1. (20 pts) Generalized Two Ray fading model: The Generalized Two Ray (GTR) fading model assumes that the received signal consists of two dominant components and many other low power diffuse multipath components. The received signal $V_r$ can be modeled as,

$$V_r = V_1 e^{j\phi_1} + V_2 e^{j\phi_2} + X + jY,$$

where $V_1$ and $V_2$ are non-negative constants, $\phi_1, \phi_2 \sim \mathcal{U}[0, 2\pi]$ or the phases of the dominant components are uniformly distributed, and $X, Y \sim \mathcal{N}(0, \sigma^2)$ or the diffuse components arises from a complex Gaussian distribution. Note that the GTR model reduced to the Rician fading model if $V_1 > 0$, $V_2 = 0$ and Rayleigh fading if $V_1 = V_2 = 0$. The model is parametrized in terms of $K, \Delta$ where,

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2},$$

$$\Delta = \frac{2V_1V_2}{V_1^2 + V_2^2}.$$

(a) (5 pts) Find the range of parameters $K$ and $\Delta$.

(b) (5 pts) Interpret the parameters $K$ and $\Delta$, i.e., describe the fading behavior modeled when,

- $K \to \infty, \Delta \to 0$
- $K \to \infty, \Delta \to 1$
- $K \to 0$
- $K > 0, \Delta \to 0$

(c) (5 pts) Show that the received signal given that the phase difference between the 2 dominant components is constant,i.e. $\phi_1 - \phi_2 = \alpha$ consists of one dominant component and the multiple diffuse low power multipath components. This implies that the GTR model reduces to the Rician fading model when the phase difference between the LoS components in constant. Find the parameter $\bar{K}$ of the equivalent Rician model as a function of $\alpha, \Delta$ and $K$.

(d) (5 pts) Explain why $\alpha \sim \mathcal{U}[0, 2\pi]$ for the GTR fading model. Use this to justify why,

$$f_{GTR}(r \mid K, \Delta) = \frac{1}{2\pi} \int_0^{2\pi} f_{Rice}(r \mid \bar{K}(\alpha, \Delta, K)) d\alpha,$$

where $f_H(r \mid \Sigma)$ is the pdf of the received signal amplitude of channel fading model $H$ with parameters $\Sigma$. 

Due: Monday (October 19), 5 pm
2. (20 pts) **Multipath and narrowband approximation in wireless channels**: In this problem we examine the narrowband approximation in the two ray model. Consider the two ray model discussed in Section 2.4.1 of the text with reflection coefficient $R = -1$, free space path loss for each ray and carrier frequency 2400 MHz. Both the transmitter and the receiver are 40 m high.

   (a) (5 pts) For a separation distance of 20 m, find the impulse response of the equivalent baseband channel in the time domain. You may want to review the Appendix A of the textbook for a description of the equivalent baseband channel.

   (b) (5 pts) Repeat the computation of the impulse response for a separation distance of 2000 m.

   (c) (10 pts) Plot the channel output in the baseband in each of the above two channels, with $\sin(\pi ft) / \pi ft$ being the channel input, and $f = 20$ MHz. What is the delay spread $T_m$ of both channels and does $T_m$ imply presence of ISI or no ISI? Find the coherence bandwidth of the channel in each case and whether this bandwidth implies flat or frequency selective fading.

3. (10 pts) **Channel scattering function** (Problem 3-15): Consider the following channel scattering function obtained by sending a 900 MHz sinusoidal input into the channel:

   \[
   S(\tau, \rho) = \begin{cases} 
   \alpha_1 \delta(\tau) & \rho = 70\text{Hz}, \\
   \alpha_2 \delta(\tau - 0.022\mu s) & \rho = 49.5\text{Hz}, \\
   0 & \text{else},
   \end{cases}
   \]

   where $\alpha_1$ and $\alpha_2$ are determined by path loss, shadowing, and multipath fading. Clearly this scattering function corresponds to a two-ray model. Assume the transmitter and receiver used to send and receive the sinusoid are located 8 m above the ground.

   (a) (5 pts) Find the distance and velocity between the transmitter and receiver.

   (b) (5 pts) For the distance computed in the previous part, is the path loss as a function of distance proportional to $d^{-2}$ or $d^{-4}$? Hint: Use the fact that the channel is based on a two-ray model.

4. (15 pts) **Wideband autocorrelation function** (Problem 3-16): Consider a wideband channel characterized by the autocorrelation function

   \[
   A_c(\tau, \Delta t) = \begin{cases} 
   \text{sinc}(W \Delta t) & 0 \leq \tau \leq 10\mu s \\
   0 & \text{else},
   \end{cases}
   \]

   where $W = 100$ Hz and $\text{sinc}(x) = \sin(\pi x) / \pi x$.

   (a) (1 pt) Does this channel correspond to an indoor channel or an outdoor channel, and why?

   (b) (2 pts) Sketch the scattering function of this channel.

   (c) (3 pts) Compute the channel’s average delay spread, rms delay spread, and Doppler spread.

   (d) (1 pt) Over approximately what range of data rates will a signal transmitted via this channel exhibit frequency-selective fading?

   (e) (2 pt) Would you expect this channel to exhibit Rayleigh or rather Rician fading statistics? Why?

   (f) (1 pt) Assuming that the channel exhibits Rayleigh fading, what is the average length of time that the signal power is continuously below its average value?

   (g) (5 pts) Assume a system with narrowband binary modulation sent over this channel. Your system has error correction coding that can correct two simultaneous bit errors. Assume also that you always make an error if the received signal power is below its average value and that you never make an error if this power is at or above its average value. If the channel is Rayleigh fading, then what is the maximum data rate that can be sent over this channel with error-free transmission? Make the approximation that the fade duration never exceeds twice its average value.
5. (15 pts) **Scattering function** (Problem 3-17): Let a scattering function \( S_c(\tau, \rho) \) be nonzero over \( 0 \leq \tau \leq .1 \) ms and \(-.1 \leq \rho \leq .1 \) Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

   (a) (2 pts) What are the multipath spread and the Doppler spread of the channel?

   (b) (3 pts) Suppose you input to this channel two identical sinusoids separated in frequency by \( \Delta f \). What is the minimum value of \( \Delta f \) for which the channel response to the first sinusoid is approximately independent of the channel response to the second sinusoid?

   (c) (5 pts) For two sinusoidal inputs to the channel \( u_1(t) = \sin 2\pi ft \) and \( u_2(t) = \sin 2\pi f(t + \Delta t) \), find the minimum value of \( \Delta t \) for which the channel response to \( u_1(t) \) is approximately independent of the channel response to \( u_2(t) \).

   (d) (5 pts) Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3-kHz bandwidth? For a cellular channel with a 30-kHz bandwidth?

6. (20 pts) **Power and bandwidth limited regimes in point-to-point AWGN**: Capacity in AWGN is given by \( C = B \log_2 \left( 1 + \frac{P}{N_0 B} \right) \), where \( P \) is the received signal power, \( B \) is the signal bandwidth, and \( N_0/2 \) is the noise PSD (for a white noise process this is constant).

   (a) (10 pts) (Problem 4-1): Find capacity in the limit of infinite bandwidth \( B \to \infty \) as a function of \( P \).

   (b) (5 pts) Write the first order approximation of capacity as \( P \) becomes small. This is the power limited regime; does doubling bandwidth affect capacity here?

   (c) (5 pts) For a large \( P \), how does the capacity scale with increasing \( P \)? This is the bandwidth limited regime; does doubling the bandwidth affect the capacity here?