Shadowing, Combined Path Loss/Shadowing, Model Parameters from Data.

Lecture Outline

- Log Normal Shadowing
- Combined Path Loss and Shadowing
- Outage Probability
- Model Parameters from Empirical Data

1. Log-normal Shadowing:
   - Statistical model for variations in the received signal amplitude due to blockage.
   - The received signal power with the combined effect of path loss (power falloff model) and shadowing is, in dB, given by
     \[ P_r(dB) = P_t(dB) + 10 \log_{10} K - 10 \gamma \log_{10}(d/d_0) - \psi(dB). \]
   - Empirical measurements support the log-normal distribution for \( \psi \):
     \[ p(\psi_{dB}) = \frac{1}{\sqrt{2\pi}\sigma_{dB}} \exp \left[ -\frac{(\psi_{dB} - \mu_{\psi_{dB}})^2}{2\sigma_{dB}^2} \right]. \]
   - This empirical distribution can be justified by a CLT argument.
   - The autocorrelation based on measurements follows an autoregressive model:
     \[ A_{\psi}(\delta) = \sigma_{\psi dB}^2 e^{-\delta/X_c} = \sigma_{\psi dB}^2 e^{-\nu \tau/X_c}, \]
     where \( X_c \) is the decorrelation distance, which depends on the environment.

2. Combined Path Loss and Shadowing
   - Linear Model:
     \[ \frac{P_r}{P_t} = K \left( \frac{d}{d_0} \right)^\gamma \psi. \]
   - dB Model:
     \[ \frac{P_r}{P_t}(dB) = 10 \log_{10} K - 10 \gamma \log_{10}(d/d_0) - \psi_{dB}. \]
     - Average shadowing attenuation: when \( K_{dB} = 10 \log_{10} K \) captures average dB shadowing, \( \mu_{\psi_{dB}} = 0 \), otherwise \( \mu_{\psi_{dB}} > 0 \) since shadowing causes positive attenuation.

3. Outage Probability under Path Loss and Shadowing
   - With path loss and shadowing, the received power at any given distance between transmitter and receiver is random.
Leads to a non-circular coverage area around the transmitter, i.e. non-circular contours of constant power above which performance (e.g. in WiFi or cellular) is acceptable.

Outage probability $P_{\text{out}}(P_{\min}, d)$ is defined as the probability that the received power at a given distance $d$, $P_r(d)$, is below a target $P_{\min}$: $P_{\text{out}}(P_{\min}, d) = p(P_r(d) < P_{\min})$.

For the simplified path loss model and log normal shadowing this becomes

$$p(P_r(d) \leq P_{\min}) = 1 - Q\left(\frac{P_{\min} - (P_t + K_{\text{dB}} - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi\text{dB}}}\right).$$

4. Model Parameters from Empirical Data:

- Constant $K_{\text{dB}}$ typically obtained from measurement at distance $d_0$.
- Power falloff exponent $\gamma$ obtained by minimizing the MSE of the predicted model versus the data (assume $N$ samples):

$$F(\gamma) = \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2,$$

where $M_{\text{measured}}(d_i)$ is the $i$th path loss measurement at distance $d_i$ and $M_{\text{model}}(d_i) = K_{\text{dB}} - 10\gamma \log_{10}(d_i)$. The minimizing $\gamma$ is obtained by differentiating with respect to $\gamma$, setting this derivative to zero, and solving for $\gamma$.

- The resulting path loss model will include average attenuation, so $\mu_{\psi\text{dB}} = 0$.
- The shadowing variance $\sigma_{\psi\text{dB}}^2$ is obtained by determining the MSE of the data versus the empirical path loss model with the minimizing $\gamma = \gamma_0$:

$$\sigma_{\psi\text{dB}}^2 = \frac{1}{N} \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2,$$

where $M_{\text{model}}(d_i) = K_{\text{dB}} - 10\gamma_0 \log_{10}(d_i)$.

- Can also solve simultaneously for $(K_{\text{dB}}, \gamma)$ via a least squares fit of both parameters to the data. Using the line equation for each data point $y_i$ that $y_i = mx_i + K_{\text{dB}}$ for $m = -10\gamma$ and $x_i = \log_{10}(d_i)$, the error of the straight line fit is

$$F(K, \gamma) = \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - (mx_i + K_{\text{dB}})]^2.$$

Main Points

- Shadowing decorrelates over its decorrelation distance, which is on the order of the size of shadowing objects.
- Combined path loss and shadowing leads to outage and non-circular coverage area (cells).
- Path loss and shadowing parameters are obtained from empirical measurements through a least-squares fit.
- Can find path loss exponent $\gamma$ by a 1-dimensional least-squares-error line fit assuming a fixed value of $K_{\text{dB}}$ from one far-field measurement (most common), or find path loss exponent $\gamma$ and $K_{\text{dB}}$ parameters simultaneously through a 2-dimensional least-squares-error line fit.