EE 359: DISCUSSION SESSION 9

FINAL REVIEW

OFDM

Spread spectrum

Final review

Multi-carrier Modulation

+ ISI combating, $R_S \leq R_c$, $N$ sub-carriers

+ Guard/buffers: reduce spectral efficiency

+ Orthogonal sub-carriers less vulnerable to timing/frequency offset

OFDM

DFT:

$X[k] = \sum_{n=0}^{N-1} x[n] e^{-2\pi \frac{kn}{N}} = \sum_{n=0}^{N-1} x[n] \omega^{nk}$

$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \omega^{nk}$

$X = Q_\omega X \\
(Q_\omega)^{-1} = \omega^l$

$X = Q_\omega^{-1} X$

Complexity $O(\log N)$ FFT algorithm

OFDM
The text on the page includes mathematical expressions and diagrams, potentially related to signal processing or linear algebra. Here are the key points:

1. **Cyclic prefix**: Cyclic prefix is a form of prefixing data in which the last part of the data is shifted to the front. It is used to mitigate the effects of frequency-selective fading in communication systems.

2. **Convolution Equation**: The equation given is $y[n] = x[n] + h[n] + v[n]$, where $y[n]$ is the output, $x[n]$ is the input, $h[n]$ is the impulse response, and $v[n]$ is the noise. This equation describes a linear time-invariant system where $y[n]$ is the output at time $n$.

3. **Linear Convolution**: The equation $y[n] = \sum_{k=0}^{\mu} h[k] \cdot x[n-k]$ represents linear convolution, where $h[k]$ is the impulse response and $x[n-k]$ is the input delayed by $k$ samples.

4. **Matrix Representation**: The text mentions the matrix representation, which is a method to represent linear systems using matrices, making it easier to solve systems of linear equations.

The diagrams illustrate these concepts with symbols and notations typical of engineering and mathematics.
Matrix representation:

\[ y = Hx + \gamma \]

\[ y = \begin{bmatrix} y_1 & y_2 & \cdots & y_n \end{bmatrix} \]

\[ x = Q_w^{-1} \begin{bmatrix} \text{[IFFT]} \\ \text{transmit} \end{bmatrix} \]

\[ y = Q_w y \begin{bmatrix} \text{[FFT receive]} \end{bmatrix} \]

\[ y = \sum X + \nu \begin{bmatrix} \text{[IFFT]} \\ \text{transmit} \end{bmatrix} \]

\[ H = Q_w \sum Q_w^H \begin{bmatrix} \text{diag}(\text{freq}) \end{bmatrix} \]

\[ \tilde{H} = \begin{bmatrix} \text{freq} \end{bmatrix} \]

\[ y = \text{H} \nu \]

\[ y = \text{H} x \]

Cons:

1. High peak to average power ratio:

[Graph showing non-linear characteristic]

\[ \text{PAPR} = \max_n \left| x(n) \right|^2 \]

\[ \text{E}_n \left| x(n) \right|^2 \]

[FFT diagram]
2) Timing & Frequency offset

SPREAD SPECTRUM

Spread a narrowband signal across a wide bandwidth. A -

- symbols indistinguishable from noise
- ISI + narrowband interference rejection
- Adaptive resource for multi-user comm.

DSSS

- Spreading code (chip sequence) $s_c(t)$ $\left[ B_s = \frac{1}{T_c} \right]$

- To multiply with our $g(t) \left[ B_s = \frac{1}{T_c} \text{ narrow band} \right]$

$T_c \ll T_s \quad \Rightarrow \quad \frac{T_c}{T_s}$ spreading

$g(t)$

$S_c(t)g(t)$

Recipient $\left( \frac{1}{T_c} \right)$

Receiver $g_r(t) = \frac{1}{T_s} \int_{t}^{t+T_s} m(t) s_c(t) \, dt$

$g_r(t)$ in time domain
Properties of \( g(t) \)

- Multipath rejection

\[ \int g(t) s_2(t - \tau) dt = s(t) \]

- Interference cancellation

\[ \int g_c(t) s_c(t) dt = s(t) \]

NARROW BAND INTERFERENCE REJECTION

[Diagram of signal and interference]

Interference reduced by spreading

\[ |S| \] rejection

Channel \( \chi(t) = s(t) + s(t - \tau) \)

\[ \chi(t) = s(t) \left[ s_2(t) + s_c(t - \tau) \right] \]

\[ \frac{1}{T_s} \int_0^{T_s} \chi(t) s(t - \tau) dt = \frac{1}{T_s} \int_0^{T_s} s(t) s(t - \tau) dt = \int_0^{T_s} s(t) s(t - \tau) \]
Large support of $S(t)$ is less to be synchronized.

