{g_1}}{f_{g_2}}\right) P \\ &= \frac{100}{500 - 100} \ln\left(\frac{500}{100}\right) \times 0.444P = 0.179P. \end{aligned}$$

The total power delivered by the cascaded cells is

$$P_T = P_1 + P_2 = 0.385P + 0.179P = 0.654P$$

$$\eta = \frac{P_T}{P} = 0.564$$

$$\boxed{\text{The overall efficiency of the two cascaded cells is 56.4\%.}}$$

Solution of Problem 14.5

Prob 14.6 Two photodiodes, each with an effective area of 10 cm^2, are exposed to bichromatic radiation having power densities of 500 W/m^2, in narrow bands one around 430 THz and the other, around 600 THz.

One diode has a bandgap energy of 1 eV, the other has 2 eV. When diode is reverse biased (in the dark), the saturation current is 10 nA.

The diodes operate at 300 K.

a. What are the short-circuit photo currents?

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The first step in the solution of this problem is to determine the photon fluxes, ϕ :

$$P = \phi h f,$$

where P is the light power density, h is Planck's constant, and , f is the frequency. Consequently

$$\phi = \frac{P}{hf}.$$

At 430 THz: $hf = 6.62 \times 10^{-34} \times 430 \times 10^{12} = 285 \times 10^{-21}$ J,

$$\phi_{430} = \frac{500}{285 \times 10^{-21}} = 1.75 \times 10^{21} \quad \text{photons m}^{-2}\text{s}^{-1}.$$

At 600 THz: $hf = 6.62 \times 10^{-34} \times 600 \times 10^{12} = 398 \times 10^{-21}$ J,

$$\phi_{600} = \frac{500}{398 \times 10^{-21}} = 1.26 \times 10^{21} \quad \text{photons m}^{-2}\text{s}^{-1}.$$

At unity quantum efficiency, each photon creates 1 electron-hole pair, provided its energy is more than that of the bandgap, W_g.

The bandgaps of the two diodes are, respectively,

$$W_{g_1} = 1 \text{ eV } \times 1.6 \times 10^{-19} \ \frac{\text{J}}{\text{eV}} = 1.6 \times 10^{-19} \quad \text{J},$$

$$W_{g_2} = 2 \text{ eV } \times 1.6 \times 10^{-19} \ \frac{\text{J}}{\text{eV}} = 3.2 \times 10^{-19} \quad \text{J},$$

Both ϕ_{430} and ϕ_{600} are larger than W_{g_1}, hence Material 1 absorbs both frequencies.

Only ϕ_{600} is larger than W_{g_2}, hence Material 2 absorbs only the higher frequency.

The short circuit current of a photodiode is given by

$$I_\nu = qA\phi_T$$

where ϕ_T is the total flux of photons absorbed. For diodes with 10 cm^2 area (10^{-3} m^2),

$$I_\nu = 1.60 \times 10^{-19} \times 10^{-3} \times \phi_T = 1.60 \times 10^{-22}\phi_T.$$

Solution of Problem 14.6

For Diode 1,

$$I_{\nu_1} = 1.6 \times 10^{-22} \times (1.75 + 1.26) \times 10^{21} = 0.482 \quad \text{A},$$

For Diode 2,

$$I_{\nu_2} = 1.6 \times 10^{-22} \times 1.26 \times 10^{21} = 0.202 \quad \text{A},$$

The short-circuit photocurrents are 0.482 and 0.202 A for, respectively, Diode 1 and Diode 2.

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b. What is the open-circuit voltage of each diode?

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The open circuit voltage is given by

$$V_{oc} = \frac{kT}{q} \ln\left(\frac{I_\nu}{I_0}\right).$$

For the diodes under consideration, $I_0 = 10^{-8}$ A, $T = 300$ K, and $kT/q = 0.026$ V,

$$V_{oc} = 0.026 \ln\left(\frac{I_\nu}{10^{-8}}\right).$$

$$V_{oc_1} = 0.026 \ln \frac{0.482}{10^{-8}} = 0.460 \quad \text{V}.$$

$$V_{oc_2} = 0.026 \ln \frac{0.202}{10^{-8}} = 0.437 \quad \text{V}.$$

The open-circuit voltages are 0.460 and 0.437 V for, respectively, Diode 1 and Diode 2.

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c. What is the maximum theoretical efficiency of each diode?

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The maximum theoretical power output from a diode is

$$P_{max\ (th)} = A\phi_T W_g \quad \text{W},$$

while the input power is

$$P_{in} = (500 + 500)A \quad \text{W}.$$

Consequently the efficiency is

$$\eta = \frac{\phi_T W_g}{1000},$$

$$\eta_1 = \frac{(1.75 + 1.26) \times 10^{21} \times 1.6 \times 10^{-19}}{1000} = 0.482,$$

$$\eta_2 = \frac{1.26 \times 10^{21} \times 3.2 \times 10^{-19}}{1000} = 0.403.$$

Solution of Problem 14.6

The efficiencies are are 48.2% and 40.3% for, respectively, Diode 1 and Diode 2.

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d. What is the maximum power each diode can deliver to a load (assume no series resistance in the diodes)?

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The voltage at maximum power output is V_m :

$$\left(1+\frac{qV_m}{kT}\right)\exp\left(\frac{qV_m}{kT}\right)=\frac{I_\nu}{I_0},$$

$$\left(1+\frac{V_m}{0.026}\right)\exp\left(\frac{V_m}{0.026}\right)=\frac{I_\nu}{10^{-8}}. \tag{1}$$

The two previously calculated values for I_ν lead to

$$\frac{I_\nu}{10^{-8}}=\begin{cases}4.82\times10^7\\2.02\times10^7\end{cases}$$

Numerical solution of Equation 1 leads to

$$V_m=\begin{cases}0.388 & \text{V},\\0.367 & \text{V}.\end{cases}$$

The current at the maximum power point is

$$I_m=I_\nu-I_0\left(\mathrm{e}^{qV_m/kT}-1\right)=I_\nu-10^{-8}\left(\mathrm{e}^{V_m/0.026}-1\right),$$

$$I_{m_1}=0.482-10^{-8}\left(\mathrm{e}^{0.388/0.026}-1\right)=0.452 \quad \text{A},$$

$$I_{m_2}=0.202-10^{-8}\left(\mathrm{e}^{0.367/0.026}-1\right)=0.189 \quad \text{A}.$$

$$P_{OUT_1}=0.452\times0.388=0.175 \quad \text{W},$$

$$P_{OUT_2}=0.189\times0.367=0.069 \quad \text{W}.$$

The output powers are are 0.175 W and 0.069 W for, respectively, Diode 1 and Diode 2.

Solution of Problem 14.6

Prob 14.7 An ideal photodiode is made of a material with a bandgap energy of 2.35 eV. It operates at 300 K and is illuminated by monochromatic light with wavelength of 400 nm. What is its maximum efficiency?

....

The frequency of a photon whose energy is W_g eV is

$$f_g = qW_g/h = 2.35 \times 1.6 \times 10^{-19}/662 \times 10^{-36} = 568 \times 10^{12} \quad \text{Hz.}$$

The frequency that corresponds to light with 400 nm wavelength is

$$f = c/\lambda = 3 \times 10^8/400 \times 10^{-9} = 750 \times 10^{12} \quad \text{Hz.}$$

Hence $f > f_g$ and the photodiode will work.

Although each photon has an energy hf it will produce only hf_g units of electric energy. Hence, assuming 100% quantum efficiency,

$$\eta = \frac{f_g}{f} = \frac{568}{750} = 0.757.$$

$$\boxed{\text{The } \textit{theoretical} \text{ efficiency is } 75.7\%.}$$

Observe that high efficiencies can be achieved with monochromatic light. The closer the light frequency is to the frequency corresponding to the bandgap energy, the higher the efficiency.

Solution of Problem 14.7

Prob 14.8 What is the short-circuit current delivered by a 10 cm by 10 cm photocell (with 100% quantum efficiency) illuminated by monochromatic light of 400 nm wavelength with a power density of 1000 W/m^2.

....

The frequency that corresponds to light with 400 nm wavelength is

$$f = c/\lambda = 3 \times 10^8/400 \times 10^{-9} = 750 \times 10^{12} \quad \text{Hz.}$$

The corresponding photon energy is

$$W_{ph} = hf = 6.62 \times 10^{-34} \times 750 \times 10^{12} = 497 \times 10^{-21} \quad \text{J.}$$

The light power density, P, is the product of the photon flux (# of photons per second per m^2) times the energy, W_{ph}, of each photon: $P = \phi W_{ph}$, hence

$$\phi = \frac{P}{W_{ph}} = \frac{1000}{497 \times 10^{-21}} = 2.0 \times 10^{21} \quad \text{photons m}^{-2}\text{s}^{-1}.$$

Each photon liberates 1 electron (100% quantum efficiency), thus 2.0×10^{21} electrons m^{-2}s^{-1} are circulated. Since the area is 10^{-2} m^2, the current is

$$I = 2.0 \times 10^{21} \times 10^{-2} \times 1.6 \times 10^{-19} = 3.22 \quad \text{A.}$$

$$\boxed{\text{The short-circuit current delivered by the photocell is 3.2 A.}}$$

Solution of Problem 14.8

Prob 14.9 Under different illumination, the cell of Problem 14.8 delivers 5 A into a short circuit. The reverse saturation current is 100 pA. Disregard any internal resistance of the photodiode. What is the open-circuit voltage at 300 K?

....

$I_0 = 10^{-10}$ A. Since there is neither internal resistance of the photodiode nor load resistance, $I_\nu = 5$ A. The open-circuit voltage is

$$V_{oc} = \frac{kT}{q} \ln \frac{I_\nu}{I_0} = 0.026 \ \ln \frac{5}{10^{-10}} = 0.64 \quad \text{V}.$$

$$\boxed{\text{The open-circuit voltage is 0.64 V.}}$$

Solution of Problem 14.9

Prob 14.10 The optical system of a solar photovoltaic system consists of a circular f:1.2† lens with a focal length, F, of 3 m.

a. When aimed directly at the sun (from the surface of Planet Earth), what is the diameter, D_i, of the solar image formed in the focal plane?

....

As seen from earth, the angular radius of the sun is 0.25° . Then

$$\frac{R_i}{F} = \tan 0.25°,$$

where R_i is the radius of the solar image and F is the focal length of the lens.

$$D_i = 2R_i = 2 \times 3 \tan 0.25° = 0.026 \quad \text{m}.$$

$$\boxed{\text{The diameter of the solar image is 2.6 cm.}}$$

---

b. What is the concentration ratio, C?

....

C is the ratio of the area of the lens (aperture) to the area of the image. Since the areas are proportional to the square of the diameters,

$$C = \left(\frac{D_a}{D_i}\right)^2.$$

Here, D_a, is the aperture diameter and is

$$D_a = \frac{F}{f} = \frac{3}{1.2} = 2.5 \text{ m}.$$

$$C = \left(\frac{2.5}{0.026}\right)^2 = 9250.$$

$$\boxed{\text{The geometrical concentration ratio is 9250.}}$$

Assume a 90% efficiency of the optical system and a perfectly clear atmosphere at noon.

c. What is the total light power, P, that falls on a photovoltaic cell exposed to the solar image?

....

† The f-number is, as in any photographic camera, the ratio of the focal length to the diameter of the lens.

Solution of Problem 14.10

The area of the lens is

$$A_L = \pi \frac{D_a^2}{4} = 4.90 \quad \text{m}^2.$$

The area of the image is

$$A_i = \pi \frac{D_i^2}{4} = 0.00053 \quad \text{m}^2.$$

The collected solar power is

$$P_{collected} = 1000 A_L = 4900 \quad \text{W}.$$

Of this, 90% reaches the image:

$$P_{image} = 0.9 \times 4900 = 4410 \quad \text{W}.$$

$$\boxed{\text{The total light power that falls on the solar image is 4410 W.}}$$

---

d. What is the power density, p?

....

The power density at the image is

$$p = \frac{P}{A_i} = \frac{4410}{0.00053} = 8.3 \times 10^6 \quad \text{W/m}^2.$$

$$\boxed{\text{The light power density at the image is } 8.3 \text{ MW/m}^2.}$$

Assume:

no conduction losses;
no convection losses;
the silicon photovoltaic cell is circular and has a diameter equal to that of the solar image. The cell intercepts all the light of the image;
the efficiency of the photocell is equal to 60% of the maximum theoretical value for a black body radiator at 5800 K.
all the power the photocell generates is delivered to an external load;
the effective heat emissivity, ϵ, is 0.4.

e. What is the power delivered to the load?

....

Solution of Problem 14.10

The maximum efficiency of a silicon photovoltaic cell exposed to 5800 K blackbody radiation is 0.43. With 60% of maximum efficiency, the cell delivers to the load

$$P_{load} = 0.6 \times 0.43 \times 4410 = 1138 \quad \text{W}.$$

$$\boxed{\text{The power delivered to the load is 1140 W.}}$$

---

f. What is the temperature of the photocell?

....

Since the light power incident on the cell is 4410 W and 1138 W are delivered to the load, a total of 3272 W have to be rejected through radiation (because the problem specifies that there is no conduction nor convection loss of heat).

$$P_{radiated} = \sigma \epsilon A_i T^4.$$

σ is the Stefan-Boltzmann constant and is equal to 5.67×10^{-8} MKS units.

$$T = \left(\frac{P_{radiated}}{\sigma \epsilon A_i} \right)^{1/4} = \left(\frac{3272}{5.67 \times 10^{-8} \times 0.4 \times 0.00053} \right)^{1/4} = 4092 \quad \text{K}$$

To cool the diode by radiation only, it would have to reach the absurd temperature of 4092 K.

---

g. What is the temperature of the photocell if the load is disconnected? If you do your calculations correctly, you will find that the concentrated sunlight will drive the cell to intolerably high temperatures. Silicon cells should operate at temperatures of 500 K or less.

....

Of course, when the load is disconnected all the 4410 W must be dissipate and, by radiation only, this would require an even higher temperature of 4376 K.

These temperatures are high enough to melt (perhaps, vaporize) the silicon. So there is no hope of cooling the diode by radiation only. An additional cooling system must be installed.

Solution of Problem 14.10

h. How much heat must be removed by a coolant to keep the cell at 500 K when no electric power is being extracted?

....

At 500 K, the cell will radiate

$$P_{radiated} = 5.67 \times 10^{-8} \times 0.4 \times 0.00053 \times 500^4 = 0.8 \quad \text{W}.$$

Clearly, radiation losses are negligible. All the heat has to be removed by the coolant.

