ISI Effects. Diversity.

Lecture Outline

- Delay Spread (ISI) Performance Effects
- Introduction to Diversity
- Selection Combining (SC) and its Performance
- Maximal Ratio Combining (MRC)
- Performance of MRC with i.i.d. Rayleigh fading

   - Delay spread exceeding a symbol time causes ISI (self-interference).
   - ISI leads to an irreducible error floor. Approximated as $P_{b,floor} \approx (\sigma T_m/T_s)^2$.
   - Without ISI compensation, avoid error floor by reducing data rate: $T_s >> T_m$ or $R \leq \log_2(M) \times \sqrt{P_{b,floor}/\sigma^2 T_m}$.

2. Introduction to Diversity
   - Basic concept is to send same information over independent fading paths.
   - Paths are combined to mitigate the effects of fading.

3. Realization of Independent Fading Paths
   - Space Diversity: Multiple antenna elements spaced apart by decorrelation distance.
   - Polarization Diversity: Two antennas, one horizontally polarized and one vertically polarized.
   - Frequency diversity: Multiple narrowband channels separated by channel coherence bandwidth.
   - Time diversity: Multiple timeslots separated by channel coherence time.

4. Array and Diversity Gain
   - Array gain is the gain in SNR from noise averaging over the multiple antennas. Gain in both AWGN and fading channels.
   - Diversity gain is the change in slope of the probability of error due to diversity. Only applies to fading channels.

5. Techniques for Combining Independent Fading Paths
   - Selection Combining: largest fading path chosen.
   - Maximal Ratio Combining: all paths cophased and summed with optimal weighting to maximize SNR at combiner output.
   - Equal Gain Combining: all paths cophased and summed with equal weighting.
We use space diversity as a reference for analysis; same analysis applies for any mechanism used to obtain independent fading paths.

6. **Selection Combining (SC) and its Performance**
   - Combiner SNR $\gamma_\Sigma$ is the maximum of the branch SNRs.
   - This gives diminishing returns, in terms of power gain, as the number of antennas increases.
   - CDF of $\gamma_\Sigma$ easy to obtain, then pdf found by differentiating.
   - Typically get 10-15 dB of gain for 2-3 antennas.

7. **Maximal Ratio Combining (MRC)**
   - Branch weights optimized to maximize output SNR of combiner.
   - Optimal weights are proportional to branch SNR.
   - Resulting combiner SNR $\gamma_\Sigma$ is sum of branch SNRs.
   - Distribution obtained by characteristic function analysis (can be hard).

8. **Performance of MRC with i.i.d. Rayleigh fading**
   - For $M$ branch diversity with i.i.d. Rayleigh fading on each branch, $\gamma_\Sigma$ is chi-squared with $2M$ degrees of freedom.
   - Can obtain $P_{out}$ and $P_s$ from this distribution.
   - For BPSK, get 15 dB gain at $10^{-3}$ BER. Larger gains obtained at lower BERs.

**Main Points**

- ISI leads to an irreducible error floor at high data rates - much work on ISI mitigation in current systems.
- Diversity is a powerful technique to overcome the effects of flat fading by combining multiple independent fading paths.
- Diversity typically entails some penalty in terms of rate, bandwidth, complexity, or size.
- Both selection combining and MRC significantly reduce the impact of fading.
- SC vs. MRC offer different levels of complexity vs. performance.
- Performance analysis of MRC greatly simplified using MGF approach.