EE356 Elementary Plasma Physics H.O. #19

Inan 19 April 2002

Spring 2002

HOMEWORK ASSIGNMENT #3
(due Friday, April 26th)

1. **Short Review Questions:**

   a. *Plasma Models.* Describe, mathematically and qualitatively, the difference between a cold plasma and a warm plasma. What are the assumptions that go into each. When is a plasma termed 'hot' and what would its mathematical model be? [HINT: read Bittencourt pp. 210-212]

   b. *Isothermal Gas.* Calculate the number density of particles at standard temperature and pressure in thermal equilibrium.

   c. *Isothermal vs. Adiabatic.* For the slight pressure perturbation that accompanies an acoustic wave travelling through a plasma, which equation of state is more appropriate in determining the relationship between pressure and number density - the isotherma or adiabatic equation?

2. **Adiabatic Equation.** Solve Problem 9.2 in Bittencourt [p. 236]. How does this compare with the adiabatic equation for an ideal gas (neutral) in 3 dimensions?

3. **Debye Length Revisited.** In Bittencourt Chapter 7, Read section 5 (p. 181) titled 'Equilibrum in the Prescence of an External Force'. This sections gives us the tools to derive the Debye Length. (a) Given a plasma of $N_i = N_e$ and temperature, T, and equation [5.16] of Lecture#5 Notes construct the Poisson equation necessary to solve for the electrostatic potential of the combination of the point charge ,$\frac{Q\delta(r)}{4\pi r^2}$ plus redistributed plasma. (b) What assumptions are necessary to approximate the equation(s) into a linear system? (c) This is a qualitative problem exploring the transient of the debye shielding formation. At time, $t = 0$, when a point charge, $Q\delta(r)$, is introduced the plasma is neutral and hence the electric field is that of free space. The plasma then reacts and the equilibrum fields are finally reached. Show that the amount of stored electrostatic energy for these two times are different. What happened to this difference in electrostatic energy?

4. **Anisotropic plasma.** The distribution of thermal (or peculiar) velocities of particles in a magnetized plasma with $\mathbf{u} = 0$ are given by the following modified Maxwell-Boltzmann distribution function:

$$f(\mathbf{w}) = N \left(\frac{m}{2\pi k_{\mathrm{B}} T_{\perp}} \right) \left(\frac{m}{2\pi k_{\mathrm{B}} T_{\parallel}} \right)^{1/2} \exp \left[\frac{-m}{2k_{\mathrm{B}}} \left(\frac{w_x^2 + w_y^2}{T_{\perp}} + \frac{w_z^2}{T_{\parallel}} \right) \right]$$

where the z-axis is the parallel direction. (a) Show that the number density of particles is given by N. (b) Show that the kinetic pressure tensor $\mathbf{\Psi}$ has only diagonal components, two of which are equal. (c) Show that the average perpendicular energy $\left(\text{i.e., } \frac{1}{2}\, m \langle v_{\perp}^2 \rangle\right)$ is equal to $k_{\mathrm{B}} T_{\perp}$, while the average parallel energy is $\frac{1}{2} k_{\mathrm{B}} T_{\parallel}$.

5. **One dimensional gas.** Consider a gas whose particles are constrained to move in the x-direction. The particle velocities are characterized by a homogeneous velocity distribution function proportional to $\left(v_x^2 + a^2\right)^{-1}$. Find the constant of proportionality in terms of a and the number density N. Find the average kinetic energy. Is this velocity distribution physically realistic?

6. **Particle flux.** Consider a gas of particles consisting of only one species and characterized by the Maxwell-Boltzmann distribution given in equation [6.8] of the Notes. Now consider a unit area perpendicular to the x-axis and show that the fraction of particles that cross this unit area in unit time from one side only, and having velocity components in the ranges dv_x,dv$_y$, and dv_z, is given by

$$\frac{1}{2\pi} \left(\frac{m}{k_{\mathrm{B}} T} \right)^2 v_x \exp \left(\frac{-mv^2}{2k_{\mathrm{B}} T} \right) dv_x \, dv_y \, dv_z$$

You can apply this result to the case of gas particles kept in an oven maintained at a high temperature, with a small orifice on one side through which the particles can escape. Assuming that the particle distribution remains Maxwellian even as particles escape, verify that the number of particles with speed v escaping through unit area of the orifice in unit time is equal to $vN/4$ where n is the number of particles in a unit volume (inside the oven) with speed v. Show that the root mean squared speed of the particles leaving the oven is greater than that of the particles inside the oven by a factor of $\sqrt{4/3}$.