At no load, the coolant has to remove 4410 W.
At full load, the coolant has to remove 3272 W.

The coolant will exit the cell at 480 K and drives a steam engine that rejects the heat at 80 C and that realizes 60% of the Carnot efficiency.

i. How much power does the heat engine deliver?

....

The Carnot efficiency of an engine operating between 480 K and $80 + 273 =$ 353 K is

$$\eta_{CARNOT} = \frac{480 - 353}{480} = 0.265$$

The engine efficiency will be $0.265 \times 0.60 = 0.16$, and its output power will be $0.16 \times 3272 = 519$ W.

The heat engine will deliver 520 W.

---

j. What is the overall efficiency of the photocell cum steam engine system?

....

The overall efficiency is now

$$\eta = \frac{1138 + 519}{4900} = 0.338.$$

The overall efficiency of the photocell plus heat engine is 33.8%.

Solution of Problem 14.10

Prob 14.11 *Treat the photo-diode of this problem as an ideal structure. Assume 100% quantum efficiency.*

A photo-diode has an area of 1 by 1 cm and is illuminated by monochromatic light with a wavelength of 780 nm and with a power density of 1000 W/m^2. At 300 K, the open circuit voltage is 0.683 V.

a. What is its reverse saturation current, I$_0$?

....

$$f = \frac{c}{\lambda} = \frac{3\times 10^8}{780\times 10^{-9}} = 385\times 10^{12} \quad \text{Hz} \quad \text{or} \quad 385 \quad \text{THz},$$

$$P = 1000\frac{\text{W}}{\text{m}^2}\times 0.01^2 \quad \text{m}^2 \quad = \quad 0.1 \quad \text{W}.$$

At 300 K, $kT/q = 0.0258$ V.

$$V_{oc} = \frac{kT}{q}\ln\frac{I_\nu}{I_0},$$

$$\frac{I_\nu}{I_0} = \exp\frac{qV_{oc}}{kT} = \exp\frac{0.683}{0.0258} = 314\times 10^9.$$

The photon flux is

$$\phi = \frac{P}{hf} = \frac{0.1}{6.62\times 10^{-34}\times 385\times 10^{12}} = 392\times 10^{15} \quad \text{photons s}^{-1}\text{cm}^{-2}.$$

With 100% quantum efficiency, each photon causes 1 free electron to appear:

$$I_\nu = q\phi = 1.6\times 10^{-19}\times 392\times 10^{15} = 0.0268 \quad \text{A},$$

$$I_0 = \frac{0.0628}{314\times 10^9} = 0.2\times 10^{-12} \quad \text{A}.$$

$$\boxed{\text{The reverse saturation current is 0.2 pA.}}$$

---

b. What is the load resistance that allows maximum power transfer?

....

The load voltage, V_m, that corresponds to the maximum output can be found from

$$\left(1+\frac{qV_m}{kT}\right)\exp\frac{qV_m}{kT} = \frac{I_\nu}{I_0}+1 \approx \frac{I_\nu}{I_0}.$$

Solution of Problem 14.11

Define

$$U \equiv \frac{qV_m}{kT},$$

$$M \equiv \frac{I_\nu}{I_0} = 314 \times 10^9.$$

Then,

$$(1+U)\exp U = 314 \times 10^9.$$

The above equation must be solved numerically. The result is $V_m \approx$ 0.601 V.

The relationship between the load voltage and the load current is

$$V_L = V_m = \frac{kT}{q}\ln\left(\frac{I_\nu - I}{I_0}\right),$$

$$I_m = I = I_\nu - I_0 \exp\frac{qV_m}{kT} = 0.0628 - 0.2 \times 10^{-12} \exp\frac{0.601}{0.0259}$$
$$= 0.0628 - 2.6 \times 10^{-3} = 60.4 \times 10^{-3} \quad \text{A},$$

$$R_L = \frac{V_m}{I_m} = \frac{0.601}{0.0604} = 10\ \Omega$$

$$\boxed{\text{The load resistance must be 10 ohms.}}$$

---

c. What is the efficiency of this cell with the load above?

....

$$P_L = V_m I_m = 0.601 \times 0.0602 = 36 \times 10^{-3} \quad \text{W},$$

$$\eta = \frac{P_L}{P} = \frac{36 \times 10^{-3}}{0.1} = 36 \times 10^{-2}.$$

$$\boxed{\text{The cell has an efficiency of 36\%.}}$$

Solution of Problem 14.11

Prob 14.12 The power density of monochromatic laser light (586 nm) is to be monitored by a 1×1 mm silicon photo-diode. The quantity observed is the short-circuit current generated by the silicon. Treat the diode as a perfect ideal device.

a. What current do you expect if the light level is 230 W/m^2?

The frequency of the laser light is

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{586 \times 10^{-9}} = 512 \quad \text{THz}.$$

The short-circuit current is

$$i_\nu = qA\phi.$$

Here ϕ is the total flux of photons. The power density is

$$P = hf\phi,$$

$$\phi = \frac{P}{hf} = \frac{230}{6.62 \times 10^{-34} \times 512 \times 10^{12}} .$$
$$= 1.6 \times 10^{-19} \times 0.001^2 \times 678 \times 10^{18} = 108 \times 10^{-6} \quad \text{A}$$

$\boxed{\text{The short-circuit current is 108 microamperes.}}$

---

b. Explain how the temperature of the semiconductor affects this current. Of course, the temperature has to be lower than that which will destroy the device (some 150 C, for silicon).

The temperature of the semiconductor affects the exact value of f_g. However since the frequency, f, of the monochromatic radiation is much larger than f_g, a small variation of the latter has absolutely no influence on i_ν.

$\boxed{\text{The temperature of the semiconductor does not affect the value of the short-circuit current.}}$

---

c. Instead of being shorted out, the diode is now connect to a load compatible with maximum electric output. Estimate the load voltage.

Solution of Problem 14.12

The efficiency of the photo-diode is

$$\eta = \frac{P_U}{P},$$

where $P_U = hf_g\phi$ is the useful electric output power of an ideal diode and $P = hf\phi$ is the total power of the incident radiation, all per square meter. Hence,

$$\eta = \frac{hf_g\phi}{hf\phi} = \frac{f_g}{f}.$$

The electric power delivered to the load is

$$P_U \times A = V_L I_L.$$

An inspection of the v-i characteristics of photo-diodes reveals that the current that delivers maximum power to a load is nearly the same as the short-circuit current, i_ν, i.e., $I_L \approx i_\nu$, in this case.

$$V_L = \frac{P_U \times A}{i_\nu} = \frac{\eta P A}{q A \phi} = \frac{\frac{f}{f_g} P}{\frac{q}{hf} P} = \frac{h}{q} f_g = 4.12 \times 10^{-15} f_g = 1.09 \quad \text{V}.$$

Notice that, to a first order, the optimum load voltage does not depend on the light power density. As the illumination goes down, the power goes down because the current drops proportionally while the voltage remains constant.

A rough estimate of the optimum load voltage can be reached by realizing that it should be very near the bandgap voltage that is 1.1 V, for silicon.

$$\boxed{\text{The load voltage that maximizes the power output is 1.09 V.}}$$

Solution of Problem 14.12

Prob 14.13 A silicon photocell being tested measures 4 by 4 cm. Throughout the tests the temperature of the device is kept at 300 K. Assume the cell has no significant series resistance. Assume 100% quantum efficiency. The bandgap energy of silicon is 1.1 eV.

Initially, the cell is kept in the dark. When a current of 100 μA is forced through it in the direction of good conduction, the voltage across the diode is 0.466 V.

Estimate the open circuit voltage developed by the cell when exposed to bichromatic infrared radiation of 412 nm and 1300 nm wavelength. The power density at the shorter wavelength is 87 W/m^2 while, at the longer, it is 93 W/m^2.

....

We note that the open-circuit voltage is

$$V_{oc} \approx \frac{kT}{q} \ln \frac{i_\nu}{I_0}.$$

As a first step in the solution of this problem let us determine the reverse saturation current, I_0, The current through a diode is

$$i = I_0 \left[\exp\left(\frac{qV}{kT}\right) - 1 \right].$$

At 300 K, $kT/q = 0.026$ V and, for $V = 0.466$, the exponential term is 61×10^6. Thus the "1" in the formula above is irrelevant.

$$I_0 = \frac{i}{61 \times 10^6} = 1.64 \quad \text{pA}.$$

Next, we must estimate the short-circuit photocurrent, i_ν.

$$i_\nu = qA\phi,$$

where A is the active area of the photocell (16×10^{-4} m^2) and ϕ is the total flux of photons with energy larger than the bandgap energy.

The bandgap energy of silicon is 1.1 eV or 176×10^{-21} joules. This corresponds to a photon frequency of

$$f = \frac{W}{h} = \frac{176 \times 10^{-21}}{6.63 \times 10^{-34}} = 267 \times 10^{12} \quad \text{Hz},$$

or a wavelength of 1120 nm.

Thus, the component of the incident illumination at 1300 nm does not interact with the silicon (the material is, essentially transparent to this

Solution of Problem 14.13

radiation). However, the shorter wavelength (412 nm or 728 THz) will create electron-hole pairs in the diode.

The power density of the active radiation is

$$hf\phi = 87 \quad \text{W m}^{-2},$$

$$\phi = \frac{87}{6.63 \times 10^{-34} \times 728 \times 10^{12}} = 1.81 \times 10^{20} \quad \text{photons s}^{-1}\text{m}^{-2},$$

$$i_\nu = 1.60 \times 10^{-19} \times 16 \times 10^{-4} \times 1.81 \times 10^{20} = 0.046 \quad \text{A}.$$

$$V_{oc} = 0.026 \ln \frac{0.046}{1.64 \times 10^{-12}} = 0.626 \quad \text{V}.$$

$$\boxed{\text{The open-circuit voltage of the photocell is 0.626 V.}}$$

Solution of Problem 14.13

Prob 14.14

a. What is the ideal efficiency of a photocell made from a semiconducting material with a bandgap energy, $W_g = 2$ eV, when illuminated by radiation with the normalized spectral power distribution given below:

f	$\partial P/\partial f$
<200 THz	0
200 to 300 THz	$0.01f-2$
300 to 400 THz	$4-0.01f$
>400 THz	0

....

The total normalized power density is

$$P = \int_{200}^{300} (0.01f - 2)df + \int_{300}^{(400)} (4 - 0.01f)df)$$
$$= 0.01 \int_{200}^{300} f df - 2 \int_{200}^{300} df + 4 \int_{300}^{400} df - 0.01 \int_{300}^{400} f df$$
$$= 0.01 \times \tfrac{1}{2} f^2 \Big|_{200}^{300} - 2(300 - 200) + 4(400 - 300) - 0.01 \times \tfrac{1}{2} f^2 \Big|_{300}^{400}$$
$$= 100 \quad \text{W/m}^2.$$

The bandgap energy can be expressed in terms of frequency,

$$f_g = \frac{W_g}{h} \quad \text{or} \quad f_g = \frac{qW_g}{h},$$

where in the first formula, W_g is in joules and, in the second, in eV.

A bandgap energy of 2 eV corresponds to 482 THz. Hence the semiconductor is transparent to all the radiation of interest and the efficiency of the photocell is zero.

$$\boxed{\text{The efficiency of the photocell is 0\%.}}$$

---

b. Repeat for a semiconductor with $W_g = 1$ eV.

....

A bandgap energy of 2 eV corresponds to 241 THz. The efficiency is

$$\eta = f_g \frac{\int_{f_g}^{400} \frac{1}{f}\frac{\partial P}{\partial f} df}{P} = 241 \frac{\int_{241}^{300} \frac{1}{f}(0.01f - 2)df + \int_{300}^{400} \frac{1}{f}(4 - 0.01f)df}{100}$$
$$= 2.41 \left(0.01 \int_{241}^{300} df - 2 \int_{241}^{300} \frac{df}{f} + 4 \int_{300}^{400} \frac{df}{f} - 0.01 \int_{300}^{400} df\right)$$
$$= 2.41 \left[0.01(300 - 241) - 2\frac{300}{241} + 4\frac{400}{300} - 0.01(400 - 300)\right]$$
$$= 0.73.$$

$$\boxed{\text{The efficiency of the photocell is 73\%.}}$$

Solution of Problem 14.14

Prob 14.15 What is the theoretical efficiency of a photocell with a 2.5 V bandgap when exposed to 100 W/m^2 solar radiation through a filter having the following transmittance characteristics:

Pass without attenuation all wavelengths between 600 and 1000 nm. Reject all else.

....

Photons in the band-pass range have energies in the range

$$W_{g_1} = \frac{hf}{q} = \frac{hc}{\lambda q} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1000 \times 10^{-9} \times 1.60 \times 10^{-19}} = 1.24 \quad \text{eV}$$

to

$$W_{g_2} = \frac{hf}{q} = \frac{hc}{\lambda q} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9} \times 1.60 \times 10^{-19}} = 2.07 \quad \text{eV}$$

All these photons have insufficient energy to interact with a material having a 2.5 eV bandgap.

$$\boxed{\text{The efficiency is } 0\%.}$$

Solution of Problem 14.15

Prob 14.16 A photodiode is exposed to radiation of uniform spectral power density ($\frac{\partial P}{\partial f}$ = constant) covering the range from 300 to 500 THz. Outside this range there is no radiation. The total power density is 2000 W/m^2.

a. Assuming 100% quantum efficiency, what is the short-circuit photo current of a diode having an active area of 1 by 1 cm?

The total power density is

$$P = \int_{300}^{500} \frac{\partial P}{\partial f} df = \frac{\partial P}{\partial f}(500 - 300) \times 10^{12} = \frac{\partial P}{\partial f} \times 200 \times 10^{12}$$

where the limits are in THz. Consequently,

$$\frac{\partial P}{\partial f} = \frac{2000}{200 \times 10^{12}} = 10 \times 10^{-12} \qquad \text{W m}^{-2}\text{s}^{-1}.$$

The total photon flux is

$$\begin{aligned} \Phi &= \frac{1}{h} \int_{300}^{500} \frac{1}{f} \frac{\partial P}{\partial f} df \\ &= \frac{10 \times 10^{-12}}{6.63 \times 10^{-34}} \ln \frac{500}{300} = 7.7 \times 10^{21} \qquad \text{photons m}^{-2}\text{s}^{-1}. \end{aligned}$$

The short-circuit current is

$$I_\nu = qA\Phi = 1.60 \times 10^{-19} \times 10^{-4} \times 7.7 \times 10^{21} = 0.124 \qquad \text{A}$$

$$\boxed{\text{The short-circuit current is 124 mA.}}$$

---

b. When exposed to the radiation in Part a of this problem, the open-circuit voltage delivered by the diode is 0.498 V. A 1.0 V voltage is applied to the diode (which is now in darkness) in the reverse conduction direction (i.e., in the direction in which it acts almost as an open circuit). What current circulates through the device. The temperature of the diode is 300 K.