Maximal linear codes

$$S_c(t) = \begin{cases} \frac{1}{2} \frac{1}{T_c} \left( 1 - \frac{N-1}{N} \right) T_c & 0 < t < T_c \\ \frac{1}{2} T_c & \text{else} \end{cases}$$

periodic \( N T_c \), \( N > T_c / T_c \)

Rate receivers

Use long spreading codes (CDMA). Each receiver can resolve multiple components.

- Rate receivers gather energy from each multipath component (synchronized to different delay components).
- SC, MRC - make use of all multiple components.

\underline{REVIEW}

1) Channel models
- Path loss
- Shadowing
- Fading

2) Performance analyses
- Capacity
2) Combating fading - diversity
3) Reading capacity - adaptive modulation.

5) MIMO
   - represent + capacity analysis
   - beam forming
   - diversity / multiplexing tradeoffs
   - MIMO receivers.

6) Combating ISI
   - OFDM
   - spread spectrum

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### Channel models

#### Path loss

- Attenuation of EM caused by diffraction due to finite transmitter/receiver size
- Free space path loss
- $2 - \lambda$ path loss
- Simplified path loss

$$\frac{P_a}{P_b} = K \left( \frac{d_b}{d_t} \right)^\gamma$$

called a 

- Shadowing

Attenuation caused by EM waves passing through buildings / trees.

$$P_{\text{dB}}(x) = P_{\text{a}}(x) + S_{\text{dB}}(x)$$

$\mathcal{N}(0, \sigma^2)$
\[ P_{\text{dB}}(N) = P_{\text{dB}}(N_0) + \frac{20 \log_{10} N}{\text{pm}^2} \]

\[ N(0, \frac{\text{pm}^2}{10^3}) \]

Shading and close positions correlated
\[(X_c - \text{decreases}) \text{ dist.}\]

**Fading models**

Attenuation caused by multiple components:

Adding up:

**Narrow band fading**

\[ \text{Rayleigh, Rician, ...} \]

**Wideband fading**

\[ c(x, t) \text{ [channel]} \]

\[ w_{SS} \text{ [channel stationary with } \text{Rician]} + US \text{ [component with delay } \tau \text{ are independent]} \]

Multipath intensity profile: \( A_c(\tau) \)

Doppler power spectrum: \( S_c(f) \)

\[ f / \tau \rightarrow \text{multipath components} \]

\[ S / t \rightarrow \text{doppler effects} \]

\[ \text{flat fading} (\text{no } 1/2) \text{ normalized} \]
2) Performance analysis

\[ C = \frac{B \log_2 (1 + \frac{P_c}{N})}{\text{data rate that can be transmitted}} \]

\[ P_c \rightarrow 0 \]

\[ \text{Mean} \]

\[ B \log_2 \left( \frac{1 + \frac{P_c}{N}}{e} \right) \]

\[ \text{Fading with CSI-R} \]

\[ B \int_0^\infty \log_2 (1 + \frac{P_c}{N}) f_Y(y) dy \]

\[ \text{Outage capacity} \]

\[ B \log_2 \left( \frac{1 + \frac{P_c}{N}}{e} \right) P(Y > Y_0) \]

\[ \text{Fading with CSI-K} \]

\[ B \int_0^\infty \log_2 \left( \frac{1 + \frac{P_c}{N}}{e} \right) f_Y(y) dy \]

\[ \text{No power adaptation} \]

\[ B \int_0^\infty \log_2 \left( \frac{1 + \frac{P_c}{N}}{e} \right) f_Y(y) dy \]

\[ \text{Waterfilling solution} \]

\[ B \left( \int_0^\infty \log_2 \left( \frac{1 + \frac{P_c}{N}}{e} \right) f_Y(y) dy \right) \]

\[ \text{No rate adaptation} \]

\[ B \log_2 \left( \frac{1 + \frac{P_c}{N}}{e} \right) \]

\[ \text{[channel inversion]} \]

\[ B \log_2 \left( 1 + \frac{1}{e} \right) \]

\[ \text{Truncated channel inversion} \]

\[ B \log_2 \left( 1 + \frac{1}{e} \left( \frac{1}{e} \right)^{10^{-3}} \right) \]

\[ \text{Output probability} \]

\[ P_o = \text{required rate falls below threshold} = P_c > P_{\text{desired}} \]

