EE359 Discussion Session 3
More Channel Models and Channel Capacity

October 19, 2016
Outline

1 Recap
2 Narrowband, wideband, coherence bandwidth & time
3 Channel capacity
Last discussion session

- Time varying channel
- Modelling multipath in general
- Narrowband assumption
  1. Multipath components overlap, i.e.
     \[ r(t) = \alpha(t)u(t) \]
  2. Characterize real and imaginary components of \( \alpha(t) \) by using CLT and AoA assumptions
  3. Envelope distributions of \( Z = |\alpha(t)| \) and joint second order moments of \( \alpha(t), \alpha(t + \Delta t) \)

This session

- Wrap up narrowband model
- What happens without narrowband assumption?
- Flat/frequency selective fading (cf. fast/slow fading)
- General channel models
- Capacity under channel models
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Difference between narrowband and wideband (time)

Figure: Narrowband $T_m \ll \frac{1}{B_u}$

Figure: Wideband $T_m \approx, \geq \frac{1}{B_u}$

Observations

- Effect of channel no longer modeled by a scalar multiple
- Need to go back to the convolution form

$$r(t) = Re \left\{ (\int_{-\infty}^{\infty} c(\tau, t)u(t - \tau)d\tau) e^{j2\pi f_c t} \right\}$$

- $c(\tau, t)$ is channel response at time $t$ to an impulse at time $t - \tau$
Difference between narrowband and wideband (frequency)

Figure: Narrowband signal (in red)

Figure: Wideband signal (in red)

Observations

- Effect of channel no longer modeled by a scalar multiple
- Need to go back to the convolution form

\[ r(t) = Re \left\{ \left( \int_{-\infty}^{\infty} c(\tau, t)u(t - \tau)d\tau \right) e^{j2\pi f_c t} \right\} \]

- \( c(\tau, t) \) is channel response at time \( t \) to an impulse at time \( t - \tau \)
A closer look at $c(\tau, t)$

**Parameters**

- $\tau$ represents spatial effects (due to scatterers, e.g.)
- $t$ represents the temporal evolution of the channel
- $c(\tau, t)$ independent of $u(t)$ but may depend on $f_c$

**Question**

What happens if channel is linear time invariant?

**An equivalent quantity**

Deterministic scattering function

$$S_c(\tau, \rho) = \mathcal{F}_t(c(\tau, t)) = \int_{-\infty}^{\infty} c(\tau, t)e^{-j2\pi\rho t} dt$$
Homework 3

Hint: Homework 3, Problem 1
- $\tau$ dependence of $S_c(\tau, \rho)$ contains information about delay induced by scatterers/reflectors
- $\rho$ dependence of $S_c(\tau, \rho)$ contains information about doppler shift

Hint: Homework 3, Problem 2
From doppler, find velocity in direction away from transmitter
From deterministic $c(\tau, t)$ to random $c(\tau, t)$

**Source of randomness**

- Randomness due to $\tau$: Large number of randomly located scatterers/reflectors
- Randomness due to $t$: Temporal character of the channel (e.g. doppler dependence)

**Modeling randomness**

- $c(\tau, t)$ is a Gaussian process in both $\tau$ and $t$
- Completely specified by $A_c(\tau_1, \tau_2, t_1, t_2) = E[c(\tau_1, t_1)c(\tau_2, t_2)]$ for all $\tau_1, \tau_2, t_1, t_2$
Modeling randomness

Some simplifying assumptions

- WSS (wide sense stationary in $t$) implies
  \[ A_c(\tau_1, \tau_2, t_1, t_2) = A_c(\tau_1, \tau_2, \Delta t), \text{ where } \Delta t = t_2 - t_1 \]

- US (uncorrelated scattering) implies
  \[ A_c(\tau_1, \tau_2, \Delta t) = A_c(\tau_1, \Delta t) \delta(\tau_1 - \tau_2) \]

Some equivalent quantities

- Statistical scattering function
  \[ S_c(\tau, \rho) = F_{\Delta t}(A_c(\tau, \Delta t)) \]