The open-circuit voltage is given by

$$V_{oc} = \frac{kT}{q} \ln \frac{I_\nu}{I_0},$$

$$I_0 = I_\nu \exp\left(-\frac{qV_{oc}}{kT}\right) = 0.124 \exp\left(-\frac{0.498}{0.026}\right) \approx 600 \times 10^{-12} \qquad \text{A.}$$

$$\boxed{\text{The reverse saturation current is about 600 pA.}}$$

Solution of Problem 14.16

Prob 14.17 The sun radiates (roughly) like a 6000 K black body. When the power density of such radiation is 1000 W/m^2, i.e., "one sun", the total photon flux is 4.46×10^{21} photons per m^2 per second. Almost exactly half of these photons have energy equal or larger than 1.1 eV (the bandgap energy, W_g, of silicon).

Consider a small silicon photodiode with a 10 by 10 cm area. When 2 V of reversed bias is applied, the resulting current is 30 nA. This is, of course, the reverse saturation current, I_0.

When the photodiode is short-circuited and exposed to black body radiation with a power density of 1000 W/m^2, a short-circuit current, I_ν, circulates.

a. **Assuming 100% quantum efficiency (each photon creates one electron-hole pair and all pairs are separated by the p-n junction of the diode), what is the value of this current?**

....

$$I_\nu = q\phi A = 1.6 \times 10^{-19} \times 0.5 \times 4.46 \times 10^{21} \times 0.1^2 = 3.57 \quad \text{A} \qquad (1)$$

$$\boxed{\text{The short-circuit current of the photodiode is 3.57 A.}}$$

---

b. **What is the open-circuit voltage of the photodiode at 300 K under the above illumination?**

....

$$V_{oc} = \frac{kT}{q} \ln \frac{I_\nu}{I_0} = 0.026 \ln \frac{3.57}{30 \times 10^{-9}} = 0.484 \quad \text{V.}$$

$$\boxed{\text{The open-circuit voltage of the photodiode is 484 mV.}}$$

Observe that the v-i characteristics of a photodiode is very steep at the high current end. In other words, the best operating current is only slightly less that the short-circuit current. This knowledge will facilitate answering the question below.

c. **Under an illumination of 1000 W/m^2, at 300 K, what is the maximum power the photodiode can deliver to a load. What is the efficiency? Do this by trial and error an be satisfied with 3 significant figures in your answer. Consider an ideal diode with no internal resistance.**

....

The load voltage is given by

$$V_L = \frac{kT}{q} \ln \left(\frac{I_\nu - I_L}{I_0} \right) = 0.026 \ln \left(\frac{3.57 - I_L}{30 \times 10^{-9}} \right), \qquad (2)$$

Solution of Problem 14.17

$$P_L = V_L I_L = 0.026 I_L \ln\left(\frac{3.57 - I_L}{30\times 10^{-9}}\right). \tag{3}$$

Now we have to try different values of I_L. We know that I_L must be smaller than 3.57 A (otherwise the formula will blow up). We know also that it should not be much less than 3.57 A.

Let's try different values in the hope of bracketing the correct one.

Trial	I_L	P_L
1	3.50	1.334
2	3.00	1.307
3	3.25	1.367
4	3.30	1.374
5	3.40	1.375
6	3.45	1.364

The maximum power is 1.37 W. Since the area of the photo diode is 0.01 m^2, it receives a total illumination of 10 W and the efficiency is 13.7%.

The maximum power delivered to the load is 1.37 W.
The corresponding efficiency is 13.7%.

d. What is the load resistance that leads to maximum efficiency?

$$R_L = \frac{P_L}{I_L^2} = \frac{1.37}{3.4^2} = 0.118 \quad \Omega. \tag{4}$$

Optimum load resistance is 118 milliohms.

---

e. Now repeat the power, efficiency and load resistance calculations for an illumination of 10,000 W/m^2.

If the illumination power density is raised by a factor of 10 while all else remains the same, then the short-circuit current increases by the same factor of 10. $I_\nu = 35.7$ A.

From Equation 3,

$$P_L = V_L I_L = 0.026 I_L \ln\left(\frac{35.7 - I_L}{30\times 10^{-9}}\right). \tag{5}$$

Again, by trial and error

Solution of Problem 14.17

Trial	I_L	P_L
1	35.0	15.44
2	33.0	15.71
3	34.0	15.78
4	34.5	15.70
5	33.5	15.77

So, maximum power occurs for $I_L = 34$ A and is 15.8 W. The efficiency is now 15.8%.

The maximum power delivered to the load is 15.8 W. The corresponding efficiency is 15.8%.

Optimum load is

$$R_L = \frac{P_L}{I_L^2} = \frac{15.8}{34^2} = 0.014 \quad \Omega. \tag{6}$$

Optimum load resistance is 14 milliohms.

---

f. Draw conclusions of what happens to the efficiency and the optimal load resistance when the power density of the illumination on a photodiode increases

....

Increasing the power density increases somewhat the efficiency (provided the temperature is kept the same) and requires much smaller load resistance.

This, by the way, illustrates the fact that when a non sun-tracking array is used (one in which the illumination changes substantially throughout the day) a fix load will not properly match the photodiodes. A load follower" that a“adjusts the load resistance to (about) its optimal value must be employed to improve the efficiency of the system.

Solution of Problem 14.17

Prob 14.18 Everything else being the same, the efficiency of a photodiode rises when:

☐

1. The operating temperature rises.

☐

2. The operating temperature falls.

☐

3. The light power density rises.

☐

4. The light power density falls.

....

Items 2 and 3 are correct.

Solution of Problem 14.18

Prob 14.19 A photodiode with a bandgap energy of $W_g = 1.4$ eV is exposed to monochromatic radiation (500 THz) with a power density $P = 500$ w/m^2. The active area of the device is 10 by 10 cm.

Treat it as an ideal device in series with an internal resistance of 2 mΩ.

All measurements were made at 298 K.

The open-circuit voltage is 0.555 V.

a. Estimate the short-circuit current.

The photon flux is

$$\phi = \frac{P}{hf} = \frac{500}{6.6 \times 10^{-34} \times 500 \times 10^{12}} = 1.51 \times 10^{21} \quad \text{photons m}^{-2}\text{s}^{-1}.$$

The short-circuit current would be

$$I_\nu = q\phi A = 1.6 \times 10^{-19} \times 1.51 \times 10^{21} \times 0.1^2 = 2.42 \quad \text{A}.$$

The internal resistance causes a 4.8 mV drop, so the diode is not really short-circuited but the drop is not large enough to alter the value of I_ν substantially.

$$\boxed{\text{The short-circuit current is 2.42 A.}}$$

---

b. How much power does the diode deliver to 200 mΩ load?

The open-circuit voltage is

$$V_{oc} = \frac{kT}{q} \ln\left(\frac{I_\nu}{I_0} + 1\right) = 0.555\ V.$$

$$0.555 = \frac{1.38 \times 10^{-23} \times 298}{1.6 \times 10^{-19}} \ln\left(\frac{2.42}{I_0} + 1\right),$$

$$\ln\left(\frac{2.42}{I_0} + 1\right) = 21.6,$$

$$I_0 = 1.01 \times 10^{-9} \quad \text{A}$$

Of course, the "1" in the above formula has no meaning.

The load voltage is

$$V_L = \frac{kT}{q} \ln\left(\frac{I_\nu - I}{I_0}\right) - R_s I$$

Solution of Problem 14.19

For a given load resistance, R_L,

$$V_L = IR_L,$$

hence,

$$I(R_L + R_s) = 0.0257 \ln\left(\frac{2.42 - I}{1.01 \times 10^{-9}}\right)$$

Since $R_L + R_s = 202\ \ \mathrm{m\Omega}$,

$$I = 0.127 \ln\left(\frac{2.42 - I}{1.01 \times 10^{-9}}\right)$$

By trial and error, it is found that $I \approx 2.32$ A. Notice that this is just a little less than the short-circuit current. The corresponding load voltage is $V_L = 2.32 \times 0.2 = 0.464$ V and the power is $P_L = 2.32 \times 0.464 = 1.08$ W.

The power delivered to the load is 1.08 W.

---

c. What is the efficiency of the device when feeding the 200 milliohm load?

....

The light power density is 500 W/m^2. Since the area of the cell is 0.01 m^2, the input power is 5 W. Thus,

$$\eta = \frac{1.08}{5} = 0.216.$$

The efficiency of the photodiode is 21.6%.

Solution of Problem 14.19

Prob 14.20 *Suggestion: To solve this problem, use a spread sheet and tabulate all the pertinent values for different hour angles (from sunrise to sunset). 5° intervals in α will be adequate*

A flat array of silicon photodiodes is set up at a place whose latitude is 32° N. The array faces south and is mounted at an elevation angle that maximizes the yearlong energy collection, assuming perfectly transparent air.

a. What is the elevation angle of the array?

From Chapter 12, it can be seen that the optimum elevation angle of a solar collector at 32° latitude is about 2° larger than the latitude.

$$\boxed{\text{The elevation angle of the array should be } 34^\circ}$$

---

b. On 15 April, 2002, how does the insolation on the array vary throughout the day? Plot the insolation, P, versus the time of day, t, in hours.

15 April is day 105 on a non-leap year. On that day, the solar declination is about

$$\delta = 23.45^\circ \sin\left[360^\circ\left(\frac{105-80}{365.25}\right)\right] = 9.8^\circ. \tag{1}$$

The instantaneous insolation on the collector is (from Chapter 12),

$$\begin{aligned} P &= P_s[\cos\epsilon\cos\chi + \sin\epsilon\sin\chi\cos(\xi-\zeta)] \\ &= 1000[\cos 34^\circ\cos\chi + \sin 34^\circ\sin\chi\cos(\xi-180^\circ)] \\ &= 829\cos\chi + 559\sin\chi\cos(\xi-180^\circ) \end{aligned} \tag{2}$$

We have to find χ and ξ as a function of time.

$$\begin{aligned} \cos\chi &= \sin\delta\sin\lambda + \cos\delta\cos\lambda\cos\alpha \\ &= \sin 9.8^\circ\sin 32^\circ + \cos 9.8^\circ\cos 32^\circ\cos\alpha \\ &= 0.0902 + 0.8357\cos\alpha. \end{aligned} \tag{3}$$

$$\sin\chi = \sqrt{1-(0.0902+0.8357\cos\alpha)^2} \tag{4}$$

$$\begin{aligned} \tan\xi &= \frac{\sin\alpha}{\sin\lambda\cos\alpha - \cos\lambda\tan\delta} \\ &= \frac{\sin\alpha}{\sin 32^\circ\cos\alpha - \cos 32^\circ\tan 9.8} \\ &= \frac{\sin\alpha}{0.5299\cos\alpha - 0.1465} \end{aligned} \tag{5}$$

Solution of Problem 14.20

Table 1 (Column 7) lists values of P versus the time of day, t, (Column 2) and the corresponding hour angle, α (Column 1) P is plotted versus t in Figure 1.

Table 1 lists values at 5° intervals of the hour angle. Actually, the table was constructed for 1° intervals and the averages listed on the last row reflect this fact.

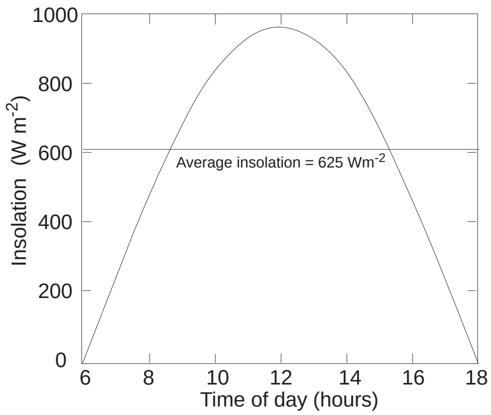


Figure 1

---

c. What is the average insolation on the collector?

....

A numerical integration performed on the data of Table 1 (last row of Column 7) reveals that $P_{ave} = 625$ W/m^2.

The average insolation on the collector is 625 W/m^2.

---

d. Assuming ideal silicon photodiodes with a reverse saturation current density of 10 nA/m^2, what is the average power delivered during the day (from sunrise to sunset) if a perfect load follower is used, i.e., if the load is perfectly matched at all the different instantaneous values of insolation? What is the average overall efficiency?

....

To solve this problem we will use equations from Chapter 14 of the text.

$$V_m = V_A \ln\left(\frac{P_{in}}{V_B J_0}\right), \tag{6}$$

where

$$V_A = 2.2885 \times 10^{-2} - 139.9 \times 10^{-6} \ln J_0 - 2.5734 \times 10^{-6} (\ln J_0)^2, \tag{7}$$

$$V_B = 4.7253 - 0.8939 \ln J_0, \tag{8}$$

Solution of Problem 14.20

For $J_0 = 10$ nA, $V_A = 2.459 \times 10^{-2}$ V and $V_B = 21.19$ V so that

$$V_m = 2.459 \times 10^{-2} \ln\left(\frac{P_{in}}{211.9 \times 10^{-9}}\right), \tag{8}$$

To find the corresponding J_m we need J_ν which can be obtained from

$$J_\nu = 0.399 P_{in}, \tag{9}$$

Now, J_m can be found from

$$J_m = J_\nu - 10^{-8}[\exp(38.94 V_m) - 1]$$

And the power delivered to the load is

$$P_L = V_m I_m. \tag{10}$$

These values are all listed in Table 1 (Columns 8, 9, 10 and 11) and a numerical integration shows that the average power delivered to the load (Bottom row of Column 11) is 128 W, leading to an overall efficiency of $128/625 = 0.205$.

The average load power is 128 W for each square meter of collector area. The overall efficiency is 20.4%.

....

e. Estimate the average power collected if the array is connected to a load whose resistance maximizes the efficiency at noon. In other words, the average power when no load-follower is used.

....