Due to - path loss (no shadowing)

\[ \text{shadowing} \]

\[ \text{indoors (slow fading)} \]
shading fading (also fading)

AVERAGE PROBABILITY OF ERROR

\[
\bar{P}_s = \mathbb{E} \, P_s(r)
\]

\[
P_s(r) = \frac{\lambda}{\pi} \mathcal{Q}\left(\sqrt{\frac{k}{2\pi} r}\right)
\]

\[
\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt
\]

\[
P_s(r) = \frac{\lambda}{\pi} \mathcal{Q}\left(\sqrt{\frac{k}{2\pi} r}\right)
\]

| $T_s \ll T_c$ | outage probability relevant
| $T_s \approx T_c$ | outage probability relevant
| $T_s \gg T_c$ | outage probability relevant

Combined $P_{out} + \bar{P}_s$

When shading causes outage:

\[
Y_s \quad \bar{P}_s \quad (\text{any other fading})
\]

\[
Y_0 : \quad \bar{P}_s > P_{\text{out-off}} \quad [\bar{P}_s = \frac{1}{2} \bar{P}_s]
\]

Point:

\[
\mathbb{P}(Y_s < Y_0) \quad \text{[Normal fading]}
\]

When fading causes outage:

\[
Y_s \quad \bar{P}_s \quad (\text{any other fading})
\]

\[
Y_0 : \quad P_s > P_{\text{out-off}} \quad [\frac{e^{-r}}{2} \text{ right}]
\]

Point:

\[
\mathbb{P}(Y_s < Y_0) \quad [e^{-Y_s}]\]
$$p_s \rightarrow 0 \quad \text{as} \quad R \rightarrow \infty$$

1) Data rate is too low with differential modulation
   - doppler (-7c)
   - $p_{floor} \approx \frac{1}{2} (\frac{R}{\sigma c})^2$

2) Data rate is too high and low in US
   - $p_{floor} \approx \left( \frac{\bar{x}_m}{T_{s}} \right)^2$

**Comparing FADING - DIVERSITY**

+ Independent fading realizations can reduce $p_s / P_{out}$

\[ \gamma_s = \max_i \gamma_i \] \quad $P_{out,i} = P_{out,1}$

+ SLC

\[ Y_s = \sum_i R_i \quad M_{I_1} (\cdot) \cdot M_{R_i} (\cdot) \quad \bar{P}_{s, Y_s} = \mathbb{d} (P_s, Y_s) \]

+ Diversity gain + array (code) gain.

**Adaptive Modulation**

- Adapt rate, power (coding, symbol time, etc) depending on CSI T

- can be used when symbol does not
\[
\begin{aligned}
\text{change rapidly} \quad & \quad \frac{1}{\text{MAN}} \quad P_b = 2 e^{-\frac{1}{x^2}} \Rightarrow M = 1 + KR \\
& \\
\quad K \uparrow \quad P_b \uparrow \quad M (\text{modulation}) \uparrow \\
& \\
K = 1 \quad \Rightarrow \quad \text{SE} = \log_2 M = C \quad P_b \neq 0 \\
& \quad \Rightarrow \log_2 (1 + KR) \\
\text{variable rate, variable power} \quad & \quad \text{maximize SE} \quad \text{s.t.} \quad \text{maximize rate} \\
\text{variable power} \quad & \quad \text{rate} \\
& \\
\text{discrete rate, variable power} \quad & \\
\text{Heuristic:} \quad & \quad M_i \leq \frac{r}{\frac{1}{K}} \leq M_{i+1} \\
\quad \Rightarrow \quad M(r) = M_i \\
& \\
\frac{P(r)}{P} = \frac{M(r) - 1}{\frac{r}{K}} \\
\text{Rate} \quad & \quad \text{Power} \\
\end{aligned}
\]
\[
\begin{bmatrix}
\vdots \\
Y_1 \\
\vdots \\
Y_{M_n}
\end{bmatrix}
= 
\begin{bmatrix}
\vdots \\
H_{11} & \cdots & H_{1M_p} \\
\vdots \\
H_{M_n1} & \cdots & H_{M_nM_p}
\end{bmatrix}
\begin{bmatrix}
\vdots \\
X_1 \\
\vdots \\
X_{M_p}
\end{bmatrix}
+ 
\begin{bmatrix}
\vdots \\
N_1 \\
\vdots \\
N_{M_p}
\end{bmatrix}
\]

