Final Exam Announcements

- Final 12/7/15 3:30-6:30pm in Thornton 102 (here)
- Covers Chapters 9.1-9.3.4, 10.1-10.5, 10.7, 12, 13.1-13.4, 14.1-14.2 (plus pre-MT material)
- Similar format to MT, but longer: open book, notes.
  - If you need a book or calculator, let us know by 12/3/14
- Practice finals posted (10 bonus points)
  - Turn in for solns, by exam for bonus pts
- Final review and discussion section: Thur 12/3 1:30-3pm, Packard 364.
OHs leading up to final exam

- **Mine**
  - Sat: 12/5 2:30-4pm
  - Mon: 12/7 10-11:30am

- **TAs:**
  - Thur 3-4 pm in Packard 364
  - Fri 2-3 pm in Packard 109, 3-4 pm outside Packard 340
  - Sat 4-5 pm outside Packard 340
  - Sun 1-2 pm outside Packard 340
Course Summary

- Signal Propagation and Channel Models
- Modulation and Performance Metrics
- Impact of Channel on Performance
- Fundamental Capacity Limits
- Flat Fading Mitigation
  - Diversity
  - Adaptive Modulation
- ISI Mitigation
  - Equalization (not covered)
  - Multicarrier Modulation/OFDM
  - Spread Spectrum
- Multiuser Systems
  - Time/frequency/code/space division
Future Wireless Networks

Ubiquitous Communication Among People and Devices

Wireless Internet access
Nth generation Cellular
Wireless Ad Hoc Networks
Sensor Networks
Wireless Entertainment
Smart Homes/Spaces
Automated Highways
All this and more…

- Hard Delay/Energy Constraints
- Hard Rate Requirements
Design Challenges

- Wireless channels are a difficult and capacity-limited broadcast communications medium.
- Traffic patterns, user locations, and network conditions are constantly changing.
- Applications are heterogeneous with hard constraints that must be met by the network.
- Energy, delay, and rate constraints change design principles across all layers of the protocol stack.
Signal Propagation

- **Path Loss**
  - Free space, 2-path,…
  - Simplified model
    \[ P_r = P_t K \left( \frac{d_0}{d} \right)^\gamma \], \( 2 \leq \gamma \leq 8 \)

- **Shadowing**
  - dB value is Gaussian
  - Find path loss exponent and shadow STD by curve fitting

- **Multipath**
  - Ray tracing
  - Statistical model
Outage Probability and Cell Coverage Area

- **Path loss**: circular cells
- **Path loss + shadowing**: amoeba cells
  - Tradeoff between coverage and interference
- **Outage probability**
  - Probability received power below given minimum
- **Cell coverage area**
  - % of cell locations at desired power
  - Increases as shadowing variance decreases
  - Large % indicates interference to other cells
Statistical Multipath Model

- Random # of multipath components, each with varying amplitude, phase, doppler, and delay
- Leads to time-varying channel impulse response

\[ c(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j\phi_n(t)} \delta(\tau - \tau_n(t)) \]

- Narrowband channel
  - No signal distortion, just a complex amplitude gain
  - Signal amplitude varies randomly (Rayleigh, Ricean, Nakagami).
  - 2\textsuperscript{nd} order statistics (Bessel function), Average fade duration
Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth

- Scattering function
  \[ s(\tau, \rho) = F_{\Delta t}[A_c(\tau, \Delta t)] \]
  - Yields delay spread/coherence BW \( (\sigma_\tau \sim 1/B_c) \)
  - Yields Doppler spread/coherence time \( (B_d \sim 1/T_c) \)
Capacity of Flat Fading Channels

- Channel Capacity
  - Maximum data rate that can be transmitted over a channel with arbitrarily small error

- Capacity of AWGN Channel: $\text{Blog}_2[1+\gamma]$ bps
  - $\gamma = \frac{P_r}{(N_0B)}$ is the receiver SNR

- Capacity of Flat-Fading Channels
  - Nothing known: capacity typically zero
  - Fading Statistics Known (few results)
  - Fading Known at RX (average capacity)

\[ C = \int_{0}^{\infty} B \log_2 (1 + \gamma) p(\gamma) d\gamma \leq B \log_2 (1 + \bar{\gamma}) \]
• **Capacity in Flat-Fading**: $\gamma$ known at TX/RX

$$C = \max_{P(\gamma) : E[P(\gamma)] = \bar{P}} \int_0^\infty B \log_2 \left(1 + \frac{\gamma P(\gamma)}{P}\right) p(\gamma) d\gamma$$