A reasonable estimate of the power delivered to the unmatched load can be obtained from Equation 80 (Chapter14) of the text:

$$P_L = \left(\frac{P_{in}}{P_{in_{max}}}\right)^2 P_{L_{max}}. \tag{11}$$

P_{in} is listed in Column 7 and $P_{in_{max}}$ is the value of P_{in} at noon. $P_{L_{max}}$ is listed in Column 11 and the various values of P_L are listed in Column 12 whose last row shows the average power of 102 W.

A fixed load (matched for the power at noon) would receive 102 W on average while perfect load follower would deliver 128 watts, a 25% improvement.

A fixed load would receive 102 W on average, compared with 128 W delivered by a perfect load follower.

Solution of Problem 14.20

Table 1

α deg	t h	$\tan\xi$	ξ deg	$\cos\chi$	$\sin\chi$	P W/m^2 Insol	V_m V	J_ν A/m^2	J_m A/m^2	P_L W/m^2 Mtchd	P_L W/m^2 Not Mtchd
1	2	3	4	5	6	7	8	9	10	11	12
-89	6.07	7.285	82.2	0.1048	0.9945	11.2	0.436	4.5	4.2	1.9	0.0
-85	6.33	9.931	84.2	0.1630	0.9866	79.9	0.485	31.9	30.3	14.7	1.4
-80	6.67	18.08	86.8	0.2353	0.9719	165.1	0.502	65.9	62.7	31.5	5.8
-75	7.00	103.3	89.4	0.3065	0.9519	248.9	0.513	99.3	94.7	48.5	13.2
-70	7.33	-27.1	92.1	0.3760	0.9266	330.9	0.520	132.0	125.9	65.4	23.3
-65	7.67	-11.7	94.9	0.4434	0.8963	410.3	0.525	163.7	156.2	82.0	35.8
-60	8.00	-7.31	97.8	0.5081	0.8613	486.5	0.529	194.1	185.3	98.0	50.4
-55	8.33	-5.2	100.9	0.5695	0.8220	558.9	0.532	223.0	212.9	113.4	66.5
-50	8.67	-3.95	104.2	0.6274	0.7787	627.1	0.535	250.2	238.9	127.9	83.7
-45	9.00	-3.1	107.9	0.6811	0.7322	690.4	0.538	275.5	263.1	141.5	101.4
-40	9.33	-2.48	112.0	0.7304	0.6830	748.5	0.540	298.6	285.3	153.9	119.2
-35	9.67	-1.99	116.6	0.7748	0.6322	800.8	0.541	319.5	305.3	165.2	136.4
-30	10.00	-1.6	122.0	0.8139	0.5810	846.9	0.543	337.9	322.9	175.2	152.6
-25	10.33	-1.27	128.3	0.8476	0.5306	886.6	0.544	353.8	338.1	183.8	167.3
-20	10.67	-0.97	135.8	0.8755	0.4832	919.5	0.545	366.9	350.6	191.0	179.9
-15	11.00	-0.71	144.7	0.8974	0.4412	945.3	0.545	377.2	360.5	196.6	190.1
-10	11.33	-0.46	155.2	0.9132	0.4075	963.9	0.546	384.6	367.6	200.6	197.7
-5	11.67	-0.23	167.1	0.9227	0.3855	975.1	0.546	389.1	371.9	203.1	202.3
0	12.00	0	180.0	0.9259	0.3778	978.9	0.546	390.6	373.3	203.9	203.9
5	12.33	0.229	192.9	0.9227	0.3855	975.1	0.546	389.1	371.9	203.1	202.3
10	12.67	0.463	204.8	0.9132	0.4075	963.9	0.546	384.6	367.6	200.6	197.7
15	13.00	0.708	215.3	0.8974	0.4412	945.3	0.545	377.2	360.5	196.6	190.1
20	13.33	0.973	224.2	0.8755	0.4832	919.5	0.545	366.9	350.6	191.0	179.9
25	13.67	1.266	231.7	0.8476	0.5306	886.6	0.544	353.8	338.1	183.8	167.3
30	14.00	1.6	238.0	0.8139	0.5810	846.9	0.543	337.9	322.9	175.2	152.6
35	14.33	1.995	243.4	0.7748	0.6322	800.8	0.541	319.5	305.3	165.2	136.4
40	14.67	2.478	248.0	0.7304	0.6830	748.5	0.540	298.6	285.3	153.9	119.2
45	15.00	3.099	252.1	0.6811	0.7322	690.4	0.538	275.5	263.1	141.5	101.4
50	15.33	3.946	255.8	0.6274	0.7787	627.1	0.535	250.2	238.9	127.9	83.7
55	15.67	5.203	259.1	0.5695	0.8220	558.9	0.532	223.0	212.9	113.4	66.5
60	16.00	7.311	262.2	0.5081	0.8613	486.5	0.529	194.1	185.3	98.0	50.4
65	16.33	11.7	265.1	0.4434	0.8963	410.3	0.525	163.7	156.2	82.0	35.8
70	16.67	27.05	267.9	0.3760	0.9266	332.4	0.520	132.6	126.5	65.7	23.5
75	17.00	-103	-89.4	0.3065	0.9519	248.9	0.513	99.3	94.7	48.5	13.2
80	17.33	-18.1	-86.8	0.2353	0.9719	165.1	0.502	65.9	62.7	31.5	5.8
85	17.67	-9.93	-84.2	0.1630	0.9866	79.9	0.485	31.9	30.3	14.7	1.4
89	17.93	-7.28	-82.2	0.1048	0.9945	11.3	0.436	4.5	4.3	1.9	0.0
						624.6				128.2	102.2

Solution of Problem 14.20

Prob 14.21 *Suggestion: To solve this problem, use a spread sheet and tabulate all the pertinent values for different hour angles (from sunrise to sunset). 5° intervals in α will be adequate*

To simplify mathematical manipulation, we will postulate a very simple (and unrealistic) spectral power distribution:

$$\frac{\partial P}{\partial f}=\begin{cases} A \text{ for } 300\,\text{THz}<f<500\ \text{THz}; \\ 0, \text{ otherwise.}\end{cases}$$

a. If $A = 10^{-12}$, what is the power density of the radiation?

....

$$P = \int_{300\times10^{12}}^{500\times10^{12}} \frac{\partial P}{\partial f} df = 10^{-12}\int_{300\times10^{12}}^{500\times10^{12}} df$$
$$= 10^{-12}(500-300)\times 10^{12} = 200 \qquad \text{W/m}^2.$$

$$\boxed{\text{The power density is 200 W/m}^2.}$$

---

b. Assuming 100% quantum efficiency, what is the short-circuit current density, J_ν?

....

The total flux of photons is

$$\Phi_T = \frac{1}{h}\int_{300\times10^{12}}^{500\times10^{12}} \frac{1}{f}\frac{\partial P}{\partial f} df = \frac{10^{-12}}{6.63\times10^{-34}}\int_{300\times10^{12}}^{500\times10^{12}} \frac{1}{f} df$$
$$= 1.51\times10^{21}\ln\frac{500\times10^{12}}{300\times10^{12}} = 771\times10^{18} \qquad \text{photons m}^{-2}\text{s}^{-1}.$$
$$J_\nu = q\Phi_T = 1.60\times10^{-19}\times 771\times10^{18} = 124 \qquad \text{A/m}^2.$$

$$\boxed{\text{The short-circuit current density is 124 A/m}^{-2}.}$$

---

c. At 300 K, assuming a reverse saturation current density of $J_0 = 10^{-7}$ A/m^2, what is the open-circuit voltage of the photocell?

....

$$V_{oc} = \frac{kT}{q}\ln\frac{J_\nu}{J_0} = 0.0259\ln\frac{124}{10^{-7}} = 0.0259\times 20.9 = 0.542 \qquad \text{V.}$$

$$\boxed{\text{The open-circuit voltage is 542 millivolts.}}$$

---

d. At what load voltage, V_m, does this photocell deliver its maximum power output?

Solution of Problem 14.21

V_m must satisfy the equation

$$\left(1+\frac{qV_m}{kT}\right)\exp\left(\frac{qV_m}{kT}\right)-\frac{J_\nu}{J_0}=0.$$

or

$$\left(1+\frac{V_m}{0.0259}\right)\exp\left(\frac{V_m}{0.0259}\right)-1.24\times10^9=0.$$

A numerical solution leads to

The maximum-power voltage is 466 millivolts.

---

e. What is the current density delivered by the photocell when maximum power is being transferred to the load?

$$V_L=\frac{kT}{q}\ln\left(\frac{J_\nu-J_m}{J_0}\right)=V_m=0.466 \quad \text{V}.$$

$$J_m=J_\nu-J_0\exp\frac{qV_m}{kT}=124-10^{-7}\exp\frac{0.466}{0.0259}=117.5 \quad \text{A/m}^2.$$

The maximum-power current is 117.5 A/m^2.

---

f. What is the efficiency of the photocell?

The power delivered to the load is

$$0.466\times117.5=54.8 \quad \text{W/m}^2 \text{ of active area of the cell.}$$

Since the illumination power density is 200 W/m^2, the efficiency is

$$\eta=\frac{54.8}{200}=0.274.$$

The efficiency is 27.4%.

---

g. What is the load resistance under the above conditions?

$$\Re_L=\frac{V_m}{J_m}=\frac{0.466}{117.5}=3.96\times10^{-3} \quad \Omega\text{m}^2.$$

This means, for instance, that if the effective area of the photocell is 1 × 1 cm, the load resistance is $R_L=39.6 \quad \Omega$.

The load resistance is 3.96 milliohms m^2.

---

h. Repeat all the above for a light power density of 2 W/m^2.

Solution of Problem 14.21

Clearly, if the photon flux is reduced to 1/100 of its previous value, the same will happen to the short-circuit current which now is 1.24 A/m^2.

The open-circuit voltage is $0.542 - 4.6 \times 0.0259 = 0.423$ V because the natural log of 100 is 4.60.

To find the new value of V_m we must again solve numerically

$$\left(1 + \frac{V_m}{0.0259}\right) \exp\left(\frac{V_m}{0.0259}\right) - 1.24 \times 10^7 = 0.$$

This yields $V_m = 0.354$ V.

$$J_m = 1.24 - 10^{-7} \exp\frac{0.354}{.0259} = 1.15 \quad \text{A/m}^2.$$

$$P_L = 0.354 \times 1.15 = 0.407 \quad \text{W/m}^2.$$

$$\eta = \frac{0.407}{2} = 0.204.$$

Thus, the efficiency decreased substantially as the level of illumination went down.

$$\Re_L = \frac{0.354}{1.15} = 0.308 \quad \Omega\text{m}^2.$$

We see that the optimum load resistance increased about 100-fold.

---

i. What would be the efficiency at these low light levels if the load resistance had the optimum value for 200 W/m^2?

....

With such a small load resistance (3.96 mΩm^2), even with the full short-circuit current density flowing to the load, the load voltage would be only $1.24 \times 0.00396 = 4.9 \times 10^{-3}$ V. This forward bias voltage essentially drives no current through the diode confirming that all the short-circuit current actually does flow through the load. The load power is then $1.24 \times 0.0049 = 0.006$ W.

The efficiency falls to $0.006/2 = 0.003$ or 0.3%.

This exercise shows that when exposed to varying level of illumination, it is important to adjust the load resistance accordingly if reasonable overall efficiencies are to be expected. In other words, a **load follower** is required for good performance.

Solution of Problem 14.21

Prob 14.22 It is hoped that high efficiency cascaded photocells can be produced at a low cost. This consists of a sandwich of two cells of different band gaps. The bottom cell (the one with the smaller band gap) cam be made using $\mathrm{CuIn_xGa_{1-x}Se_2}$, known as CIGS. This material has a band gap of about 1 eV and has been demonstrated as yielding cells with 15% efficiency.

The question here is what band gap of the top cell yields the largest efficiency for the combined cascaded cells. Assume radiation from a black body at 6000 K. Assume no losses, i.e., consider only the theoretical efficiency.

....

In the text, we derived an expression for the theoretical efficiency of a photocell exposed to black body radiation:

$$\eta_{th_t} = 1780\frac{V_{g_t}}{T}\int_{\frac{qV_{g_t}}{kT}}^{\infty}\frac{x^2}{e^x-1}dx \tag{1}$$

The definite integral, tabulated in Appendix A of Chapter 14, will be represented as $\mathrm{Int}(\frac{qV_{g_t}}{kT})$.

Using $T = 6000$ K,

$$\eta_{th_t} = 0.2967V_{g_t}\mathrm{Int}(1.932V_{g_t}) \tag{2}$$

This would be the efficiency of the top cell as a function of its band gap voltage, V_{g_t}.

The efficiency of the bottom cell is

$$\eta_{th_b} = 1780\frac{V_{g_b}}{T}\int_{\frac{qV_{g_b}}{kT}}^{\frac{qV_{g_t}}{kT}}\frac{x^2}{e^x-1}dx \tag{3}$$

Notice that the upper limit of the integral is not infinity as in the formula in the Text because the top cell cuts off all radiation above $\frac{qV_{g_t}}{kT}$.

