EE359 Discussion Session 3
More Channel Models and Channel Capacity

October 14, 2015
Outline

1. Recap
2. Narrowband, wideband, coherence bandwidth & time
3. Channel capacity
Last discussion session

- Time varying channel
- Modelling multipath in general
- Narrowband assumption
  1. Multipath components overlap, i.e.
     \[ r(t) = \alpha(t) u(t) \]
  2. Characterize real and imaginary components of \( \alpha(t) \) by using CLT and AoA assumptions
  3. Envelope distributions of \( Z = |\alpha(t)| \) and joint second order moments of \( \alpha(t), \alpha(t + \Delta t) \)

This session

- Wrap up narrowband model
- What happens without narrowband assumption?
- Flat/frequency selective fading (cf. fast/slow fading)
- General channel models
- Capacity under channel models
Problem 1

Generalization of Rician fading to two line of sight components
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Difference between narrowband and wideband (time)

Figure: Narrowband $T_m \ll \frac{1}{B_u}$

Figure: Wideband $T_m \approx, \geq \frac{1}{B_u}$

Observations

- Effect of channel no longer modeled by a scalar multiple
- Need to go back to the convolution form

$$r(t) = Re \left\{ \left( \int_{-\infty}^{\infty} c(\tau, t)u(t - \tau)d\tau \right) e^{j2\pi f_c t} \right\}$$

- $c(\tau, t)$ is channel response at time $t$ to an impulse at time $t - \tau$
Difference between narrowband and wideband (frequency)

Figure: Narrowband signal (in red)  Figure: Wideband signal (in red)

Observations

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\[ r(t) = Re \left\{ \left( \int_{-\infty}^{\infty} c(\tau, t) u(t - \tau) d\tau \right) e^{j2\pi f_c t} \right\} \]

- \( c(\tau, t) \) is channel response at time \( t \) to an impulse at time \( t - \tau \)
A closer look at $c(\tau, t)$

Parameters
- $\tau$ represents spatial effects (due to scatterers, e.g.)
- $t$ represents the temporal evolution of the channel
- $c(\tau, t)$ independent of $u(t)$ but may depend on $f_c$

Question
What happens if channel is linear time invariant?

An equivalent quantity
Deterministic scattering function

$$S_c(\tau, \rho) = \mathcal{F}_t(c(\tau, t)) = \int_{-\infty}^{\infty} c(\tau, t)e^{-j2\pi \rho t} dt$$
Hint: Homework 3, Problem 2

- $\tau$ dependence of $S_c(\tau, \rho)$ contains information about delay induced by scatterers/reflectors
- $\rho$ dependence of $S_c(\tau, \rho)$ contains information about doppler shift

Hint: Homework 3, Problem 3

From doppler, find velocity in direction away from transmitter
From deterministic $c(\tau, t)$ to random $c(\tau, t)$

### Source of randomness
- Randomness due to $\tau$: Large number of randomly located scatterers/reflectors
- Randomness due to $t$: Temporal character of the channel (e.g. doppler dependence)

### Modeling randomness
- $c(\tau, t)$ is a Gaussian process in both $\tau$ and $t$
- Completely specified by $A_c(\tau_1, \tau_2, t_1, t_2) = E[c(\tau_1, t_1)c(\tau_2, t_2)]$ for all $\tau_1, \tau_2, t_1, t_2$
Modeling randomness

Some simplifying assumptions

- WSS (wide sense stationary in \( t \)) implies
  \[ A_c(\tau_1, \tau_2, t_1, t_2) = A_c(\tau_1, \tau_2, \Delta t), \text{ where } \Delta t = t_2 - t_1 \]
- US (uncorrelated scattering) implies
  \[ A_c(\tau_1, \tau_2, \Delta t) = A_c(\tau_1, \Delta t)\delta(\tau_1 - \tau_2) \]