• **Optimal Rate and Power Adaptation**

$$\frac{P(\gamma)}{P} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \text{else} \end{cases}$$

$$\frac{C}{B} = \int_{\gamma_0}^\infty \log_2 \left(\frac{\gamma}{\gamma_0}\right) p(\gamma) d\gamma.$$  

• The instantaneous power/rate only depend on $p(\gamma)$ through $\gamma_0$
Channel Inversion

- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
    - Close to optimal
Frequency Selective Fading Channels

- For time-invariant channels, capacity achieved by water-filling in frequency
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
  - Each subband has width $B_c$
  - Independent fading in each subband
  - Capacity is the sum of subband capacities

$$\frac{1}{|H(f)|^2}$$
Linear Modulation in AWGN: MPSK and MQAM

- ML detection induces decision regions
  - Example: 8PSK

- $P_s$ depends on
  - # of nearest neighbors
  - Minimum distance $d_{\text{min}}$ (depends on $\gamma_s$)
  - Approximate expression

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$
Linear Modulation in Fading

- In fading $\gamma_s$ and therefore $P_s$ random
- Metrics: outage, average $P_s$, combined outage and average.

$$\overline{P_s} = \int P_s(\gamma_s) p(\gamma_s) d\gamma_s$$
Moment Generating Function Approach

- Simplifies average $P_s$ calculation
- Uses alternate Q function representation
- $\overline{P_s}$ reduces to MGF of $\gamma_s$ distribution
- Closed form or simple numerical calculation for general fading distributions
- Fading greatly increases average $P_s$. 
Doppler Effects

- High doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation
  - Increasing power does not reduce error
- Error floor depends on $f_D T_b$ as

$$P_{floor} = \frac{1 - J_0(2\pi f_D T_b)}{2} \approx 0.5(\pi f_D T_b)^2$$
Delay Spread (ISI) Effects

- Delay spread exceeding a symbol time causes ISI (self interference).

- ISI leads to irreducible error floor: $\overline{P}_{b,\text{floor}} \approx (\sigma_{T_m}/T_s)^2$
  - Increasing signal power increases ISI power

- ISI imposes data rate constraint: $T_s >> T_m \ (R_s << B_c)$

$$R \leq \log_2(M) \times \sqrt{\overline{P}_{b,\text{floor}}/\sigma_{T_m}^2}$$
Diversity

- Send bits over independent fading paths
  - Combine paths to mitigate fading effects.

- Independent fading paths
  - Space, time, frequency, polarization diversity.

- Combining techniques
  - Selection combining (SC)
  - Maximal ratio combining (MRC)

- Can have diversity at TX or RX
  - In TX diversity, weights constrained by TX power
Selection Combining

- Selects the path with the highest gain
- Combiner SNR is the maximum of the branch SNRs.
- CDF easy to obtain, pdf found by differentiating.
- Diminishing returns with number of antennas.
- Can get up to about 20 dB of gain.
MRC and its Performance

- With MRC, $\gamma_\Sigma = \Sigma \gamma_i$ for branch SNRs $\gamma_i$
  - Optimal technique to maximize output SNR
  - Yields 20-40 dB performance gains
  - Distribution of $\gamma_\Sigma$ hard to obtain

- Standard average BER calculation
  $$\overline{P}_s = \int \int \cdots \int P_s(\gamma_\Sigma) p(\gamma_\Sigma) d\gamma_\Sigma = \int \int \cdots \int P_s(\gamma_\Sigma) p(\gamma_1) \cdot p(\gamma_2) \cdots \cdot p(\gamma_M) d\gamma_1 d\gamma_2 \cdots d\gamma_M$$
  - Hard to obtain in closed form
  - Integral often diverges

- MGF Approach:
  $$\overline{P}_s = \frac{\alpha M}{\pi} \int_0^{\pi/2} \prod_{i=1}^M M_{\gamma_i} \left[ \frac{-0.5 \beta_M}{\sin^2 \phi} \right] d\phi.$$
  - TX diversity has same gains as RX diversity
Variable-Rate Variable-Power MQAM