Again, using $T = 6000$ K and letting $V_{g_b} = 1$ V,

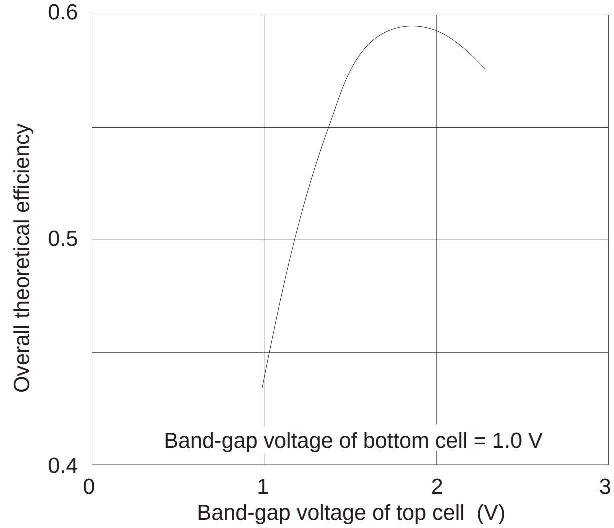
$$\begin{aligned}\eta_{th_b} &= 0.2967\int_{1.932}^{\frac{qV_{g_t}}{kT}}\frac{x^2}{e^x-1}dx = 0.2967\left(\int_{1.932}^{\infty}\frac{x^2}{e^x-1}dx - \int_{\frac{qV_{g_t}}{kT}}^{\infty}\frac{x^2}{e^x-1}dx\right)\\ &= 0.2967\left(\mathrm{Int}(1.932) - \mathrm{Int}(1.932V_{g_t})\right) = 0.2967(1.4611 - \mathrm{Int}(1.932V_{g_t}))\\ &= 0.4335 - 0.2967\mathrm{Int}(1.932V_{g_t})\end{aligned} \tag{4}$$

The overall efficiency, η, is the sum of the efficiency of the two cells,

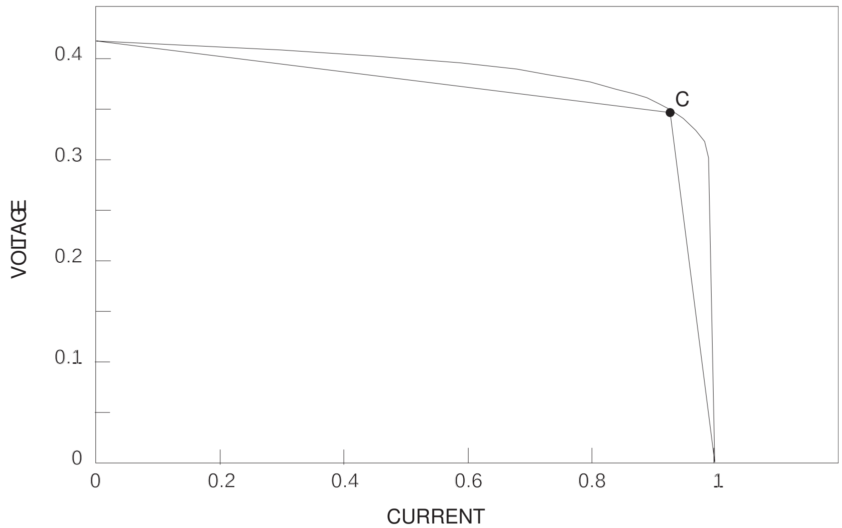
$$\begin{aligned}\eta_{th} &= \eta_{th_t} + \eta_{th_b} = 0.2967V_{g_t}\mathrm{Int}(1.932V_{g_t}) + 0.4335 - 0.2967\mathrm{Int}(1.932V_{g_t})\\ &= 0.4335 + 0.2967\mathrm{Int}(1.932V_{g_t}) \times (V_{g_t} - 1).\end{aligned} \tag{5}$$

Solution of Problem 14.22

To find the maximum efficiency, use the graph,



Solution of Problem 14.22



Prob 14.23 **The V-I characteristic of a photocell is describable by a rather complex mathematical formula which can be handled with a computer but which is too complicated for an in-class exam. To simplify handling, we are adopting, rather arbitrarily, a simplified characteristic consisting of two straight lines as shown in the figure above. The position of the point, C, of maximum output varies with the I_ν/I_0 ratio. Empirically,**

$$V_C = V_{OC}\left(0.7 + 0.0082\ln\frac{I_\nu}{I_0}\right) \tag{1}$$

and

$$I_C = I_\nu\left(0.824 + 0.0065\ln\frac{I_\nu}{I_0.}\right) \tag{2}$$

Consider now silicon photodiodes operating at 298 K. These diodes form a panel, 1 m^2 in area, situated in Palo Alto (latitude 37.4° N, longitude 125° W). The panel faces true south and has an elevation of 35°. In practice, the panel would consist of many diodes in a series/parallel connection. In the problem, here, assume that the panel has a single enormous photodiode.

Calculate the insolation on the surface at a 1130 PST and at 1600 PST on October 27. Assume clear meteorological conditions.

To avoid a lot of calculation, assume that the true solar time is equal to the PST.

a. Calculate the insolation on the collector at the two moments mentioned.

....

Solution of Problem 14.23

The elevation of the surface is $\epsilon = 35°$.
October 27 is day 300.

$$\delta = 23.44° \sin\left(\frac{360}{365.25}(300-80)\right) = -6.1°. \tag{3}$$

At 1130,

$$\alpha = 15(11:30 - 12:00) = -7.5°. \tag{4}$$

$$\cos\chi = \sin 37.4° \sin(-6.1°) + \cos 37.4° \cos(-6.1°) \cos(-7.5°) = 0.7186 \tag{5}$$

$$\chi = 44.06°. \tag{6}$$

$$\tan\xi = \frac{\sin(-7.5°)}{\sin 37.4° \cos(-7.5°) - \cos 37.4° \tan(-6.1°)} = -0.18997 \tag{7}$$

$$\xi = 169.2° \tag{8}$$

$$\begin{aligned} P &= 1000\big[0.7186 \cos 35° + \sin 35° \sin 44.06° \cos(169.2° - 180°)\big] \\ &= 980 \quad \text{W/m}^2. \end{aligned} \tag{9}$$

At 1600,

$$\alpha = 15(16:00 - 12:00) = 60°. \tag{10}$$

$$\cos\chi = \sin 37.4° \sin(-6.1°) + \cos 37.4° \cos(-6.1°) \cos(60°) = 0.3304 \tag{11}$$

$$\chi = 70.71°. \tag{12}$$

$$\tan\xi = \frac{\sin 60°}{\sin 37.4° \cos 60° - \cos 37.4° \tan(-6.1°)} = 2.2287 \tag{14}$$

$$\xi = 245.8° \tag{15}$$

$$\begin{aligned} P &= 1000\big[0.3304 \cos 35° + \sin 35° \sin 70.71° \cos(245.8° - 180°)\big] \\ &= 493 \quad \text{W/m}^2. \end{aligned} \tag{16}$$

$$\boxed{\text{The insolations are 980 W/m}^2 \text{ and 493 W/m}^2.}$$

---

b. What are the short-circuit currents (I_ν) under the two illuminations? Consider the sun as a 6000 K black body.

....

In Example 14.2 of the Text, we found that the flux of photons having energies greater that 1.1 eV when the power density of light from a 6000 K black body is 1000 W/m^2 is $\phi_g = 2.49 \times 10^{21}$ photons m^{-2}s^{-1}. This would yield a short-circuited current density of $J_\nu = q\phi_g = 1.6 \times 10^{-19} \times 2.49 \times 10^{21} = 398.4$ A/m^2. Since the panel has an area of 1 m^2, $I_\nu = 398.4$ A.

The short-circuit current is proportional to the light power density, hence, for the two insolations calculated above, we will have,

$$I_\nu = \begin{cases} 390 & \text{A} \\ 196 & \text{A.} \end{cases} \tag{17}$$

Solution of Problem 14.23

$$\boxed{\text{The short-circuit currents are 390 A and 196 A.}}$$

---

c. When exposed to the higher of the two insolations, the open-circuit voltage of the photodiode is 0.44 V.
What is the power delivered to a load at 1130 and at 1600? The resistance of the load is, in each case, that which maximizes the power output for that case. What are the load resistances? What are the efficiencies?

....

For maximum power, the operating point must be Point C in the figure. We need to know the I_ν/I_0 ratio.

$$V_{OC} = \frac{kT}{q} \ln \frac{I_\nu}{I_0}, \tag{18}$$

$$\frac{I_\nu}{I_0} = \exp \frac{q}{kT} V_{OC} = \exp \frac{0.44}{0.026} = 22.4 \times 10^6 \tag{19}$$

Since $I_\nu = 390$ A,

$$I_0 = \frac{390}{22.4 \times 10^6} = 17.4 \times 10^{-6} \quad \text{A.} \tag{20}$$

<u>At 1130,</u>

$$V_C = 0.44(0.7 + 0.0082 \ln 22.4 \times 10^6 = 0.369 \quad \text{V} \tag{21}$$

$$I_C = 390(0.824 + 0.0065 \ln 22.4 \times 10^6 = 364 \quad \text{A} \tag{22}$$

$$P_L = 0.369 \times 364 = 134 \quad \text{W.} \tag{23}$$

$$R_L = \frac{0.369}{364} = 1.01 \times 10^{-3} \quad \Omega \tag{24}$$

$$\eta = \frac{134}{980} = 0.137 \tag{25}$$

<u>At 1600,</u>

$$\frac{I_\nu}{I_0} = \frac{196}{17.4 \times 10^{-6}} = 11.4 \times 10^6 \tag{26}$$

$$V_{OC} = 0.026 \times \ln 11.4 \times 10^6 = 0.42 \tag{27}$$

$$V_C = 0.42(0.7 + 0.0082 \ln 11.4 \times 10^6 = 0.350 \quad \text{V} \tag{28}$$

$$I_C = 196(0.824 + 0.0065 \ln 11.4 \times 10^6 = 182 \quad \text{A} \tag{29}$$

$$P_L = 0.35 \times 182 = 63.7 \quad \text{W.} \tag{30}$$

$$R_L = \frac{0.35}{182} = 1.92 \times 10^{-3} \quad \Omega \tag{31}$$

$$\eta = \frac{63.7}{493} = 0.129 \tag{32}$$

Time	*Load power (W)*	*Load resistance (milliohms)*	*Efficiency*
1130	*134*	*1.01*	*13.7%*
1600	*63.7*	*1.92*	*12.9%*

Solution of Problem 14.23

---

d. Suppose that at 1600 the load resistance used were the same as that which optimized the 1130 out put. What are the power in the load and the efficiency?

....

With a load resistance of 1.01 milliohms, the photodiode would operate at a point other than Point C. It will operate at a voltage smaller than V_C. The straight line V-I characteristic in that region is

$$V = 4.9 - 0.025I \tag{33}$$

The corresponding current can be found from

$$R_L = \frac{4.9 - 0.025I}{I} = 0.00101 \quad \Omega, \tag{34}$$

from which $I = 188.4$ A.

$$V_L = 4.9 - 0.025 \times 188.4 = 0.19 \quad \text{V}, \tag{35}$$

$$P_L = 0.19 \times 188.4 = 35.8 \quad \text{W}. \tag{36}$$

$$\eta = \frac{35.8}{493} = 0.073. \tag{37}$$

It can be seen that without a load follower to adjust the load resistance, the efficiency will fall substantially.

Time	*Load power (W)*	*Load resistance (milliohms)*	*Efficiency*
1130	*134*	*1.01*	*13.7%*
1600	*35.8*	*1.01*	*7.3%*

	Load power (W)	*Efficiency*
Matched load	*197.7*	*13.4%*
Fixed load	*169.8*	*11.5%*

---

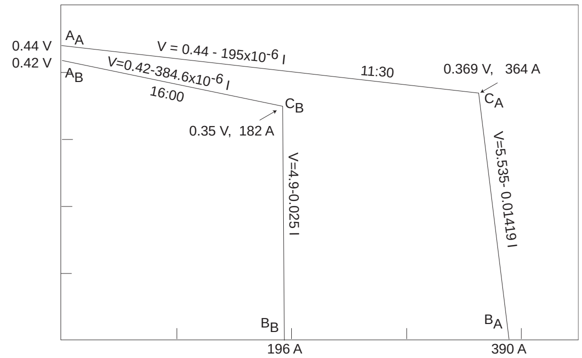
e. Let the load resistance be the same at both 1130 and 1600 but, unlike Question d, not necessarily the resistance that optimizes the output at 1130. The idea is to operate the panel at slightly lower efficiency at 1130 and at somewhat higher efficiency than that of Question d at 1600 in the hope that the overall efficiency can be improved.

What is the value of this common load resistance?

....

$V = 5.535 - 0.01419I$

Solution of Problem 14.23



The V-I characteristics and the mathematical expressions describing the four straight line segments are shown above and and repeated here:

Time	Segment	Equation
1130	A_A to C_A	$V = 0.44 - 195 \times 10^{-6} I$
1130	C_A to B_A	$V = 5.535 - 0.01419 I$
1600	A_B to C_B	$V = 0.42 - 384.6 \times 10^{-6}$
1600	C_B to B_B	$V = 4.9 - 0.025 I$

We observer that at 1130 the optimum R_L is 1.01 milliohms and that at 1600 it is 1.92 milliohms. We want to operate the 1130 case at a somewhat higher resistance and the 1600 case at a slightly lower resistance so that these two load resistance become the same. This means that we have to move the 1130 operation point towards a point to the left of Point C and at 1600, to the right. This puts us on segments A_A to C_A and C_B to B_B.

The load resistance is V/I,

$$R = \frac{0.44 - 195 \times 10^{-6} I_A}{I_A} = \frac{4.9 - 0.025 I_B}{I_B} \tag{38}$$

Solve for I_A,

$$I_A = \frac{88000 I_B}{980000 - 4961 I_B} \tag{39}$$

The total power delivered by the panel is

$$\begin{aligned} P_T &= P_A + P_B = R(I_A^2 + I_B^2) \\ &= \frac{4.9 - 0.025 I_B}{I_B}\left[\left(\frac{88000 I_B}{980000 - 4961 I_B}\right)^2 + I_B^2\right] \end{aligned} \tag{40}$$

$$\frac{dP_T}{dI_B} = 0, \tag{41}$$

yields

$$I_B = \begin{cases} 101.272 & A \\ 179.723 & A \\ 194.386 & A \\ 215.241 & A. \end{cases}$$

Solution of Problem 14.23

Since I_B cannot exceed 196 A, the last value, above, is not valid. Introducing the values of I_B into Equation 39, we get

$$I_A = \begin{cases} 18.66 \quad A \\ 178.922 \quad A \\ 1092.96 \quad A \\ -215.705 \quad A. \end{cases}$$

From Equation 38,

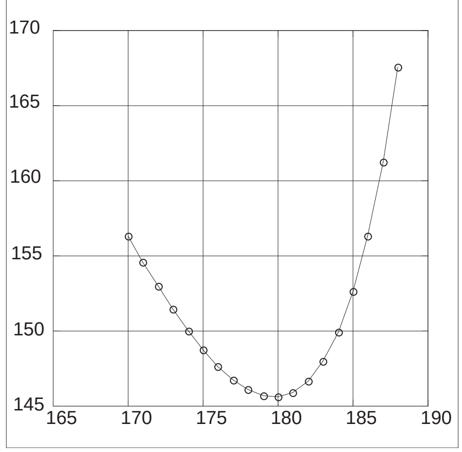
$$R = \begin{cases} 0.02338 \quad \Omega \\ 0.002264 \quad \Omega \\ 0.0002071 \quad \Omega \\ -0.00223 \quad \Omega. \end{cases}$$

And from $P_T = R(I_A^2 + I_B^2)$,

$$P_T = \begin{cases} 247.926 \quad W \\ 145.617 \quad W \\ 255.81 \quad W \\ -207.52 \quad W. \end{cases}$$

Solution #	I_B	I_A	V_A	V_B
1	101.3	18.7	0.436	**2.37**
2	**179.7**	**178.9**	**0.405**	**0.407**
3	194.4	**1093**	0.227	0.04
4	**215.2**	**-215.7**	**0482**	**-0.481**

All solutions are unacceptable. Solutions 1, 3, and 4 are out of range of the characteristics as indicated by the bold type. Solution 2 is within the range but happens to be a minimum, not a maximum, as shown in the figure below that is a plot of the total power in the load as a function of the current I_B. The plot stops at $I_B = 188.4$ A because, at higher values of I_B, I_A is out of range.



