Recap

Wideband coherence B/W is time

Channel capacity

\[ r(t) = R \left\{ \left( \sum_{n=1}^{\infty} a_n e^{-j2\pi f_n t} - b_n \right) u(t - T_n) e^{-j2\pi f_n t} \right\} \]

Narrowband

\[ u(t - T_n) \approx u(t) \quad T_n \ll \frac{1}{f_n} \]

\[ r(t) = \alpha(t) u(t) \]

\[ \downarrow \text{complex scalar} \]

\[ \mathbf{N}(t) \text{ is large, CLT applicable.} \]

\[ f_n T_n \gg \Phi_n \]

\[ \mathbf{N}(t) \rightarrow \text{gaussian.} \]

\[ \mathbf{E} \mathbf{N}(t), \quad \mathbf{E} \mathbf{N}(t) \mathbf{N}(t)^+ \]

Further assignments.
On $\tilde{\mathbb{R}} \cup (0, 2\pi]$, $\Theta_n, \Xi_n$ for very slowly (practically constant)

$$E[R(\Xi_n) R[\lambda (4+\tau)]]$$

$$|Z| = |\lambda (4)|, \quad |Z|^2 = |\lambda (4)|^2.$$
Narrowband

\[ T_m \approx T_u^{-1} \]

\[ \lessapprox T_u \approx B_u^{-1} \]

Wideband fading.

\[ T_m \approx T_u \approx B_u^{-1} \]
Observations:

1. Effect is not multiplication by scalar.

2. Convolution:

\[ s_2(t) \ast f(t) = \int_{-\infty}^{\infty} s_2(\tau) f(t-\tau) d\tau \]

- Channel response \( c(\tau, t) \)

- Key concept:

  - Multipath / scatterers / delay spread
  - Temporal variations (Doppler)

Deterministic channel:

\[ c(\tau, t) \]

\[ s_2(\tau, z) = \int c(\tau, t) dt \]

Scattering
Random channels

$$(\tau, t) \rightarrow \text{gaussian process in } \tau, t.$$ 

What's required: $\mathbb{E} \{ c(\tau, t) \} = 0 \text{ mean }$

$$A_c(\tau, t, t_1, t_2) = \mathbb{E} \left\{ c(\tau_1, t_1) c(\tau_2, t_2) \right\} \neq 0 \quad t_1 \neq t_2,$$

2 simplifying assumptions.

1) $c(\tau, t)$ is wide sense stationary in $t$.

$$A_c(\tau_1, \tau_2, t, t_1, t_2, \Delta t) = A_c(\tau_1, \tau_2, t, \Delta t)$$

2) Uncorrelated scattering.

$$A_c(\tau_1, \tau_2, \Delta t) = A_c(\tau_1, \Delta t) A_c(\tau_2, \Delta t)^\dagger$$

$$[c(\tau, t) \text{ is not stationary in } \tau]$$

$$[\text{Different multipath components}\]$$

$\Delta t \rightarrow 0$ characterization.
\[ C(f,t) = \mathcal{F}_t [c(\tau, t)] \]

\[ A_c(f_1, f_2, \Delta t) = \mathcal{F}_t [C(f_1, t) (\Delta f, \Delta t)] \]

\[ = A_c(\Delta f, \Delta t) \]

What does \( S_c(\Delta f, \Delta t) \) or \( A_c(\Delta f, \Delta t) \) say?

- **Temporal variation / doppler** \( \frac{\Delta f}{\Delta t} \)
- **Delay spread** \( \Delta f / \Delta t \)
- **Multipath intensity profile**: Fix \( \frac{\Delta f}{\Delta t} \), study \( \Delta f \)
- **Doppler power spectrum**: Fix \( \Delta f, \) study \( \frac{f}{\Delta t} \)
Coherence bandwidth.

Delay spread $T_m$ var. $\sim \frac{1}{B_c}$

Coherence b/w $\leq B_c \approx T_m^{-1}$

[Signal separated by more than $B_c$ experience independent fading]

Small $B_c$ (coherence b/w)

$T_m \rightarrow \text{large}$.

Frequency selectivity

Coherence time.

$\Sigma_c (2 = 0, 5)$

$\frac{B_n}{\text{Doppler spread}}$
Coherence time \( T_c \approx B_d^{-1} \) (kHz)

Small \( T_c \) (coherence time)

Large Doppler spread.

Changes changes rapidly (fast-fading)

Coherence \( B_r \)

\( B_r \) [fast fading]

[slow fading]

Coherence time \( T_c \)

A 2 \( \rightarrow \) Deterministic channel - delay spread/signal BW.
$A_3 \rightarrow$ looking at doppler of components, find velocity of receiver moving away

$\phi_4 \rightarrow (f), (g)$

Level crossing.

$\begin{pmatrix} 121, \frac{d}{dt} |_{21} \end{pmatrix}$

Avg fade duration.

$$\frac{e^{-1}}{\sqrt{2\pi} \int_0^f}$$

$$\approx \frac{\beta_D}{\beta/2}$$

Q.5: signals $\beta_D$ — independent fading.

$T_c$ — independent fading.

---

**Capacity**

**Defn:** Maximum data rate that can be transmitted with vanishing error through a wireless channel.
Channel models

AWGN: \[ y_i = x_i + n_i, \quad n_i \sim \mathcal{N}(0,1) \]
\[ E_{n_i}^2 = P. \]

Fading: \[ y_i = \sqrt{r_i} x_i + n_i; \]
\[ r_i \sim \text{fading distribution}. \]

Multi-user.

\[ r_1 \quad r_1 \]
\[ R_1 \quad R_2 \]
\[ r_2 \quad R_2 \]

With the knowledge of the channel:

+ Receiver / Transmitter know distribution

+ Receiver knows CSI

+ Receiver knows CSI

\[ P(x_i) \quad P(y_i|x_i) \]
\[ x_i \quad \rightarrow \quad y_i \]
\[ C = B \log_2 \left( 1 + \frac{P_e}{N_0 B} \right) \]

\( \downarrow \) bandwidth

\( \downarrow \) SNR

\( P_e \rightarrow \text{received power} \)

\( N_0 B \rightarrow \text{noise power} \)

Q6 \rightarrow Taylor series expansions.

\[
\begin{align*}
\log (1 + \text{large}) & \approx \log \text{large} \\
\log_e (1 + \text{small}) & \approx \frac{\text{small} - \text{small}}{2}
\end{align*}
\]