Some equivalent quantities

- Statistical scattering function
  \[ S_c(\tau, \rho) = \mathcal{F}_{\Delta t}(A_c(\tau, \Delta t)) \]
- Autocorrelation function of Fourier transforms of \( c(\tau_1, t_1) \) and \( c(\tau_2, t_2) \) w.r.t. \( \tau_1 \) and \( \tau_2 \), i.e. if \( C(f, t) = \mathcal{F}_\tau(c(\tau, t)) \),
  \[ A_C(f_1, f_2, t_1, t_2) \equiv A_C(\Delta f, \Delta t) \text{ under WSSUS} \]
$S_c(\tau, \rho)$ or $A_C(\Delta f, \Delta t)$ in more detail

- $\rho$ or $\Delta t$ captures the doppler or other time varying effects
- $\tau$ or $\Delta f$ captures the delay spread due to scatterers/reflectors

List of things we can derive from $S_c(\tau, \rho)$ or $A_C(\Delta f, \Delta t)$

- Multipath intensity profile (fix $\rho$ and vary $\tau$)
- Doppler power spectrum (fix $\tau$ and vary $\rho$)
Multipath intensity profile $S_c(\tau, \rho = \rho_0)$

Idea
Track energy in multipath component delayed by $\tau$

Delay spread $T_m$ and coherence bandwidth $B_c$
- Single number $T_m$ representing “spread” of multipath
- Coherence bandwidth $B_c \approx \frac{1}{T_m}$

Small $B_c$ corresponds to ...
- Flat/frequency selection fading? (Hint: Is $T_m$ large or small?)
Doppler power spectrum \( S_c(\tau = \tau_0, \rho) \)

**Idea**

Track energy at frequency \( \rho \)

**Doppler spread \( B_D \) and coherence time \( T_c \)**

- Doppler spread \( B_D \) representing how fast/slow the channel variations are
- Coherence time \( T_c \approx \frac{1}{B_D} \)

**Small \( T_c \) corresponds to ...**

- Fast/slow fading?
Takeaway from $S_c(\tau, \rho)$ or $A_C(\Delta f, \Delta t)$

Coherence time and bandwidth
i.e. $T_c$ and $B_c$

- $B_c$ high, flat fading
- $B_c$ low, freq. sel. fading
- $T_c$ low, fast fading
- $T_c$ high, slow fading
Homework 3

Hints for Homework 3, Problem 5

1. Frequency selective fading when doppler $B_D \ll B_u$
2. Rayleigh fading if energy spread uniformly with $\tau$
3. Average fade duration with power $P_r$: $\frac{e^{\rho^2} - 1}{\sqrt{2\pi}f_D\rho}$

Hints for Homework 3, Problem 5

4. Independence when separation in the corresponding domain (time or frequency) is greater than the corresponding coherence time
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## Capacity

### Definition

Maximum data rate that can be sent through channel with vanishing error probability

### Channel models

- **AWGN channel:** \( y[i] = x[i] + n[i], n[i] \sim \mathcal{N}(0, 1), E[x[i]^2] = P \)
- **AWGN with fading:** \( y[i] = \sqrt{\gamma[i]} x[i] + n[i], \gamma[i] \sim \text{fading distribution} \)
- **Multiple users:** Capacity region

### Channel knowledge

- CSIT (channel state info. at transmitter) and CSIR (CSI at receiver)
- CSI distribution known at transmitter or receiver
- CSI information available (fully or partially) at transmitter or receiver
Achieving capacity $C$

\[ C = B \log_2 (1 + \gamma), \]

where $B$ is the bandwidth of the channel and $\gamma = \frac{P}{N_0 B}$ is the SNR.

- Achieved by employing gaussian codebooks with infinite delay
- Need CSI $\gamma$ at both transmitter and receiver
Homework 3, Problem 6

- Use Taylor’s series expansion or similar analysis
- Investigate effect of power and bandwidth on capacity