receive antennas 
\(Y\) 
\(Y_{ij} = i^{th} \text{ receive } \) 
\(H_{ij} = i^{th} \text{ transmit} \)

\[Y = HX + N\]

\[\sum_{i=1}^{M_t} |x_i|^2 = \|X\|^2 \leq P \quad \text{(Transmit power constraint)}\]

Parallel decomposition

\[H = UV^H\]

\[X = V^HY\quad \text{tranmit precoding}\]

\[\tilde{Y} = UY\quad \text{receive shaping}\]

\[\tilde{Y} = \tilde{X} + \tilde{N}\]

\[r = \min(M_n, M_p)\]

CSIR \& CSIR - water filling

CSIR - ergodic capacity

Beamforming

Combine signals from antennas to use a single channel with high SNR

\[\tilde{Y} = \bar{H} [N, \tilde{X} + \tilde{N}]\]

Capacity optimal if \(\tilde{Y} \rightarrow X_1 \gg \lambda_2 \cdots \lambda_r\)

\[
\begin{bmatrix}
\lambda_1 & \\ & \ddots \\
& & \lambda_r
\end{bmatrix}
\]

\[\sum_{i=1}^{M_n} |x_i|^2 \leq P \quad \text{(Transmit power constraint)}\]

Diversity / Multiplexing trade off
Diversity / Multiplexing Trade-off

Multiplexing gain: \[ \alpha = \lim_{R \to \infty} \frac{\text{Rate}}{\log Y} \]
Diversity gain: \[ d = \lim_{R \to \infty} \frac{\log P_e}{\log \alpha} \]

\[ d = (M_0 - \lambda) (M_0 - \lambda) \]
\[ \lambda = 0 \]
\[ d = M_0 M_0 \]
High reliability: \[ \lambda = \min (M_0, M_0) \]
Highest rate: \[ d = 0 \]

MIMO receivers:
\[ X_i \in \mathbb{C} \quad X \sim (\mathbb{C}_i, \mathbb{C}) \]
\[ X \in \mathbb{C}^{M_0} \]

ML:
\[ X_{ML} = \arg\max_{X \in \mathbb{C}^{M_0}} P(Y | X, H) \]
\[ = \arg\min_{X \in \mathbb{C}^{M_0}} \| Y - H X \|_2^2 \]

ZF:
\[ X_{ZF} = \arg\min_{X \in \mathbb{C}^{M_0}} \| H X - Y \|_2^2 \]
\[ \Rightarrow X_{ZF} : \mathbb{H}^4 \text{ mapped to a point in the constellation.} \]

MMSE:
\[ X_{MMSE} = \arg\min_{X \in \mathbb{C}^{M_0}} \mathbb{E} \left[ \| X - Y \|_2^2 \right] \]
\[ E \left[ x \mid y \right] = \left( \frac{1}{n} \sum_{i=1}^{n} x_i \right) + \frac{1}{n} y \]

\( y_{	ext{linear}} = Ay + b \)

\( (A, b) = \argmin_{A, b} \frac{1}{n} \sum_{i=1}^{n} (Ay_i + b - x_i)^2 \)

\( y_{	ext{linear}} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2 \cdot y \)

**Sphere decoder:** \( y_{	ext{sphere}} = \argmin_{x} \| y - x \| \_2 \)

\( x \in \mathcal{X} \)

\( \mathcal{X} \subseteq \mathbb{R}^m \)

\( y - x > 0 \)

\( \lambda \uparrow \) complexity \( \uparrow \) (close to \( ML \))

\( \lambda \) too small = use.

\( \| y - \hat{x} \| \_2 = \sqrt{\sum_{i=1}^{m} (y_i - \hat{x}_i)^2} \)

\( \hat{x} = \argmin_{x} \sum_{i=1}^{m} (y_i - x_i)^2 \)

\( x \) in plane.