Uncoded Data Bits → Delay → \( \gamma(t) \) → Point Selector → One of the M(\( \gamma \)) Points → \( \gamma(t) \) → To Channel

- \( \log_2 M(\gamma) \) Bits
- M(\( \gamma \))-QAM Modulator
- Power: \( S(\gamma) \)

\( M(\gamma) \)-QAM Points:
- BSPK
- 4-QAM
- 16-QAM

Goal: Optimize \( S(\gamma) \) and \( M(\gamma) \) to maximize \( EM(\gamma) \)
Optimal Adaptive Scheme

- **Power Water-Filling**
  \[
  \frac{S(\gamma)}{\bar{S}} = \begin{cases} 
  \frac{1}{\gamma_0} - \frac{1}{\gamma K} & \gamma \geq \frac{\gamma_0}{K} = \gamma_K \\
  0 & \text{else}
  \end{cases}
  \]

- **Spectral Efficiency**
  \[
  \frac{R}{B} = \int_{\gamma_k}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_K} \right) p(\gamma) d\gamma.
  \]

*Equals Shannon capacity with an effective power loss of K.*
Constellation Restriction

\[
M(\gamma) = \frac{\gamma}{\gamma K^*}
\]

\[
M_1 = \frac{\gamma_1}{K^*} = M_1
\]

**Power adaptation:**

\[
\frac{P_j(\gamma)}{P} = \begin{cases} 
(M_j - 1)/\gamma K & \gamma_j \leq \gamma < \gamma_{j+1}, j > 0 \\
0 & \gamma < \gamma_1
\end{cases}
\]

**Average rate:**

\[
\frac{R}{B} = \sum_{j=1}^{N} \log_2 M_j p(\gamma_j \leq \gamma < \gamma_{j+1})
\]

Performance loss of 1-2 dB
Practical Constraints

- Constant power restriction
  - Another 1-2 dB loss

- Constellation updates
  - Need constellation constant over $10-100T_s$
  - Use Markov model to obtain average fade region duration

- Estimation error and delay *(not on final)*
  - Lead to imperfect CSIT (assume perfect CSIR)
  - Causes mismatch between channel and rate
  - Leads to an irreducible error floor
MIMO systems have multiple \( M \) transmit and receiver antennas.

Decompose channel through transmit precoding \((x=V\tilde{x})\) and receiver shaping \((\tilde{y}=U^Hy)\)

\[ y = Hx + n \quad \text{H=U}\Sigma V^H \quad \tilde{y} = \Sigma \tilde{x} + \tilde{n} \quad \tilde{y}_i = \sigma_i \tilde{x} + \tilde{n}_i \]

Leads to \( R_H \leq \min(M_t, M_r) \) independent channels with gain \( \sigma_i \) (\( i^{th} \) singular value of \( H \)) and AWGN

Independent channels lead to simple capacity analysis and modulation/demodulation design
Capacity of MIMO Systems

- Depends on what is known at TX and RX and if channel is static or fading

- For static channel with perfect CSI at TX and RX, power water-filling over space is optimal:
  - In fading waterfill over space (based on short-term power constraint) or space-time (long-term constraint)

- Without transmitter channel knowledge, capacity metric is based on an outage probability
  - $P_{out}$ is the probability that the channel capacity given the channel realization is below the transmission rate.
Beamforming

- Scalar codes with transmit precoding

\[ y = u^H H v x + u^H n \]

- Transforms system into a SISO system with diversity.
  - Array and diversity gain
  - Greatly simplifies encoding and decoding.
  - Channel indicates the best direction to beamform
  - Need “sufficient” knowledge for optimality of beamforming

- Precoding transmits more than 1 and less than \( R_H \) streams
  - Transmits along some number of dominant singular values
Diversity vs. Multiplexing

- Use antennas for multiplexing or diversity

- Diversity/Multiplexing tradeoffs (Zheng/Tse)

\[
\lim_{SNR \to \infty} \frac{\log P_e(SNR)}{\log SNR} = -d
\]

\[
\lim_{SNR \to \infty} \frac{R(SNR)}{\log SNR} = r
\]

\[
d^*(r) = (M_t - r)(M_r - r)
\]
How should antennas be used?