The figure indicates that there is no mathematical maximum for P_T; as I_B grows beyond 180 A, the total output power rises monotonically reaching

Solution of Problem 14.23

the highest value just as I_A reaches the maximum permissible value. The greatest output power is that for $I_B = 188.4$ A.

$$\boxed{\text{In this example, the highest output with a fixed load occurs when the load is optimized for 1130 hours.}}$$

Solution of Problem 14.23

Prob 14.24

a. What is the ideal (theoretical) efficiency of a gallium phosphide photocell exposed to the radiation of a 6000 K black body? For your information: the corresponding efficiency for silicon is 43.8%.

....

The band-gap energy in gallium phosphide is 2.24 eV. The lower limit of integration in Equation 25 of the Text is

$$\frac{qV_g}{kT} = \frac{1.6 \times 10^{-19} \times 2.24}{1.38 \times 10^{-23} \times 6000} = 4.33 \tag{1}$$

From Appendix A of Chapter 16, one finds, by interpolation, that the integral has a value of 0.3888 when its lower limit is 4.33.

$$\eta_{theoretical} = 1780\frac{V_g}{T} \times 0.3888 = 0.258. \tag{2}$$

$$\boxed{\text{The theoretical efficiency of a gallium phosphide photocell exposed to a 6000-K blackbody is 25.8\%.}}$$

---

b. What is the efficiency of an ideal silicon photocell when illuminated by monochromatic light with a frequency of 266 THz?

....

The energy of a 266 Thz photon is

$$W_{266} = \frac{hf}{q} = \frac{6.626 \times 10^{-24} \times 266 \times 10^{12}}{1.6 \times 10^{-19}} = 1.102 \quad \text{eV}. \tag{3}$$

$$\boxed{\text{Since 1.102 eV is an energy just above that of the band-gap energy of silicon, the efficiency is, essentially, 100\%,}}$$

---

c. What is the efficiency of an ideal silicon photocell when illuminated by monochromatic light with a frequency of 541.6 THz?

....

The energy of the illuminating photon is hf_{ill}. The input power is

$$P_{in} = \phi h f_{ill}. \tag{4}$$

The load power is

$$P_L = \phi W_g = \phi h f_g, \tag{5}$$

where f_g is the band-gap frequency of silicon and is 266×10^{12} Hz.

The efficiency is

$$\eta = \frac{P_L}{P_{in}} = \frac{266 \times 10^{12}}{541.6 \times 10^{12}} = 0.491. \tag{6}$$

Solution of Problem 14.24

$\boxed{\text{The efficiency is 49.1\%.}}$

---

d. A real silicon photocell measuring 10 by 10 cm is exposed to 6000-K blackbody radiation with a power density of 1000.0 W/m^2. The temperature of the cell is 310 K. The measured open-circuit voltage is 0.493 V. When short-circuited, the measured current is 3.900 A. The power that the cell delivers to a load depends, of course, on the exact resistance of this load. By properly adjusting the load, the power can maximized. What is this maximum power?

....

See Example 13.2 in the Text. Under the circumstances of this question, the flux of photons with energy above the 1.1 eV band-gap is

$$\phi_g = 2.49 \times 10^{21} \quad \text{photons m}^{-2}\text{s}^{-1}. \tag{7}$$

The resulting I_ν, i.e., the short-circuit current if the cell had no series resistance would be

$$I_\nu = q\phi_g A = 1.6 \times 10^{-19} \times 2.49 \times 10^{21} \times 0.01 = 3.984 \quad \text{A}. \tag{8}$$

Observe that I_ν differs from the measured short-circuit current which is 3.900 A. The reason is that the cell has an internal resistance, R_s. We must calculate the value of the latter.

The open-circuit voltage is

$$V_{oc} = 0.493 = \frac{kT}{q}\ln\left(\frac{I_\nu}{I_0}\right) = \frac{1.38 \times 10^{-23} \times 310}{1.6 \times 10^{-19}}\ln\left(\frac{3.984}{I_0}\right) \tag{9}$$

$$0.493 = 0.02674\ln\left(\frac{3.984}{I_0}\right) \tag{10}$$

From the above, $I_0 = 39.1 \times 10^{-9}$ amperes.

A short-circuited photo diode with a *series* resistance, R_s, is equivalent to a resistanceless diode with a *load* resistance, R_s. The load voltage of the later is

$$V_{L_{sc}} = \frac{kT}{q}\ln\left(\frac{I_\nu - I_{Lsc}}{I_0}\right), \tag{11}$$

and the load current is

$$I_{L_{sc}} = \frac{V_{L_{sc}}}{R_s}. \tag{12}$$

$$R_s = \frac{V_{L_{sc}}}{I_{L_{sc}}} = \frac{1}{I_{L_{sc}}}\frac{kT}{q}\ln\left(\frac{I_\nu - I_{L_{sc}}}{I_0}\right) \tag{13}$$

$I_{L_{sc}}$ was given as 3.900 A, hence

$$R_s = \frac{1}{3.9} 0.02674 \ln\left(\frac{3.984 - 3.9}{39.1 \times 10^{-9}}\right) = 0.09997 \quad \Omega \tag{14}$$

With an arbitrary load resistance, the load power is

$$P_L = V_L P_L = 0.0267 I_L \ln\left(\frac{3.984 - I_L}{39.1 \times 10^{-9}}\right) - 0.09997 I_L^2. \tag{15}$$

$$\frac{dP_L}{dI_L} = 0.02674 \ln \frac{3.984 - I_L}{39.1 \times 10^{-9}} - \frac{0.02674 I_L}{3.984 - I_L} - 2 \times 0.09997 I_L = 0. \tag{16}$$

The solution is $I_{L_m} = 2.195$ A, and the corresponding V_{L_m} is

$$V_{L_m} = 0.02674 \ln \frac{3.984 - 2.195}{39.2 \times 10^{-9}} - 2.1952 \times 0.9997 = 0.2522 \quad .V \tag{17}$$

Consequently, the power delivered to the load is

$$P_{L_m} = I_{L_m} V_{L_m} = 2.195 \times 0.2522 = 0.5536 \quad \text{W.} \tag{18}$$

$$\boxed{\text{The photodiode delivers 0.554 watts to the load.}}$$

Solution of Problem 14.24

Prob 14.25 Consider a solar cell made of semiconducting nanocrystals with a bandgap energy of $W_g = 0.67$ eV. What is the theoretical efficiency when the solar cell is exposed to the radiation of a 6000-K black body? Assume that photons with less than 3.3 W_g create each one single electron/hole pair and that those with more than 3.3 W_g create 2 electron/hole pairs each owing to impact ionization. ....

Photons with energy < 0.67 eV do not interact with the solar cell.

Photons with energy > 0.67 eV create, each, one exciton. Let the flux of these photons be ϕ_1. The electric power density derived from these photons is

$$P_1 = \phi_{g_1} W_g. \tag{1}$$

In addition, photos with energy $> 3.3W_g$ eV, create an additional exciton through impact ionization. Let the flux of these photons be ϕ_2. The electric power density derived from these photons is

$$P_2 = \phi_{g_2} W_g. \tag{2}$$

The total electric power density will, of course, be

$$P = P_1 + P_2. \tag{3}$$

We have to calculate ϕ_1 and ϕ_2.

$$\phi_g = \frac{A}{h}\left(\frac{kT}{h}\right)^3 \int_X^\infty \frac{x^2}{e^x - 1} dx, = 2.949 \times 10^{75} A \int_X^\infty \frac{x^2}{e^x - 1} dx \tag{4}$$

where $X = W_{edge}/(kT) = W_{edge}/(1.38 \times 10^{-23} \times 6000) = 12.07 \times 10^{18} W_{edge}$.

W_{edge} is the energy of the a photon at the low frequency edge of the band considered. For the band starting at 0.67 eV, $X = X_1 = 12.08 \times 10^{18} \times 1.6 \times 10^{-19} \times 0.67 = 1.296$, and for the band starting at $3.3 \times 0.67 = 2.21$ eV, $X_2 = 4.276$

$$\phi_{g_1} = 2.949 \times 10^{75} A \int_{1.296}^\infty \frac{x^2}{e^x - 1} dx, \tag{5}$$

$$\phi_{g_2} = 2.949 \times 10^{75} A \int_{4.276}^\infty \frac{x^2}{e^x - 1} dx, \tag{6}$$

The definite integrals can be evaluated either from the table in Appendix A to Chapter 13 or by using the approximate analytical expression in the same appendix.

Solution of Problem 14.25

The values are 1.873 for the lower limit of 1.295, and 0.409 for the lower limit of 4.274. Thus

$$\phi_{g_1} = 5.524 \times 10^{75} A, \tag{7}$$

$$\phi_{g_2} = 1.206 \times 10^{75} A, \tag{8}$$

and

$$P_1 = 5.519 \times 10^{75} Aq \times 0.67 = 591.7 \times 10^{54} A \tag{9}$$

$$P_2 = 1.205 \times 10^{75} Aq \times 0.67 = 129.2 \times 10^{54} A \tag{10}$$

$$P = (591.7 + 129.2) \times 10^{54} A = 722.4 \times 10^{54} A. \tag{11}$$

The incident light power is

$$P_{in} = \left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15} A = 1.584 \times 10^{57} A, \tag{12}$$

consequently the efficiency is

$$\eta = \frac{P}{P_{in}} = \frac{722.4 \times 10^{54} A}{1.584 \times 10^{57} A} = 0.456. \tag{13}$$

$$\boxed{\text{The efficiency is } 45.6\%}$$

Solution of Problem 14.25

Prob 14.26 The Solar Power Satellites proposed by NASA would operate at 2.45 GHz. The power density of the beam at ionospheric heights (400 km) was to be 230 W/^{2}. The collector on the ground was designed to use dipole antennas with individual rectifiers of the Schottky barrier type. These dipoles were dubbed rectennas.

The satellites would have been geostationary (they would be on a 24-h equatorial orbit with zero inclination and zero eccentricity).

a. Calculate the orbital radius of the satellites.

....

The satellites are in a geostationary orbit, meaning that the orbit must be circular—the centrifugal force, mv^2/r, must exactly balance the centripetal gravitational attraction, $\gamma mM/r^2$,

$$\frac{mv^2}{r} = \gamma\frac{mM}{r^2} \quad \therefore \quad v^2 = \gamma\frac{M}{r}. \tag{1}$$

where m is the mass of the satellite, M is the mass of earth, r is the distance from the center of earth, and γ is the gravitational constant.

The length of the orbit is

$$L = 2\pi r, \tag{2}$$

and the duration must be 24 h or 86,400 s. Hence the velocity is

$$v = \frac{2\pi r}{86,400} \tag{3}$$

Introducing this into Equation 1,

$$\frac{4\pi^2 r^2}{86,400^2} = \gamma\frac{M}{r} \quad \therefore \quad r = \left(\frac{86,400^2 M\gamma}{4\pi^2}\right) = 42,250,000 \quad \text{m.} \tag{4}$$

The orbital radius is 42,250 km (35,879 km above sea level).

---

b. Calculate the microwave power density on the ground at a point directly below the satellite (the sub-satellite point). Assume no absorption of the radiation by the atmosphere.

....

Compared with the satellite-to-ground distance, the height of the ionosphere is almost negligible. Hence, the quick answer is that the power density on the ground is the same as at the ionosphere—230 W/m^2.

If greater accuracy is desired:

$$P_{ground} = P_{ionosphere}\left(\frac{35,479}{35,879}\right)^2 = 225 \quad \text{W/m}^2, \tag{5}$$

The power density at the subsatellite point is 225 W/m^2.

Solution of Problem 14.26

c. The total power delivered to the load is 5 GW. The rectenna system has 70% efficiency. Assume uniform power density over the illuminated area. What is the area that the ground antenna farm must cover?

....

If the system is to deliver 5 MW to the power lines and if the conversion efficiency of the rectenna is 70%, then a total of 7.14 MW must be received on the ground. With a power density of 225 W/m^2, the required area is

$$A = \frac{7.14 \times 10^9}{225} = 31.7 \times 10^6 \quad \text{m}^2. \tag{6}$$

This corresponds to a circle of 6360 m in diameter.

The rectenna area must be a circle with 6.36 km diameter.

---

d. A dipole antenna abstracts energy from an area given in the Text. How many dipoles must the antenna farm use?

....

The area from which a simple antenna intercepts energy is

$$A_{dipole} = 1.64\frac{\lambda^2}{4\pi} = 1.64 \times \frac{(3 \times 10^8/2.45 \times 10^9)^2}{4\pi} = 1.96 \times 10^{-3} \quad \text{m}^2. \tag{7}$$

The number of dipoles that must be used is

$$N = \frac{31.7 \times 10^6}{1.96 \times 10^{-3}} 16 \times 10^9 \tag{8}$$

The extraordinarily large number of required dipoles is 16 billion.

---

e. Assuming (very unrealistically) that the only part of each rectenna that has any mass is the dipole itself, and assuming that the half wave dipole is made of extremely thin aluminum wire, only 0.1 mm in diameter, what is the total mass of aluminum used in the antenna farm?

....

2.45 GHz corresponds to a wavelength of 12.2 cm. The mass of each dipole is $0.122/2 \times \pi \times (1 \times 10^{-4})^2/4 \times 3500 = 1.66 \times 10^{-6}$ kg per dipole. 3500 is the number of kg of 1 cubic meter of aluminum.

The 16 billion dipoles mass 53,600 kg.

The rectenna uses 53.6 tons of aluminum.

Solution of Problem 14.26

f. How many watts must each dipole deliver to the load?

Each dipole must deliver $5 \times 10^9/16 \times 10^9 = 0.31$ W.

$$\boxed{\text{Each dipole must deliver 0.31 W to the load.}}$$

---

g. If the impedance of the rectenna is 70 ohms, how many volts does each dipole deliver?

$$V = \sqrt{RP_L} = \sqrt{70 \times 0.31} = 4.6 \quad \text{V.} \tag{9}$$

$$\boxed{\text{Each dipole must deliver 4.6 volts.}}$$

Solution of Problem 14.26

Prob 14.27 The Solar Power Satellite radiates 6 GW at 2.45 GHz. The transmitting antenna is mounted 10 km from the center of gravity of the satellite. What is the torque produced by the radiation?