- Autocorrelation function of Fourier transforms of $c(\tau_1, t_1)$ and $c(\tau_2, t_2)$ w.r.t. $\tau_1$ and $\tau_2$, i.e. if $C(f, t) = F_\tau(c(\tau, t))$,
  \[ A_C(f_1, f_2, t_1, t_2) \equiv A_C(\Delta f, \Delta t) \text{ under WSSUS} \]
$S_c(\tau, \rho)$ or $A_C(\Delta f, \Delta t)$ in more detail

- $\rho$ or $\Delta t$ captures the doppler or other time varying effects
- $\tau$ or $\Delta f$ captures the delay spread due to scatterers/reflectors

List of things we can derive from $S_c(\tau, \rho)$ or $A_C(\Delta f, \Delta t)$

- Multipath intensity profile (fix $\rho$ and vary $\tau$)
- Doppler power spectrum (fix $\tau$ and vary $\rho$)
Multipath intensity profile \( S_c(\tau, \rho = \rho_0) \)

**Idea**

Track energy in multipath component delayed by \( \tau \)

**Delay spread** \( T_m \) and coherence bandwidth \( B_c \)

- Single number \( T_m \) representing “spread” of multipath
- Coherence bandwidth \( B_c \approx \frac{1}{T_m} \)

**Small** \( B_c \) corresponds to ...

- Flat/frequency selection fading? (Hint: Is \( T_m \) large or small?)
Doppler power spectrum $S_c(\tau = \tau_0, \rho)$

**Idea**
Track energy at frequency $\rho$

**Doppler spread $B_D$ and coherence time $T_c$**
- Doppler spread $B_D$ representing how fast/slow the channel variations are
- Coherence time $T_c \approx \frac{1}{B_D}$

**Small $T_c$ corresponds to ...**
- Fast/slow fading?
Takeaway from $S_c(\tau, \rho)$ or $A_C(\Delta f, \Delta t)$

Coherence time and bandwidth
i.e. $T_c$ and $B_c$

$T_c$ high, slow fading
$B_c$ high, flat fading
$B_c$ low, freq. sel. fading
$T_c$ low, fast fading
$T_c$ high, slow fading
Hints for Homework 3, Problem 3

1. Frequency selective fading when doppler $B_D \ll B_u$
2. Rayleigh fading if energy spread uniformly with $\tau$
3. Average fade duration with power $P_r$: $\frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_D \rho}$

Hints for Homework 3, Problem 4

1. Independence when separation in the corresponding domain (time or frequency) is greater than the corresponding coherence time
Outline

1. Recap

2. Narrowband, wideband, coherence bandwidth & time

3. Channel capacity
Capacity

**Definition**
Maximum data rate that can be sent through channel with vanishing error probability

**Channel models**
- **AWGN channel**: \( y[i] = x[i] + n[i], n[i] \sim \mathcal{N}(0, 1), E[x[i]^2] = P \)
- **AWGN with fading**: \( y[i] = \sqrt{\gamma[i]} x[i] + n[i], \gamma[i] \sim \text{fading distribution} \)
- Multiple users: **Capacity region**

**Channel knowledge**
- CSIT (channel state info. at transmitter) and CSIR (CSI at receiver)
- CSI distribution known at transmitter or receiver
- CSI information available (fully or partially) at transmitter or receiver
Achieving capacity \( C \)

\[
C = B \log_2(1 + \gamma),
\]

where \( B \) is the bandwidth of the channel and \( \gamma = \frac{P}{N_0 B} \) is the SNR.

- Achieved by employing gaussian codebooks with infinite delay
- Need CSI \( \gamma \) at both transmitter and receiver
## Homework 3, Problem 5
- Use Taylor’s series expansion or similar analysis
- Investigate effect of power and bandwidth on capacity

## Homework 3, Problem 6
Use the AWGN capacity formula to compute an achievable multiuser rate pair. Recognizing signal and noise power is key.