- Use antennas for multiplexing:
  - High-Rate Quantizer → ST Code High Rate → Decoder
  - Error Prone

- Use antennas for diversity
  - Low-Rate Quantizer → ST Code High Diversity → Decoder
  - Low \( P_e \)

Depends on end-to-end metric: \textit{Solve by optimizing app. metric}
MIMO Receiver Design

- **Optimal Receiver:**
  - Maximum likelihood: finds input symbol most likely to have resulted in received vector
  - Exponentially complex # of streams and constellation size

- **Linear Receivers**
  - Zero-Forcing: forces off-diagonal elements to zero, enhances noise
  - Minimum Mean Square Error: Balances zero forcing against noise enhancement

- **Sphere Decoder:**
  - Only considers possibilities within a sphere of received symbol.
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

$$\hat{x} = \arg \min | y - Hx |^2$$

ML Decoding

Sphere Decoding

$$\hat{x} = \arg \min_{x:|y-Hx|<r} | y - Hx |^2$$
Other MIMO Design Issues

• Space-time coding (*not covered on final*):
  - Map symbols to both space and time via space-time block and convolutional codes.
  - For OFDM systems, codes are also mapped over frequency tones.

• Adaptive techniques:
  - Fast and accurate channel estimation
  - Adapt the use of transmit/receive antennas
  - Adapting modulation and coding (*not covered on final*)

• Limited feedback (*not covered on final*):
  - Partial CSI introduces interference in parallel decomp: can use interference cancellation at RX
  - TX codebook design for quantized channel
Multicarrier Modulation

- Divides bit stream into N substreams
- Modulates substream with bandwidth B/N
  - Separate subcarriers
  - $B/N < B_c$ → flat fading (no ISI)
- Requires N modulators and demodulators
  - Impractical: solved via OFDM implementation

![Diagram of Multicarrier Modulation]
Overlapping Substreams

- Can have completely separate subchannels
  - Required passband bandwidth is $B$.

- OFDM overlaps substreams
  - Substreams (symbol time $T_N$) separated in RX
  - Minimum substream separation is $B_N$.
  - Total required bandwidth is $B/2$ (for $T_N = 1/B_N$)

\[
\text{B/N}
\]

\[
f_0 \quad f_{N-1}
\]
FFT Implementation of OFDM

- Use IFFT at TX to modulate symbols on each subcarrier
- Cyclic prefix makes linear convolution of channel circular, so no interference between FFT blocks in RX processing
- Reverse structure (with FFT) at receiver

\[ R \text{ bps} \]

**TX**

- QAM Modulator
  - Serial To Parallel Converter
  - IFFT
  - Add cyclic prefix and Parallel To Serial Convert

\[ X_0 \]

\[ X_0 \]

\[ X_{N-1} \]

\[ X_{N-1} \]

\[ \cos(2\pi f_c t) \]

\[ h(t) \]

\[ n(t) \]

**RX**

- \( \cos(2\pi f_c t) \)
- LPF
- A/D
- Remove cyclic prefix and Serial to Parallel Convert

\[ X \]

\[ y_0 \]

\[ y_{N-1} \]

\[ Y_0 \]

\[ Y_{N-1} \]

\[ Y_i = H_i X_i + n_i \]

- FFT
- Parallel To Serial Convert
- QAM Modulator

\[ R \text{ bps} \]
OFDM Design Issues

- **Timing/frequency offset:**
  - Impacts subcarrier orthogonality; self-interference

- **Peak-to-Average Power Ratio (PAPR)**
  - Adding subcarrier signals creates large signal peaks
  - Solve with clipping or PAPR-optimized coding

- **Different fading across subcarriers**
  - Mitigate by precoding (fading inversion), adaptive modulation over frequency, and coding across subcarriers
MIMO-OFDM

- Send OFDM symbol along each spatial dimension
  - MIMO diversity-capacity benefits, OFDM removes ISI
  - Can adapt across time, space, and frequency

- OFDM can be represented by a matrix:
  - Represents DFT as a matrix: $y = \hat{H}x + v$, $\hat{H}$ circulant
  - Then vector $Y = \Lambda X + v_Q$ for $\Lambda$ an $N \times N$ diagonal matrix
  - Cyclic prefix added after DFT