....

The toque exerted by the energy radiated is

$$\Upsilon = Fd, \tag{1}$$

where F is the force due to the recoil of the emitted electrons and d is the distance between the antenna and the center of gravity of the satellite.

The force is

$$F = \frac{P}{c} \tag{2}$$

where P is the radiated power and c is the velocity of light. Hence,

$$F = \frac{6 \times 10^9}{3 \times 10^8} = 20 \quad \text{N.} \tag{3}$$

The torque is

$$\Upsilon = 20 \times 10,000 = 200 \quad \text{Nm.} \tag{4}$$

The torque is 200 Nm. This is a small torque but not negligible. Attitude-control rockets must be used to counteract the torque.

Solution of Problem 14.27

Prob 14.28 Compare the amount of energy required to launch a mass, m, from the surface of the earth to the energy necessary to launch the same mass from the surface of the moon. "Launch" here means placing the mass in question an infinite distance from the point of origin. Consult the *Handbook of Chemistry and Physics* **(CRC) for the pertinent astronomical data.**

....

The gravitational force that a planet of mass, M, exerts on a mass, m, is

$$F = \gamma m M \frac{1}{r^2} \tag{1}$$

where γ is the gravitational constant and r is the distance from the center of the planet. Consequentially, the energy to move the mass from the surface of the planet all the way to infinity is

$$W = \gamma m M \int_{\rho}^{\infty} \frac{dr}{r^2} = -\gamma m M \left. \frac{1}{r} \right|_{\rho}^{\infty} = \frac{\gamma m M}{\rho}, \tag{2}$$

Here, ρ is the radius of the planet.

The pertinent data are

Gravitational constant	γ	6.67×10^{-11}	$m^2s^{-2}kg^{-1}$
Mass of earth	M_E	5.98×10^{24}	kg
Mass of moon	M_M	7.35×10^{22}	kg
Radius of earth	ρ_E	6371	km
Radius of moon	ρ_M	1738	km

The ratio of the energy to eject a mass m from the surface of the earth to the corresponding energy from the surface of the moon is

$$\frac{W_E}{W_M} = \frac{\gamma m M_E}{\rho_E} \frac{\rho_M}{\gamma m M_M} = \frac{M_E \rho_M}{M_M \rho_E} = \frac{6.98 \times 10^{24} \times 1738}{7.35 \times 102 \times 6371} = 22.2 \tag{3}$$

$$\boxed{\text{It takes 22.2 times more energy to launch a mass from the surface of the earth than from the surface of the moon.}}$$

Solution of Problem 14.28

(EE-293B 06/07 FN) **Prob 14.29 Consider a simple spectral distribution:**
For $f < 300$ **THz,** $\partial P/\partial f = 0$**,**
For $f = 300$ **THz,** $\partial P/\partial f = A$**,**
for $300 \leq f \leq 500$**,** $\partial P/\partial f = af$**,**
for $f > 500$**,** $\partial P/\partial f = 0$**.**
The total power density is $P = 1000$ **W/m**^{2}**.**

a - A silicon diode having 100% quantum efficiency is exposed to this radiation. What is the short-circuit current density delivered by the diode?

....

First, we have to determine the value of the constant A. At $f = 300$ THz,

$$\frac{\partial P}{\partial f} = 300 \times 10^{12} a = A, \tag{1}$$

hence

$$a = \frac{A}{300 \times 10^{12}}. \tag{2}$$

Between 300 and 500 THz,

$$\frac{\partial P}{\partial f} = \frac{Af}{300 \times 10^{12}}. \tag{3}$$

The total power density is

$$\begin{aligned} P &= \int_{300\ THz}^{500\ THz} \frac{\partial P}{\partial f} df = \frac{A}{300 \times 10^{12}} \int_{300\ THz}^{500\ THz} f df \\ &= \frac{A}{600 \times 10^{12}} \left[\left(500 \times 10^{12}\right)^2 - \left(300 \times 10^{12}\right)^2 \right] \\ &= 266.7 \times 10^{12} A = 1000 \quad \text{W/m}^2. \end{aligned} \tag{5}$$

$$A = 3.75 \times 10^{-12} \quad \text{W m}^{-2}\text{Hz}^{-1}. \tag{6}$$

To calculate the short-circuit current density, we have to know the flux of photons incident of the diode. To this end, we will adapt Equation 13 of the book:

$$\begin{aligned} \phi = \frac{1}{h} \int_{300\ THz}^{500\ THz} \frac{1}{f} \frac{\partial P}{\partial f} df = \frac{3.75 \times 10^{-12}}{300 \times 10^1 0^{12} h} \int_{300\ THz}^{500\ THz} df \\ = 18.88 \times 10^6 [500 - 300] \times 10^{12} = 3.78 \times 10^{21} \quad \text{photons/m}^2 \text{per s.} \end{aligned} \tag{7}$$

Solution of Problem 14.29

The short-circuit current density is

$$J_\nu = q\phi = 1.6 \times 10^{-19} \times 3.78 \times 10^{21} = 604 \quad \text{A/m}^2. \tag{8}$$

$\boxed{\text{The short-circuit current density is 604 A per square meter.}}$

---

b - At 300 K, the reverse saturation current density, J_0, of the diode is 40 μA/m^2. What is the open-circuit voltage generated by the diode?

$$V_{oc} = \frac{kT}{q} \ln\left(\frac{I_\nu}{I_0}\right) = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln \frac{604}{40 \times 10^{-6}} = 0.428 \quad \text{V}. \tag{9}$$

$\boxed{\text{The open-circuit voltage is 0.428 V.}}$

---

c - If the photodiode has an effective area of 10 by 10 cm, what load resistance will result in the largest possible output power?

Maximum power occurs when the load voltage is J_m, as given by

$$\left(1 + \frac{qV_m}{kT}\right) \exp\left(\frac{qV_m}{kT}\right) = \frac{J_\nu}{J_0} + 1 \tag{10}$$

Call

$$x \equiv \frac{qV_m}{kT}, \tag{11}$$

then

$$(1 + x)\exp(x) = 15.1 \times 10^6. \tag{12}$$

The numerical solution of the above leads to $x = 13.83$ or $V_m = 0.358$ V.

The corresponding J_m is

$$\begin{aligned} J_m &= J_\nu - J_0 \left[\exp\left(\frac{qV_m}{kT}\right) - 1\right] \\ &= 604 - 40 \times 10^{-6} \times \left[\exp(13.83) - 1\right] = 563 \quad \text{A/m}^2. \end{aligned} \tag{13}$$

Since the active area of the diode is 0.01 m^2, the current is 5.63 A, and the load resistance is $I = V_m/I_m = 0.358/5.63 = 0.064 \quad \Omega$.

$\boxed{\text{The optimum load resistance is 64 milliohms,}}$

Solution of Problem 14.29

Prob 14.30

a – Five identical photodiodes are connected in series and feed a single-cell water electrolyzer.

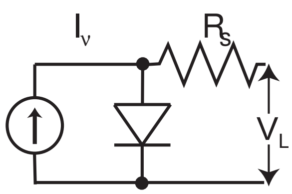
The whole system operates at 300 K, and the photodiode is exposed to a light power density that causes a photon flux of 2.5×10^{21} photons per second per square meter to interact with the device. Quantum efficiency is 100%.

For each photodiode, the reverse saturation current, I_0, is 0.4 μA. and the series resistance is 10 mΩ. The active area of the diode is 10 by 10 cm.

The electrolyzer can be represented by a 1.6 V voltage source, in series with a 100 mΩ resistance.

What is the hydrogen production rate in grams/day?

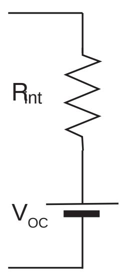
....



The circuit model of a photodiode is the one shown in he figure. When delivering a current, I_L, it will develop a voltage, V_L:

$$V_L = \frac{kT}{q} \ln\left(\frac{I_\nu - I_L}{I_0} + 1\right) - R_s I_L.$$

The voltage delivered by 5 identical photocells in series is, of course,



$$V_{ph} = 5\left[\frac{kT}{q} \ln\left(\frac{I_\nu - I_L}{I_0} + 1\right) - R_s I_L\right].$$

This voltage is applied to an electrolyzer whose circuit model is shown in the next figure.

The combined circuit model has an equation,

$$5\left[\frac{kT}{q} \ln\left(\frac{I_\nu - I_L}{I_0} + 1\right) - R_s I_L\right] = I_L R_{int} + V_{oc}. \tag{4.1}$$

$$I_\nu = qAphi_g = 1.6 \times 10^{-19} \times 10^{-3} \times 2.5 \times 10^{21} = 4 \quad \text{A}.$$

Introducing the given numerical values,

$$5\left[0.026 \ln\left(\frac{4 - I_L}{4 \times 10^{-7}} + 1\right) - 0.01 I_L\right] = 0.1 I_L + 1.6.$$

The only unknown is I_L. A numerical solution reveals that $I_L = 2.43$ A.

Solution of Problem 14.30

The hydrogen production rate, $\dot{N}$, is

$$\dot{N} = \frac{I_L}{q n_e N_0} = \frac{2.43}{1.6 \times 10^{-19} \times 2 \times 6.02 \times 10^{26}} = 1.26 \times 10^{-8} \quad \text{kmoles/s of } H_2.$$

This corresponds to 1.09×10^{-3} kmoles of H_2 per day or 2.2 g H_2 per day.

The production rate is 2.2 grams per day.

---

b – What photon flux is sufficient to just start the electrolysis?

As the photon flux is reduced, the current through the electrolyzer diminishes. The photon flux that causes $I_L \to 0$ is at the limit of hydrogen production.

Setting $I_L = 0$, in Equation 4.1,

$$5\left[\frac{kT}{q} \ln\left(\frac{I_\nu}{I_0} + 1\right)\right] = V_{oc} = 5\left[0.026 \ln\left(\frac{I_\nu}{4 \times 10^{-7}} + 1\right)\right] = 1.6.$$

The solution is $I_\nu = 0.0886$ A.
This leads to

$$\phi_g = \frac{I_\nu}{qA} = \frac{0.0886}{1.6 \times 10^{-19} \times 10^{-3}} = 55 \times 10^{18} \quad \text{photons/s.}$$

The photon flux is 55×10^{18} photons/ s.

Solution of Problem 14.30

Prob 14.31 What is the frequency at which a human body radiates the most heat energy per 1 Hz band width?

Normal body temperature of a human is 37 C or 310 K.
Considered as a black body, the heat radiation peaks at

$$f_{peak} = 59 \times 10^9 T = 59 \times 10^9 \times 310 = 18.3 \quad \text{THz.} \tag{1}$$

The peak of heat radiation from a human body occurs at 18.3 THz.

Solution of Problem 14.31

Prob 14.32 **The ideal photocells can exceed the black body spectrum efficiency if used in a configuration called "cascade". Thus the ideal efficiency of silicon cells exposed to a 6000-K black body radiation is 43.8%. If in cascade with an ideal cell with a 1.8 eV band-gap, the efficiency is 56%. Clearly, using more cells the efficiency goes up further.**

What is the efficiency of a cascade arrangement consisting of an infinite number of cells, the first having a band-gap of 0 eV and each succeeding one having a band-gap infinitesimally higher than that of the preceding cell? The cell with the highest band-gap is on top (nearest the light source).

....

$$\boxed{\text{The efficiency is 100\%.}}$$

Solution of Problem 14.32

Prob 14.33 A high-precision photometer (300 K) equipped with a very narrow band-pass filter made the following measurements:

Light power density in a 1 MHz-wide band centered around 200 THz: 2.0×10^{-11} **W/m^2**

Light power density in a 1 MHz-wide band centered around 300 THz: 2.7×10^{-11} **W/m^2**

Assuming the radiation came from a black body, what is the temperature of the latter?

....

Because the units of $\frac{\partial P}{\partial f}$ are W m^{-2} Hz^{-1}, we have to divide the power densities given (which are per MHz-wide band) by 10^6:

At 200 THz, $\frac{\partial P}{\partial f} = \frac{2.0\times10^{-11}}{10^6} = 2.0 \times 10^{-17}$ W m^{-2} Hz^{-1},

at 300 THz, $\frac{\partial P}{\partial f} = \frac{2.7\times10^{-11}}{10^6} = 2.7 \times 10^{-17}$ W m^{-2} Hz^{-1}. The spectral power density of a black body radiator is

$$\frac{\partial P}{\partial f} = A\frac{f^3}{\exp\left(\frac{hf}{kT}\right) - 1} \tag{1}$$

where A is a parameter that reflect the total intensity (total power density) of the radiation. We can write a pair of simultaneous equations:

$$\begin{cases} A\frac{(200\times10^{12})^3}{\exp\left(\frac{9609}{T}\right)-1} = 2.0 \times 10^{-17} \\ A\frac{(300\times10^{12})^3}{\exp\left(\frac{14,412}{T}\right)-1} = 2.7 \times 10^{-17} \end{cases} \tag{2}$$

The numerical solution of the above leads to $T = 6140$ K.

$\boxed{\text{The temperature of the radiator is 6140 K.}}$

Observe that if only a single photometer measurement were made, then the solution would be undetermined, because for a single equation, any positive value of T will be the correct solution, provided we chose a fitting values of A.

Solution of Problem 14.33

Prob 14.34 A silicon diode, operating at 300 K, is exposed to 6000-K black body radiation with a power density of 1000 W/m^2. Its efficiency is 20% when a load that maximizes power output is used. Estimate the open-circuit voltage delivered by the diode.

....

From Example 14.2 in the text, $\phi_g = 2.49 \times 10^{21}$ photons m^{-2}s^{-1}. Consequently,

$$J_\nu = q\phi_g = 1.6 \times 10^{-19} \times 2.49 \times 10^{21} = 398 \approx 400 \quad \text{A}, \tag{1}$$

and

$$\eta = \frac{P_L}{P_{in}} = \frac{V_m I_m}{P_{in}}, \tag{2}$$

but, $J_m \approx J_\nu \approx 400$ A/m^{-2},

$$0.2 = \frac{400 V_m}{1000} \tag{3}$$

from which

$$V_m = 0.5 \quad \text{V}. \tag{4}$$

Since

$$\left(\frac{qV_m}{kT}\right) = \left(\frac{1.6 \times 10^{-19} \times 0.5}{1.38 \times 10^{-23} \times 300}\right) = 19.3, \tag{5}$$

$$\frac{J_\nu}{J_0} = \left(1 + \frac{qV_m}{kT}\right) \exp\left(\frac{qV_m}{kT}\right) = 20.3 \times \exp(19.3) = 4.89 \times 10^9, \tag{6}$$

$$V_{oc} = \frac{kT}{q} \ln\left(\frac{J_\nu}{J_0}\right) = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln 4.89 \times 10^9 = 0.577 \quad \text{V}. \tag{7}$$

$$\boxed{\text{The open-circuit voltage is 0.577 V.}}$$

....

