Announcements:
- HW posted, due Friday
- MT exam grading done;
  - Can pick up from Julia or during TA discussion section tomorrow.
  - Distribution can be picked up at the same time.

Massive MIMO Capacity
MIMO Capacity in Fading
MIMO Beamforming
Diversity/Multiplexing Tradeoffs
MIMO Receiver Design
Review of Last Lecture

- Practical constraints in adaptive modulation
  - Constellation update rate
    \[ \bar{\tau}_j = \frac{\pi_j}{N_{j+1} + N_j} > T >> T_M \]
    For typical Dopplers, constant for 10s to 100s of symbol times

- Estimation error
  - Estimation error at TX can be caused by estimator and/or delay in the feedback path
  - Causes mismatch of adaptive power and rate to actual channel
  - Can lead to large errors
Review of Last Lecture (Ctd)

- MIMO systems have multiple TX and RX antennas; system model defined via matrices and vectors

- Channel decomposition: TX precoding, RX shaping

\[ y = Hx + n \]

\[ H = U\Sigma V^H \]

\[ \tilde{y} = \Sigma \tilde{x} + \tilde{n} \]

\[ \tilde{y}_i = \sigma_t \tilde{x} + \tilde{n}_i \]

- MIMO Channel Capacity
  - For static channel with perfect CSI at TX and RX, power water-filling over space is optimal
  - Without transmitter channel knowledge, capacity metric is based on an outage probability, \( P_{out} = p(C(H) < R) \)
Beamforming

- Scalar codes with transmit precoding

\[
y = u^H H v x + u^H n
\]

- Transforms system into a SISO system with diversity.
  - Array and diversity gain
  - Greatly simplifies encoding and decoding.
  - Channel indicates the best direction to beamform
  - Need “sufficient” knowledge for optimality of beamforming
Diversity vs. Multiplexing

- Use antennas for multiplexing or diversity

\[ d^*(r) = (M_t - r)(M_r - r) \]

- Diversity/Multiplexing tradeoffs (Zheng/Tse)

\[
\lim_{SNR \to \infty} \frac{\log P_e(SNR)}{\log SNR} = -d \\
\lim_{SNR \to \infty} \frac{R(SNR)}{\log SNR} = r
\]
How should antennas be used?

- Use antennas for multiplexing:
  - High-Rate Quantizer → ST Code High Rate → High Rate Decoder → Error Prone
  - Use antennas for diversity:
  - Low-Rate Quantizer → ST Code High Diversity → Low $P_e$

Depends on end-to-end metric: Solve by optimizing app. metric
MIMO Receiver Design

**Optimal Receiver:**
- Maximum likelihood: finds input symbol most likely to have resulted in received vector
- Exponentially complex # of streams and constellation size

**Linear Receivers**
- Zero-Forcing: forces off-diagonal elements to zero, enhances noise
- Minimum Mean Square Error: Balances zero forcing against noise enhancement

**Sphere Decoder:**
- Only considers possibilities within a sphere of received symbol.
  - If minimum distance symbol is within sphere, optimal, otherwise null is returned

\[
\hat{x} = \arg \min_x |y - Hx|^2
\]

\[
\hat{x} = \arg \min_{x:|y - Hx| < r} |y - Hx|^2
\]
Main Points

- **Capacity of MIMO systems**
  - Without TX CSI, at high SNR/large arrays, capacity increases linearly with the number of TX/RX antennas
  - With TX and RX channel knowledge, water-fill power over space or space-time to achieve capacity

- **Beamforming** transforms MIMO system into a SISO system with TX and RX diversity.
  - Beamform along direction of maximum singular value

- **MIMO** introduces diversity/multiplexing tradeoff
  - Optimal use of antennas depends on application

- **MIMO RX design** trades complexity for performance
  - ML detector optimal - exponentially complex
  - Linear receivers balance noise enhancement against stream interference