- MIMO-OFDM matrix representation: $y = Hx + v$
  - Dimensions are $H$: $NM_r x (N+\mu)M_t$; $x$: $(N+\mu)M_t$; $y,v$: $M_r N$
  - Extends matrix representation of OFDM (example in HW)
Direct Sequence Spread Spectrum

- Bit sequence modulated by \textit{chip} sequence
- Spreads bandwidth by large factor (K)
- Despread by multiplying by $s_c(t)$ again ($s_c(t)=1$)
- Mitigates ISI and narrowband interference
  - ISI mitigation a function of code autocorrelation
- Must synchronize to incoming signal
ISI and Interference Rejection

- **Narrowband Interference Rejection \((1/K)\)**

  - Info. Signal
  - Receiver Input
  - Despread Signal

- **Multipath Rejection (Autocorrelation \(\rho(\tau)\))**

  - Info. Signal
  - Receiver Input
  - Despread Signal

- **Short codes repeat every \(T_s\), so poor multipath rejection at integer multiples of \(T_s\)**

- **Otherwise take a partial autocorrection**
Spreading Code Design

- Autocorrelation determines ISI rejection
  - Ideally equals delta function

- Would like similar properties as random codes
  - Balanced, small runs, shift invariant (PN codes)

- Maximal Linear Codes
  - No DC component
  - Max period \( (2^n-1)T_c \)
  - Linear autocorrelation
  - Recorrelates every period
  - Short code for acquisition, longer for transmission
  - In SS receiver, autocorrelation taken over \( T_s \)
  - Poor cross correlation (bad for MAC)
Synchronization

- Adjusts delay of $s_c(t-\tau)$ to hit peak value of autocorrelation.
  - Typically synchronize to LOS component
- Complicated by noise, interference, and MP
- Synchronization offset of $\Delta t$ leads to signal attenuation by $\rho(\Delta t)$
- Synchronize with long codes for better performance
RAKE Receiver

- **Multibranch receiver**
  - Branches synchronized to different MP components

- These components can be coherently combined
  - Use SC, MRC, or EGC
Multiuser Channels: Uplink and Downlink

Uplink (Multiple Access Channel or MAC): Many Transmitters to One Receiver.

Downlink (Broadcast Channel or BC): One Transmitter to Many Receivers.

Uplink and Downlink typically duplexed in time or frequency
Bandwidth Sharing

- Frequency Division
  - OFDMA

- Time Division

- Code Division
  - Code cross-correlation dictates interference
  - Multiuser Detection

- Space (MIMO Systems)

- Hybrid Schemes
Code Division via DSSS

- Interference between users mitigated by code cross correlation

\[
\hat{x}(t) = \int_0^{T_h} \alpha_1 s_1(t)s_{c1}(t)\cos^2(2\pi f_c t) + \alpha_2 s_2(t-\tau)s_{c2}(t-\tau)s_{c1}(t)\cos(2\pi f_c t)\cos(2\pi f_c (t-\tau)) dt
\]
\[
= .5\alpha_1 d_1 + .5\alpha_2 d_2 \int_0^{T_h} s_{c1}(t)s_{c2}(t) dt = .5d_1 + .5d_2 \cos(2\pi f_c \tau) \rho_{12}(\tau)
\]

- In downlink, signal and interference have same received power

- In uplink, “close” users drown out “far” users (near-far problem)
OFDMA and SDMA

- **OFDMA**
  - Implements FD via OFDM
  - Different subcarriers assigned to different users

- **SDMA (space-division multiple access)**
  - Different spatial dimensions assigned to different users
  - Implemented via multiuser beamforming (e.g. zero-force beamforming)
  - Benefits from multiuser diversity
Megathemes of EE359

- The wireless vision poses great technical challenges
- The wireless channel greatly impedes performance
  - Low fundamental capacity; Channel is randomly time-varying.
  - Flat fading and ISI must be compensated for.
- Compensate for flat fading with diversity or adaptive mod.
- MIMO provides diversity and/or multiplexing gain
- A plethora of ISI compensation techniques exist
  - Various tradeoffs in performance, complexity, and implementation.
  - OFDM and spread spectrum are the dominant techniques
  - OFDM works well with MIMO: basis for 4G Cellular/WiFi systems due to flexibility in adapting over time/space/frequency
- How best to share the limited spectrum among multiple users remains a major challenge in wireless system design