Another solution:

$$V_{oc} = \frac{kT}{q} \ln\left(\frac{J_\nu}{J_0}\right) \tag{8}$$

We need to find J_0. Use Table 14.5 in the text which shows, in its body, the efficiency of a loss-less photodiode (with a number of different values of J_0) when subjected to a light power density, P, from a 6000-K black body. From the $P = 1000$ W/m^2 row, we obtain $J_0 = 0.26$ nA/m^2 (by interpolation). This leads to $V_{oc} = 0.610$ V.

This result differs from the previous one because, in our first solution, we overestimated J_m by making it equal to J_ν, i.e., equal to 398 A. The second solutions uses (implicitly) the correct value of $J_m = 380.2$ A.

Solution of Problem 14.34

Prob 14.35 A GaAs photodiode, operating at 39 C, is exposed to 5500 K black body radiation with a power density of 675 W/m^2. The open-circuit voltage of the device is 0.46 V. What is the efficiency of the photodiode when delivering energy to a 2-milliohm m^2 load?

....

$$P_{in} = A\left(\frac{kT_s}{h}\right)^4 \frac{\pi^4}{15} = 675. \tag{1}$$

from which, $A = 5.943 \times 10^{-55}$. We will be using T_s as the temperature of the source (5500 K), and T as the temperature of the device (312 K).

The band-gap voltage of GaAs is 1.35 V, hence,

$$\begin{aligned}\phi_g &= \frac{A}{h}\left(\frac{kT_s}{h}\right)^3 \int_{\frac{qV_g}{kT_s}}^{\infty} \frac{x^2}{e^x-1}dx \\ &= \frac{5.943\times10^{-55}}{6.62\times10^{-34}}\left(\frac{1.38\times10^{-23}\times5500}{6.62\times10^{-34}}\right)^3 \int_{2.846}^{\infty}\frac{x^2}{e^x-1}dx. \end{aligned} \tag{2}$$

The value of the integral is (from Appendix A, by interpolation), 0.9376. The flux of photons with energy higher than the GaAs band-gap is $\phi_g = 1.28 \times 10^{21}$ photons m^{-2}s^{-1}. This leads to a short-circuit current of

$$I_\nu = q\phi_g = 1.6\times10^{-19}\times1.28\times10^{21} = 204.8 \quad \text{A/m}^2. \tag{3}$$

The open-circuit voltage is

$$V_{oc} = \frac{kT}{q}\ln\frac{I_\nu}{I_0} = 0.46 \quad \text{V}, \tag{4}$$

$$I_0 = 7.718\times10^{-6} \quad \text{A/m}^2. \tag{5}$$

Using $\Re_L$ as the specific restistance of the load (i.e., the number of ohms per meter square of active diode area), the load voltage is,

$$\begin{aligned} V_L &= \frac{kT}{q}\ln\left(\frac{I_\nu - I_L}{I_0}+1\right) \\ &= \frac{1.38\times10^{-23}\times312}{1.6\times10^{-19}}\ln\left(\frac{204.8-I_L}{3.90\times10^{-6}}+1\right) = \Re_L I_L \end{aligned} \tag{6}$$

$$0.02691\ln\left(\frac{204.8-I_L}{7.718\times10^{-6}}+1\right) = 0.002 I_L \tag{7}$$

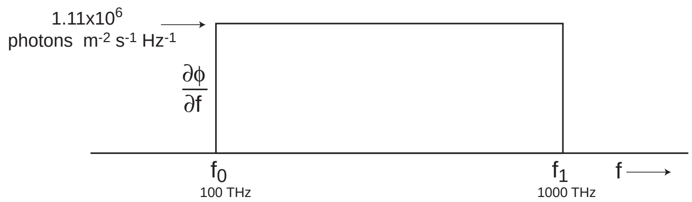
The numerical solution is $I_L = 192.4$ A/m^2. The corresponding load voltage is 0.3845 V and the load power is 73.98 W/m^2. This corresponds to an efficiency of

$$\eta = \frac{73.98}{675} = 0.11. \tag{7}$$

$$\boxed{\text{The efficiency is } 11\%}$$

Solution of Problem 14.35

Prob 14.36



Consider radiation having the spectral distribution above. Observe that the plot is of photon flux versus frequency, not of power density versus frequency as usual. The flux is:

$f < 100$ THz, $\partial\phi/\partial f = 0$,
$100 < f < 1000$ THz, $\partial\phi/\partial f = 1.11 \times 10^6$ photons m^{-2} s^{-1} Hz^{-1},
$f > 1000$ THz, $\partial\phi/\partial f = 0$,

a – What is the total flux of photons?

....

The total flux of photons is

$$\begin{aligned}\phi &= \int_{f_0}^{f_1} \frac{\partial\phi}{\partial f} df = \frac{\partial\phi}{\partial f}\int_{f_0}^{f_1} df = \frac{\partial\phi}{\partial f}(f_1 - f_0)\\ &= 1.11\times10^6(10^{15}-10^{14}) = 10^{21} \quad \text{photons m}^{-2}\ \text{s}^{-1}\ \text{Hz}^{-1} \qquad (1)\end{aligned}$$

$$\boxed{\text{The total flux of photons is } 10^{21} \text{ photons m}^{-2}\ \text{s}^{-1}.}$$

---

b – What is the total power density of the radiation (W/m^2)?

....

$$dP = hf\ d\phi \qquad (2)$$

$$\frac{dP}{df} = hf\ \frac{d\phi}{df} \qquad (3)$$

$$dP = hf\ \frac{\partial\phi}{\partial f}df \qquad (4)$$

$$\begin{aligned}P &= h\frac{\partial\phi}{\partial f}\int_{f_0}^{f_1} f\ df = h\frac{\partial\phi}{\partial f}\times\tfrac{1}{2}\ f^2\Big|_{f_0}^{f_1} = h\frac{\partial\phi}{\partial f}\times\tfrac{1}{2}\times\left(f_1^2-f_0^2\right)\\ &= 6.63\times10^{-34}\times1.11\times10^6\times\tfrac{1}{2}\times\left[\left(10^{15}\right)^2-\left(10^{14}\right)^2\right]\\ &= 325 \quad \text{W/m}^2 \qquad (5)\end{aligned}$$

Solution of Problem 14.36

The total power density is 325 W/m^2.

---

c – What is the ideal efficiency of a diode exposed to the above radiation if its band gap energy is just slightly more than $10^{14}h$ joules? And if it is just a tad less than $10^{15}h$ joules? ....

$$P_L = \frac{\partial \phi}{\partial f}(f_1 - f_g)hf_g = 1.1 \times 10^6 \times (10^{15} - f_g)hf_g \tag{6}$$

For $f_g = f_0 = 10^{14}$,

$$P_L = 1.1 \times 10^6 \times (10^{15} - 10^{14})10^{14} \times 6.63 \times 10^{-34} = 65.6 \quad \text{W.} \tag{7}$$

$$\eta = \frac{65.6}{325} = 0.202. \tag{8}$$

For $f_g = f_0 = 10^{15}$,

$$P_L = 1.1 \times 10^6 \times (10^{15} - 10^{15})10^{14} \times 6.63 \times 10^{-34} = 0 \quad \text{W.} \tag{9}$$

$$\eta = 0. \tag{10}$$

For f_g =100 THz, the efficiency is 20.2%
For f_g =1000 THz, the efficiency is 0.

---

d – What band gap energy causes the ideal photo diode to attain maximum efficiency when illuminated by the radiation we are discussing? What is that efficiency? .... From Equation 6,

$$\begin{aligned}\frac{\partial P_L}{\partial f_g} &= \frac{\partial}{\partial f_g}\left(1.1 \times 10^6 \times 6.631 \times 10^{-34}\right) \times (10^{15} - f_g)f_g \\ &= 7.29 \times 10^{-28}\left(10^{15} - \tfrac{1}{2}f_g\right) = 0\end{aligned} \tag{11}$$

$$f_g = 500 \quad \text{THz.} \tag{12}$$

$$P_L = 1.1 \times 10^6 \times (10^{15} - 500 \times 10^{12}) \times 500 \times 10^{12} \times 6.63 \times 10^{-34} = 182.3 \quad \text{W.} \tag{12}$$

$$\eta = \frac{82.3}{325} = 0.561. \tag{13}$$

A maximum efficiency of 56.1% is achieved
at a band gap of $500 \times 10^{12}h$ joules.

Solution of Problem 14.36

---

e – If it were possible to split the spectrum into two regions, one extending from 100 THz to 500 THz and the other from 500 THz to 1000 THz and if one were to use two independent photodiodes, one with a band gap of 300×10^{12}h joules exposed to the lower of the two bands mentioned, and the other, with a band gap of 750× 10^{12}h exposed to the higher of the two bands mentioned, what would the combined output and efficiency of the system be?

....

The flux of photons between 300 and 500 THz is

$$\phi_{300/500} = 1.11 \times 10^6 \times \left(500 - 300\right) \times 10^{12} = 2.22 \times 10^{20} \quad \text{photons sec}^{-1}\text{m}^2. \tag{14}$$

and between 750 and 1000 THz is

$$\phi_{750/1000} = 1.11 \times 10^6 \times \left(1000 - 750\right) \times 10^{12} = 2.78 \times 10^{20} \quad \text{photons sec}^{-1}\text{m}^2. \tag{15}$$

The output of the photodiode with band gap $300 \times 10^{12}h$ joules, when exposed to the radiation between 100 and 500 THz is

$$\begin{aligned} P_{L_{100/500}} &= \phi_{300/500} W_{g_1} = 2.22 \times 10^{20} \times 300 \times 10^{12} \times 6.63 \times 10^{-34} \\ &= 44 \quad \text{W m}^{-2}. \end{aligned} \tag{16}$$

The output of the photodiode with band gap $750 \times 10^{12}h$ joules, when exposed to the radiation between 500 and 1000 THz is

$$\begin{aligned} P_{L_{500/1000}} &= \phi_{500/1000} W_{g_1} = 2.78 \times 10^{20} \times 750 \times 10^{12} \times 6.63 \times 10^{-34} \\ &= 138 \quad \text{W m}^{-2}. \end{aligned} \tag{17}$$

The combined output is 182 W/m^2. Since the total radiation power density is 364 W m^{-2}.

$$\eta = \frac{182}{364} = 0.50. \tag{18}$$

$$\boxed{\text{The efficiency is 50\%.}}$$

---

f – What is the efficiency of an ideal diode with a band gap energy of 250×10^{12}h when exposed to the radiation under discussion? Then, assume that for each photon with energy equal or larger than 3×250×10^{12}h one additional electron becomes available to the output. What is the efficiency, now? ....

....

Solution of Problem 14.36

Let ϕ_{g_1} be the flux of photons with more energy than $250 \times 10^{12} h$ joules and ϕ_{g_2} be the flux of photons with more than $3 \times 250 \times 10^{12} h$ joules.

$$\phi_{g_1} = 1.11 \times 10^6 \times (1000 - 250) \times 10^{12} = 8.25 \times 10^{20} \quad \text{photons m}^2\text{s}^{-1}. \quad (19)$$

and

$$\phi_{g_2} = 1.11 \times 10^6 \times (1000 - 750) \times 10^{12} = 2.75 \times 10^{20} \quad \text{photons m}^{-2}\text{s}^{-1}. \quad (19)$$

<u>Without carrier multiplication.</u>
The power delivered to the load is

$$P_{L_1} = \phi_{g_1} h f_{g_1} = 8.25 \times 10^{20} \times 6.63 \times 10^{-34} \times 250 \times 10^{12} = 137 \quad \text{W/m}^2. \quad (20)$$

The corresponding efficiency is

$$\eta = \frac{137}{364} = 0.376. \quad (21)$$

<u>With carrier multiplication.</u>
The power delivered to the load is

$$\begin{aligned} P_L &= (\phi_{g_1} + \phi_{g_2}) \times 250 \times 10^{12} h \\ &= (8.25 + 2.75) \times 10^{20} \times 250 \times 10^{12} \times 6.63 \times 10^{-34} \\ &= 182 \quad \text{W m}^{-2}. \end{aligned} \quad (22)$$

The corresponding efficiency is

$$\eta = \frac{182}{364} = 0.50. \quad (23)$$

The efficiency is 37.6% without carrier multiplication, and 50% with carrier muiltiplication.

Solution of Problem 14.36

Prob 14.37 A high-precision photometer (300 K) equipped with a very narrow band-pass filter made the following measurements:

Light power density in a 1 MHz-wide band centered around 200 THz: 2.0×10^{-11} **W/m**^{2}

Light power density in a 1 MHz-wide band centered around 300 THz: 2.7×10^{-11} **W/m**^{2}

This is not black body radiation. What is the short-circuit current density delivered by a silicon photodiode ($V_g = 1.1$ **V) exposed to the radiation passed by the two filters?**

$$\Phi = \frac{P}{hf} \tag{1}$$

$$\Phi_{200} = \frac{2 \times 10^{-11}}{200 \times 10^{12} \times 6.6 \times 10^{-34}} = 152 \times 10^6 \quad \text{photons m}^{-2}\ \text{s}^{-1}. \tag{2}$$

$$\Phi_{300} = \frac{2.7 \times 10^{-11}}{300 \times 10^{12} \times 6.6 \times 10^{-34}} = 136 \times 10^6 \quad \text{photons m}^{-2}\ \text{s}^{-1}. \tag{3}$$

The total flux of photons is $152 + 136 \times 10^6 = 288 \times 10^6$ photons m^{-2} s^{-1}.

The current is

$$J_\nu = q\Phi = 1.6 \times 10^{-19} \times 288 \times 10^6 = 46 \times 10^{-12} \quad \text{A}. \tag{4}$$

$\boxed{\text{The short-circuit current is 46 pA.}}$

Solution of Problem 14.37

Solution of Problem 14.37