EE359 – Lecture 10 Outline

• **Announcements:**
  
  • Project proposals due Friday **midnight** (post, email link)
  
  • Midterm will be Nov. 10 6-8pm
    
    • No HW due that week.
    
    • Exam open book/notes, covers thru Chp. 7.
    
    • Midterm review date/time TBD. Brief in-class summary as well
    
    • SCPD students can take exam on campus or remotely
    
    • More MT announcements next week (practice MTs)

• **Average $P_s (P_b)$**

• **MGF approach for average $P_s$**

• **Combined average and outage $P_s$**

• **Effects of delay spread on error probability**
Review of Last Lecture

- Focus on linear modulation
- $P_s$ approximation in AWGN:
  - Nearest neighbor error dominates
- Probability of error in fading is random
  - Characterized by outage, average $P_s$, combination
- Outage probability

\[
P_s \approx \alpha M Q \left( \sqrt{\beta M \gamma_s} \right)
\]

- Probability $P_s$ is above target; Probability $\gamma_s$ below target
- Fading severely degrades performance
**Average \( P_s \)**

- Expected value of random variable \( P_s \)
- Used when \( T_c \sim T_s \)
- Error probability much higher than in AWGN alone
- Rarely obtain average error probability in closed form
  - Probability in AWGN is Q-function, double infinite integral

\[
\overline{P}_s = \int P_s(\gamma_s) p(\gamma_s) d\gamma_s
\]
Alternate Q Function Representation

- Traditional Q function representation
  \[ Q(z) = p(x > z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx, \quad x \sim N(0,1) \]
  - Infinite integrand, argument in integral limits
  - Average \( P_e \) entails infinite integral over \( Q(z) \)

- Craig’s representation:
  \[ Q(z) = \frac{1}{\pi} \int_{0}^{\pi/2} e^{-z^2/(\sin^2 \varphi)} \, d\varphi \]
  - Very useful in fading and diversity analysis

\[
\overline{P}_s = \frac{\alpha}{\pi} \int_{0}^{\pi} M_{\gamma_s} \left( \frac{-g}{\sin^2 x} \right) dx
\]
\( M_{\gamma_s} \text{ is MGF of fading distribution} \)
\( \gamma_s, g \text{ depends on modulation} \)
Combined outage and average $P_s$

- Used in combined shadowing and flat-fading
- $\overline{P}_s$ varies slowly, locally determined by flat fading
- Declare outage when $\overline{P}_s$ above target value
Delay Spread (ISI) Effects

- Delay spread exceeding a symbol time causes ISI (self interference).

- ISI leads to irreducible error floor: 
  \[ \overline{P}_{b, floor} \approx (\sigma T_m / T_s)^2 \]
  - Increasing signal power increases ISI power

- ISI imposes data rate constraint: 
  \[ T_s >> T_m \quad (R_s << B_c) \]
  \[ R \leq \log_2(M) \times \sqrt{\overline{P}_{b, floor} / \sigma^2_{T_m}} \]
Main Points

- Fading greatly increases average $P_s$ or required power for a given target $P_s$ with some outage
- Alternate Q function approach simplifies $P_s$ calculation, especially its average value in fading
  - Average $P_s$ becomes a Laplace transform.
- In fast/slow fading, outage due to shadowing, probability of error averaged over fast fading pdf
- Need to combat flat fading or waste lots of power
  - Adaptive modulation and diversity are main techniques to combat flat fading: adapt to fading or remove it
- Delay spread causes an irreducible error floor at high data rates